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TRIVIALITY OF SIMPLE FIBER-PRESERVING ACTIONS OF TORI ON HILBERT-CUBE-FIBER BUNDLES

VO THANH LIEM[1]

ABSTRACT. Let σ denote the standard based-free action of a torus T on the Hilbert cube Q. It will be shown that every simple fiber-preserving action of T on $Q \times B$, where B is a finite-dimensional, compact metric absolute retract, is fiber-preserving equivalent to the diagonal action $\sigma \times \mathrm{id}_B$.

1. Introduction and definitions. Let Q denote the Hilbert cube $I_1 \times I_2 \times \cdots$, and let G be a topological group. An action α: $G \times M \to M$ of G on a topological space M is said to be *semifree* if, for each $x \in M$, the isotropy group of x is either G of $\{e\}$. For a finite or compact Lie group G, it is known that the metric cone of G, cone(G), is a compact absolute retract (AR). The left translation on G defines a natural semifree action of G on cone(G) that induces a semifree action, the diagonal action of G on $\prod_{n=1}^{\infty} \mathrm{cone}(G) \approx Q$ [**We**]. This action has a unique fixed point. Following [**B-We**], we call it the *standard based-free action* σ of G on Q whose orbit space is denoted by Q/σ. An action λ of G on $Q \times B$ is said to be fiber-preserving (f.p.) if, for each $g \in G$, we have $p_B \circ \lambda(g, \cdot) = p_B(\cdot)$, where p_B: $Q \times B \to B$ is the projection. Two actions α and β: $G \times M \to M$ are said to be *equivalent* if there is a homeomorphism f: $M \to M$ such that

$$f(\alpha(g, x)) = \beta(g, f(x))$$

for all $x \in M$ and $g \in G$. A f.p. action λ on $Q \times B$ is said to be *simple* if the restriction of λ to each fiber $Q \times \{b\}$ is equivalent to σ and has fixed point set $\{(0, b)\}$.

In [**L**], the following result has been proved: "*If B is a finite-dimensional compact metric AR, then every simple f.p. action of a finite group G on $Q \times B$ is equivalent to the diagonal action $\sigma \times \mathrm{id}_B$*".

The purpose of this paper is to establish a similar result for simple f.p. actions of tori on $Q \times B$, where B is a finite-dimensional, compact metric AR, as stated in the abstract or Theorem 6 below.

In this paper, we set $I = [0, 1]$, $Q_0 = Q - \{0\}$. For basic notions and results in Q-manifold theory, we refer to [$\mathbf{Ch_1}$]. A map p: $X \to B$ is a Hurewicz fibration if p has the homotopy covering extension property (HCEP) with respect to every

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topological space. The following is a special case of the main result in [**Ch$_2$**] that we will use

LEMMA 0. *Let p: $X \to B$ be a Hurewicz fibration, where B is a f.d. compact AR and the fibers are Q-manifolds, then the composition $p \circ p_X$: $X \times [0,1) \to X \to B$ is a locally trivial fiber bundle.*

Given α: $G \times Q \times B \to Q \times B$ a simple f.p. action of G over B, we will define E to be its orbit space and E_0 its regular orbit space $E \setminus (\{0\} \times B)$ (we identify the fixed point set $\{0\} \times B$ with its image in the orbit space). Let p: $E \to B$ denote the natural map induced from the projection map p_B: $Q \times B \to B$. The *fiberwise-reduced cone* $C(E, p)$ of E over B is defined to be the quotient $(E \times I)/\mathfrak{D}$, where $\mathfrak{D}$ is the decomposition of $E \times I$ whose nondegenerate elements are all closed subsets of the form $(p^{-1}(b) \times \{1\}) \cup (b \times I)$ for $b \in B$. Let d_1 denote a metric on $C(E, p)$ which extends a metric, say d_1, in E. Note that, besides being endowed with the metric d_1, the open subset $E_0 \times [0,1)$ of $C(E, p)$ also inherits the product metric d_2 of $d_1 \mid E_0$ and the usual metric on $[0,1)$. Given a $\delta > 0$, a δ-subset of a metric space is a subset of diameter less than δ.

OBSERVATIONS. Let K be a compact subset of E_0 and let ε be a positive number. Then, there is a $\delta > 0$ such that every δ-subset of $(E_0 \times [0,1), d_2)$ intersecting K is an ε-subset of $(E_0 \times [0,1), d_1)$. Consequently, if ρ: $K \times I \to C(E, p)$ is a δ-homotopy (i.e. $\operatorname{diam}_2 \rho(x \times I) < \delta$) in $(E_0 \times [0,1), d_2)$ with $\rho(x, 0) = x$ for all x in K, then $\operatorname{diam}_1 \rho(x \times I) < \varepsilon$ in $(E_0 \times [0,1), d_1)$ for all x in K. In particular, for the projection p_E: $C(E, p) \to E$ and a δ-subset A of $(E_0 \times [0,1), d_2)$ intersecting K, then $\operatorname{diam}_1 p_E(A) < \varepsilon$ in (E, d_1). Similar properties also hold true when d_1 and d_2 are interchanged.

Given an open cover β of a space Y, a homomotopy H: $X \times I \to Y$ is said to be *β-homotopy* if for each $x \in X$, the track $H(x \times I)$ is contained in a member of β; a map f: $X \to Y$ is said to be a *β-equivalence* if there is a map g: $Y \to X$ such that $f \circ g$ is β-homotopic to id_Y and $g \circ f$ is $f^{-1}(\beta)$-homotopic to id_X, where $f^{-1}(\beta)$ denotes the open cover $\{f^{-1}(U) \mid U \in \beta\}$ of X.

2. Results and details of proofs. In the proof of the following proposition, we refer to p. 266 of [**Do**] for the definition of section extension property (SEP), numerable open covering, etc.

PROPOSITION 1. *If λ is a simple f.p. action of a finite group or a compact Lie group G on $Q \times B$, then the natural map p: $E_0 \to B$ is a Hurewicz fibration.*

PROOF. Since E_0 and B are metric spaces, we only have to show that p has the HCEP for the class of all metric spaces [**Du**, Corollary 2.3, p. 396]. Assume that X is a metric space. Given a commutative diagram

$$\begin{array}{ccc} X \times \{0\} & \xrightarrow{h} & E_0 \\ \downarrow & & \downarrow p \\ X \times I & \xrightarrow{\phi} & B \end{array}$$

we will show that ϕ has a lifting Φ: $X \times I \to E_0$ with $\Phi(x, 0) = h(x)$ for all x in X. Equivalently, if we define

(1) $R = \{(x, w) \in X \times E_0^I \mid w(0) = h(x) \text{ and } pw(t) = \phi(x, t)\}$, and

(2) q: $R \to X$ by $q(x, w) = x$,

then we will show that there is a numerable open cover $\{V_\lambda \mid \lambda \in \Lambda\}$ of X such that the restriction $q_\lambda = q \mid q^{-1}(V_\lambda)$: $q^{-1}(V_\lambda) \to V_\lambda$ has SEP; hence, q has a cross-section by Theorem 2.7 of **[Do]**.

Let $\{U_\lambda \mid \lambda \in \Lambda\}$ be a covering of E_0 consisting of contractible open subsets. Let $V_\lambda = h^{-1}(U_\lambda)$, $\lambda \in \Lambda$, then $\{V_\lambda \mid \lambda \in \Lambda\}$ is numerable since X is a metric space. Let π: $Q_0 \times B \to E_0$ denote the orbit map. Then, $\pi \mid \pi^{-1}(U_\lambda)$: $\pi^{-1}(U_\lambda) \to U_\lambda$ is a trivial fiber bundle with fiber G **[Br**, Theorem 5.8, p. 88]; hence, there is a lifting $\tilde{h}_\lambda$: $V_\lambda \times \{0\} \to \pi^{-1}(U_\lambda) \subset Q_0 \times B$ of $h_\lambda = h \mid (V_\lambda \times \{0\})$. Observe that the pair $(\phi_\lambda, \tilde{h}_\lambda)$, where $\phi_\lambda = \phi \mid (V_\lambda \times I)$, has the HCEP; consequently, so does the pair $(\phi_\lambda, h_\lambda)$. Therefore, the map q_λ: $q^{-1}(V_\lambda) \to V_\lambda$ has SEP by Lemma 4.5 of **[Do]**. So, the proof of the proposition is complete.

For the sake of simplicity, let F_b denote $p^{-1}(b)$, X_b its reduced cone at $(0, b)$ and $F_{b,0}$ the regular orbit space $F_b \cap E_0$ over b for each $b \in B$; let Y_b denote $(Q/\sigma) \times \{b\}$. Let d denote the product metric on $Q/\sigma \times B$. Given $b, c \in B$, let $\varepsilon_{b,c}$: $Y_b \to Y_c$ be the natural homeomorphism defined by $\varepsilon_{b,c}(x, b) = (x, c)$, then $d(\varepsilon_{b,c}(x, b), (x, b)) = d_B(b, c)$ for each $(x, b) \in Y_b$.

From the above proposition and Lemma 0, it follows that the composition $p \circ p_{E_0}$: $E_0 \times [0, 1) \to B$ is a trivial fiber bundle whose fiber is homeomorphic to Q_0/σ **[B-We]**. Therefore, $E_0 \times [0, 1)$ is f.p. homeomorphic to $(Q_0/\sigma) \times B$ over B. Consequently, its fiberwise-one-point compactification $C(E, p)$ is f.p. homeomorphic to $(Q/\sigma) \times B$ over B. Let h denote such a homeomorphism in the following Lemmas 2–5.

LEMMA 2. *Given an $\varepsilon > 0$, there is a $\delta > 0$ such that if $d_B(b, c) < \delta$, then $d_1(h^{-1}\varepsilon_{b,c}h(x), x) < \varepsilon$ for all $x \in X_b$ and $d_1(h^{-1}\varepsilon_{c,b}h(z), z) < \varepsilon$ for all $z \in X_c$.*

PROOF. From the uniform continuity of h^{-1}, there is a $\delta > 0$ ($\delta < \varepsilon$) such that $d(h^{-1}(y), h^{-1}(y')) < \varepsilon$ for every pair of $y, y' \in (Q/\sigma) \times B$ with $d(y, y') < \delta$. Now, if $d_B(b, c) < \delta$, then for each $x \in X_b$ we have $d(\varepsilon_{b,c}h(x), h(x)) = d_B(b, c) < \delta$; hence, $d_1(h^{-1}\varepsilon_{b,c}h(x), x) = d_1(h^{-1}\varepsilon_{b,c}h(x), h^{-1}h(x)) < \varepsilon$ as we desired. The proof of the second inequality is the same.

LEMMA 3. *Given an $\varepsilon > 0$, there is a δ ($0 < \delta < \varepsilon$) such that if $d_B(b, c) < \delta$ and if A is a δ-subset of (X_b, d_1), then $h^{-1}\varepsilon_{b,c}h(A)$ is an ε-subset of $(C(E, p), d_1)$.*

PROOF. From Lemma 1, choose a $\delta < \varepsilon/3$ such that if $d_B(b, c) < \delta$, then $d_1(h^{-1}\varepsilon_{b,c}h(x), x) < \varepsilon/3$ for each $x \in X_b$. Then, for $x, y \in X_b$ with $d_1(x, y) < \delta$, we have

$$\begin{aligned} d_1\big(h^{-1}\varepsilon_{b,c}h(x), h^{-1}\varepsilon_{b,c}h(y)\big) &\leq d_1\big(h^{-1}\varepsilon_{b,c}h(x), x\big) + d_1(x, y) \\ &\quad + d_1\big(y, h^{-1}\varepsilon_{b,c}h(y)\big) \\ &< \varepsilon. \end{aligned}$$

LEMMA 4. *Fix a point $b \in B$ and a compact subset K of $F_b - \{b\}$. Given an $\varepsilon > 0$, then there is a $\delta > 0$ such that if $d_B(b, c) < \delta$, there are maps f: $F_c \to F_b$ and g: $F_b \to F_c$ having the following properties*:

(a) $d_1(f(x), x) < \varepsilon$ *for all* $x \in f^{-1}(K)$, *and* $d_1(g(y), y) < \varepsilon$ *for all* $y \in K$.

(b) $g \circ f | f^{-1}(K)$ *is* ε*-homotopic to the inclusion* $f^{-1}(K) \subset F_c$.

(c) $f \circ g | K$ *is* ε*-homotopic to the inclusion* $K \subset F_b$.

SUBLEMMA. *Let M be a compact subset of E_0. Given an $\varepsilon > 0$, there is an $\varepsilon_1 < \varepsilon$ such that if $d_B(b, c) < \varepsilon_1$ and if A is an ε_1-subset of (X_c, d_1) intersecting M, then $p_E h^{-1} \varepsilon_{c,b} h(A)$ is an ε-subset of F_c.*

PROOF. Let η be a positive number such that the closure P of the η-neighborhood $N_\eta(M)$ of M is a compact subset of E_0. Choose μ $(0 < \mu < \eta)$ such that $\operatorname{diam}_1 p_E(R) < \varepsilon$ for each μ-subset R of $(P \times [0, \mu], d_1)$. Then, choose ε_1 $(0 < \varepsilon_1 < \mu/2)$ such that if $d_B(b, c) < \varepsilon_1$, then

(*) $\operatorname{diam}(h^{-1}\varepsilon_{c,b}h(A)) < \mu/2$ with respect to d_1 and d_2 for every ε_1-subset A of (X_c, d_1) intersecting M (see Lemma 3 and Observations), and

(**) $d_1(h^{-1}\varepsilon_{c,b}h(x), x) < \mu/2$ and $d_2(h^{-1}\varepsilon_{c,b}h(x), x) < \mu/2$ for every $x \in F_c \cap M$ (see Lemma 2 and Observations).

Now, if A is an ε_1-subset of (X_c, d_1) intersecting M, then $h^{-1}\varepsilon_{c,b}h(A)$ intersects $N_{\mu/2}(M) \times [0, \mu/2]$ by (**). Combining this fact and (*), we observe that the set $R = h^{-1}\varepsilon_{c,b}h(A)$ is a μ-subset of $(N_\mu(M) \times [0, \mu], d_1) \subset (P \times [0, \mu], d_1)$. Therefore, $\operatorname{diam}_1(p_E h^{-1}\varepsilon_{c,b}h(A)) < \varepsilon$ by the choice of μ.

Let us return to the proof of Lemma 4. Given an $\varepsilon > 0$, we will choose inductively the positive numbers $\varepsilon > \varepsilon_1 > \delta_2 > \delta_1 > \delta_E > \delta$ as follows: We can assume that ε is so small that $N_{2\varepsilon}(K) \cap N_{2\varepsilon}(B) = \varnothing$, and let M be the closure of $N_{2\varepsilon}(K)$.

(1) Choose ε_1, from the Sublemma, such that if H: $Z \times I \to (E_0 \times [0, 1), d_1)$ is an ε_1-homotopy with $H(z \times I) \cap M \neq \varnothing$ and $H(z \times I) \subset X_b$ for all $z \in Z$, then $(p_E h^{-1}\varepsilon_{b,c}h) \circ H$ is an ε-homotopy in (E, d_1) when $d_B(b, c) < \varepsilon_1$.

(2) Choose δ_2 such that every δ_2-homotopy F: $Z \times I \to (E_0 \times [0, 1), d_2)$ with $F(z \times I) \cap M \neq \varnothing$ for all $z \in Z$ is an ε_1-homotopy in $(E_0 \times [0, 1), d_1)$ (from Observations).

(3) Choose δ_1 such that every δ_1-subset of $(E_0 \times [0, 1), d_1)$ intersecting M is a δ_2-subset of $(E_0 \times [0, 1), d_2)$ (from Observations).

(4) Choose δ_E such that if A is a δ_E-subset of $(E_0 \times [0, 1), d_1)$ intersecting M, then $p_E(A)$ is a $(\delta_1/2)$-subset of (E, d_1).

(5) Choose δ such that if $d_B(b, c) < \delta$, then $d_1(h^{-1}\varepsilon_{c,b}h(x), x) < \delta_E$ for all $x \in X_c$ and $d_1(h^{-1}\varepsilon_{b,c}h(y), y) < \delta_E$ for all $y \in X_b$ (from Lemma 2).

With this choice of δ, we now define $f = p_E h^{-1}\varepsilon_{c,b}h$ and $g = p_E h^{-1}\varepsilon_{b,c}h$ and show that f and g have all required properties if $d_B(b, c) < \delta$.

(a) For $x \in f^{-1}(K) \subset F_c \subset X_c$, then $d_1(h^{-1}\varepsilon_{c,b}h(x), x) < \delta_E$ by (5); hence, $d_1(p_E h^{-1}\varepsilon_{c,b}h(x), p_E(x)) < \delta_1/2$ by (4); consequently, $d_1(f(x), x) < \delta_1/2 < \varepsilon$. Similarly, $d_1(g(y), y) < \varepsilon$ for $y \in K \subset F_b$.

(b) Observe that if $x \in f^{-1}(K)$, then

$$\begin{aligned} d_1(p_E h^{-1}\varepsilon_{c,b}h(x), h^{-1}\varepsilon_{c,b}h(x)) &\leq d_1(p_E h^{-1}\varepsilon_{c,b}h(x), x) + d_1(x, h^{-1}\varepsilon_{c,b}h(x)) \\ &= d_1(f(x), x) + d_1(x, h^{-1}\varepsilon_{c,b}h(x)) \\ &< \delta_1/2 + \delta_E < \delta_1 \end{aligned}$$

(from the proof of (a) and by (5)). Hence, $d_2(f(x), h^{-1}\varepsilon_{c,b}h(x)) < \delta_2$ by (3), since $f(x) \in M$. Therefore, f is δ_2-homotopic along $[0, 1)$ to $h^{-1}\varepsilon_{c,b}h$ in $(F_{b,0} \times [0, 1), d_2) \subset X_b$, say H. Consequently, H is an ε-homotopy in (X_b, d_1) by (2). Finally, $(p_E h^{-1}\varepsilon_{b,c}h) \circ H$ is an ε-homotopy by (1) from $g \circ f$ to

$$p_E h^{-1}\varepsilon_{b,c}hh^{-1}\varepsilon_{c,b}h = p_E|f^{-1}(K) = \text{the inclusion } f^{-1}(K) \subset F_c.$$

(c) The proof of (c) is the same as that of (b).

The proof of Lemma 4 is now complete.

LEMMA 5. *Fix an element b of B. Given an open cover β of F_b and an open subset U of F_b whose closure K is contained in $F_{b,0}$, then there is a $\delta > 0$ such that, if $c \in B$ with $d_B(b, c) < \delta$, then there is a map f: $F_c \to F_b$ which is a β-equivalence over U.*

PROOF. Let λ be a positive number such that every λ-subset of F_b intersecting the compactum K is contained in a member of β meeting K. Let δ be a positive number from Lemma 4 corresponding to $\varepsilon = \lambda/4$; and let f and g be two maps satisfying (a), (b) and (c) in Lemma 4.

Now, it is clear that $f \circ g|K$ is β-homotopic to the inclusion $K \subset F_{b,0}$ by (c). Moreover, there is a $(\lambda/4)$-homotopy H from $g \circ f|f^{-1}(K)$ to the inclusion $f^{-1}(K) \subset F_{c,0}$ by (b); then $f \circ H$ is a $(3\lambda/4)$-homotopy by (a), and it is a β-homotopy since each tract intersects K. Therefore, f is a β-equivalence over U as we desired.

THEOREM 6. *Let B be a finite-dimensional compact metric AR. If λ is a simple f.p. action of the torus T^n on $Q \times B$, then* (1) *the map p: $E \to B$ is a locally trivial fiber bundle, and* (2) *λ is f.p. equivalent to the diagonal action $\sigma \times \mathrm{id}_K$.*

PROOF. The proof will be similar to that of Theorem 3.2 in [**L**].

(1) Since the homeomorphism group $\mathrm{Homeo}(Q/\sigma)$ is locally contractible [**L**, Theorem 2.2], we will only have to show that the map p is completely regular; i.e. given an element $b \in B$ and an $\varepsilon > 0$, we will find a $\delta > 0$ such that, for each $c \in B$ with $d_B(b, c) < \delta$, there is a homeomorphism ϕ: $F_c \to F_b$ such that $d_1(\phi(x), x) < \varepsilon$ for each $x \in F_c$.

Following [**B-We**], we will use the following notation in this proof. Let L_n denote the orbit space $(T * T * \cdots * T)/\alpha$ of the induced diagonal action α (from the left translation of T on itself) on the join of n copies of T. Then, L_n is naturally embedded in L_{n+1}, and let M_n denote the mapping cylinder

$$\mathrm{Map}(L_1 \to L_2 \to \cdots \to L_n).$$

Define $M_\infty = \bigcup_1^\infty M_n$. It is proved in [**B-We**] that F_b is homeomorphic to the one-point compactification of $M_\infty \times Q$.

Given an $\varepsilon > 0$, there is an integer m such that $F_b - (M_m \times Q)$ is contained in $N_{\varepsilon/2}(0, b)$, the $(\varepsilon/2)$-neighborhood of $(0, b)$. Let ε' $(0 < \varepsilon' < \varepsilon/2)$ be chosen such that $M_{m+2} \times Q$ misses $N_{\varepsilon'}(0, b)$.

Let β be an open cover of F_b as in Theorem 3.6 in [F] such that every β-equivalence f: $N \to F_{b,0}$ (N is a Q-manifold) over the interior of $M_{m+2} \times Q$ is $(\varepsilon'/2)$-homotopic to a map $\bar{f}$: $N \to F_{b,0}$ having the following properties:

(a) $\bar{f} | \bar{f}^{-1}(M_{m+1} \times Q)$ is an open embedding, and

(b) $\bar{f}(x) = f(x)$ for all $x \in N - f^{-1}(M_{m+2} \times Q)$.

Now, choose δ $(0 < \delta < \varepsilon'/2)$ as in Lemma 5 corresponding to the open cover β and $U = \operatorname{int} M_{m+2} \times Q$. Let b and c be in B with $d_B(b, c) < \delta$, and let f: $F_c \to F_b$ be a β-equivalence given by Lemma 5, and let $\bar{f}$ be as in the above paragraph. Observe that $\bar{f}^{-1}(L_{m+1/2} \times Q)$ is bicollared in $F_{c,0}$ and it is contained in $N_\varepsilon(0, c)$. Then, as in the proof of Theorem 3.2 in [L], since $F_{c,0}$ and $F_{b,0}$ are of the homotopy type of the Eilenberg-Mac Lane space $K(Z^n, 2)$, we can extend $\bar{f} | \bar{f}^{-1}(M_{m+1/2} \times Q)$ to a homeomorphism ϕ: $F_{c,0} \to F_{b,0}$, and finally from F_c onto F_b. Finally, it can be verified that $d(\phi(x), x) < \varepsilon$ for all $x \in F_c$. Therefore, the proof of (1) is complete.

(2) By (1), we now identify E_0 with $(Q_0/\sigma) \times B$. Since the principal T-bundle p_1: $Q_0 \times B \to E_0 = (Q_0/\sigma) \times B$ is universal and B is contractible the action is classified by the projection map $E_0 = Q_0/\sigma \times B \to Q_0/\sigma \times \{b_0\}$ for any $b_0 \in B$ and is therefore equivalent to $\sigma \times \mathrm{id}_B$ by a fiber-preserving, equivariant homeomorphism g: $Q_0 \times B \to Q_0 \times B$, which must extend to a homeomorphism of $Q \times B$.

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Department of Mathematics, University of Alabama, University, Alabama 35486