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CORRECTION TO "ZERO-ONE LAWS FOR STABLE MEASURES"

R. M. DUDLEY[1] AND MAREK KANTER

ABSTRACT. A law on a vector space (e.g. the plane) with all one-dimensional marginals stable (of index less than 1) need not be stable, by examples of David Marcus. An error in the authors' proof to the contrary is noted.

In our paper [**3**], Theorems 4 and 5 assert that for certain vector spaces S and F in duality, if μ is a probability law on S such that for every $f \in F$, the one-dimensional marginal $\mu \circ f^{-1}$ is stable, then μ is stable.

The proof shows that all the $\mu \circ f^{-1}$ are stable with the same index γ where $0 < \gamma \leqslant 2$. The next step in the proof is incorrect, but can be repaired for $\gamma > 1$ by letting $k(f) = \int f\, d\mu$, which is linear in f, with $f - k(f)$ having a strictly stable law.

For $\gamma < 1$ the assertion is false. David Marcus [**6**] has given a counterexample, with $S = F = \mathbf{R}^2$. For $\gamma = 1$ the question appears to be open.

Other results in [**3**], including the zero-one laws themselves, did not depend on Theorems 4 and 5 and are unaffected.

Giné and Hahn [**4**] show that μ is stable if in addition all its two-dimensional marginals are infinitely divisible.

We take this opportunity to make some other small corrections and remarks on [**3**]. In [**3**, (2)] the stable laws are nondegenerate. The question posed after Theorem 1 remains open as far as we know. For continuity of a universally measurable real linear form on a complete metric linear space, perhaps not locally convex, see [**2**, Theorem 2], or in the Borel measurable case [**1**, Chapter 1, Theorem 4, p. 23]. On p. 250, line 4 of [**3**], $\mu * (\mu \circ m_r^{-1}) = \cdots$.

Hilbert first proved "Waring's theorem" (cf. [**5**, pp. 297–298, 315]).

In the last paragraph of the proof of Theorem 7 [**3**], replace $r_1, \ldots, r_k$ by $r_0, \ldots, r_k$, $1 \leqslant j \leqslant k$ by $0 \leqslant j \leqslant k$, etc.

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DEPARTMENT OF MATHEMATICS, 2-245, MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASSACHUSETTS 02139

2134 GRANT AVE., BERKELEY, CALIFORNIA 94704