CORRECTIONS AND ADDITIONS TO
“A GENERALIZATION OF A THEOREM OF AYOUB AND CHOWLA”

DON REDMOND

Let \( \chi_1 \) and \( \chi_2 \) be characters modulo \( q_1 \) and \( q_2 \), respectively, and let

\[
f(n) = \sum_{d|n} \chi_1(d) \chi_2(n/d).
\]

In [3], I estimated the sum

\[
\sum_{n \leq x} f(n) \log(x/n).
\]

Unfortunately, as was pointed out to me in a letter by A. Ivic of Beograd, the error term claimed in [3] is incorrect. The error lies in the estimate (3.14) and the best that one can claim is

\[
\sum_{n \leq x} f(n) \log(x/n) = C_1(\chi_1, \chi_2) x \log x + C_2(\chi_1, \chi_2) x + C_3(\chi_1, \chi_2) \log x
\]

\[
+ C_4(\chi_1, \chi_2) + O(x^{-1/4}),
\]

as \( x \to +\infty \), where the constants \( C_j(\chi_1, \chi_2) \), \( 1 \leq j \leq 4 \), are as stated in [3]. This error term is the same as obtained in [1 and 2], however, we still have achieved a uniform calculation of the constants \( C_j(\chi_1, \chi_2) \), \( 1 \leq j \leq 4 \).

Let \( k \geq 2 \) be a positive integer. Then another generalization is to consider \( k \) characters \( \chi_j \) of modulus \( q_j \), \( 1 \leq j \leq k \), and let

\[
f_k(n) = \sum_{d_1\cdots d_k=n} \chi_1(d_1) \cdots \chi_k(d_k).
\]

Then, in the same way as above, I obtain

\[
\sum_{n \leq x} f_k(n) \log^{k-1}(x/n) = xP_{1,k-1}(\log x) + P_{2,k-1}(\log x) + O(x^{-(k-1)/2k}),
\]

where \( P_{1,k-1}(u) \) and \( P_{2,k-1}(u) \) are polynomials of degree \( k - 1 \), which arise is the calculation of the residues. Indeed, if

\[
F_k(s) = \sum_{n=1}^{+\infty} f_k(n)n^{-s},
\]
then
\[ xP_{1,k-1}(\log x) = \text{res}(x^sF_k(s)s^{-k}, s = 1) \]
and
\[ P_{2,k-1}(\log x) = \text{res}(x^sF_k(s)s^{-k}, s = 0). \]

The actual calculation would be carried out in the same manner as in §§3 and 4 of [3].

The case \( q_1 = \cdots = q_2 = 1 \), that is, when \( f_k(n) = d_k(n) \), the \( k \)-fold divisor function, was obtained by A. Ivic and mentioned in the letter referred to above. It was his result that suggested this generalization to me.

**References**


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