A NEGATIVE ANSWER TO THREE QUESTIONS ON $K$-PRIMITIVE RINGS

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ABSTRACT. It is shown that three questions on $K$-primitive rings posed by Kezlan in [3] have a negative answer.

We shall use the terminology of [3, 4]. Kezlan has noticed [3] that every strongly $K$-primitive ring is a right Ore domain. In fact the following characterization of strongly $K$-primitive rings is true.

PROPOSITION 1. A ring $R$ is (right) strongly $K$-primitive if and only if $R$ is a (right) Ore domain and $R$ is (right) bounded.

Let us notice that Proposition 1 corresponds to the second part of Question 3 of [3] and the counterexample given there.

If $\sigma$ is an automorphism of a field $D$ then $D[[t, \sigma]]$ denotes a $\sigma$-twisted power series algebra and $D((t, \sigma))$ denotes a twisted Laurent series algebra (see e.g. [2]).

THEOREM 2. Let $\sigma$ be an automorphism of infinite order of a field $D$. Then $D[[t, \sigma]]$ is a left and right strongly $K$-primitive ring which together with the center of its quotient ring $Q$ does not generate $Q$.

PROOF. Since every one-sided ideal of a domain $D[[t, \sigma]]$ is a two-sided ideal, $D[[t, \sigma]]$ is left and right strongly $K$-primitive. It is well known and easy to see that $D((t, \sigma))$ is a left and right quotient ring of $D[[t, \sigma]]$ and its center $F$ is the fixed subfield of $\sigma$ acting on $D$. Hence $FD[[t, \sigma]] = D[[t, \sigma]] \neq D((t, \sigma))$.

The above theorem gives us a negative answer to Question 2 of [3].

Let $J$ denote the Jacobson radical. We shall need the following

THEOREM 3 [1]. Let $k$ be a field and $R$ a $J$-radical $k$-algebra contained in a skew field $K$. Then $K$ can be embedded in a skew field $L$ which contains a simple $J$-radical $k$-subalgebra containing $R$.

The next theorem gives the negative answer to the first part of Question 3 and hence also to Question 4 posed by Kezlan.

THEOREM 4. For any field $k$ there exists a $k$-algebra which is a right and left Ore domain and is not $K$-primitive.
Proof. Let $K_0 = k((t))$, $R_0 = k[[t]]t$. Notice that $R_0$ is a right and left Ore domain. Since $R_0$ is $J$-radical then, by Theorem 3, there exists a skew field $K_1$ containing $K_0$ and simple $J$-radical $k$-algebra $R_1$ containing $R_0$. Let $K_2 = K_1((t))$, $R_2 = R_1 + tK_1[[t]]$. Then $R_2$ is a $J$-radical right and left Ore domain. Continuing in this way we get an ascending sequence $(K_i)_{i=0}^{\infty}$ of division algebras and an ascending sequence $(R_i)_{i=0}^{\infty}$ of their $J$-radical subalgebras such that $R_i$ is a right and left Ore domain for even $i$ and $R_i$ is a simple ring for odd $i$. Then $\bigcup R_i \subseteq \bigcup K_i$ is a right and left Ore domain which is a simple $J$-radical ring. From [4] it follows that $\bigcup R_i$ is not a $K$-primitive ring.

References


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