SHORTER NOTES

The purpose of this department is to publish very short papers of unusually elegant and polished character, for which there is no other outlet.

INVARINACE OF COMPLEMENTARY DOMAINS
OF A FIXED POINT SET
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Introduction. The following useful result seems not to be in the literature. It has a simple but perhaps nonobvious proof.

Proposition. Let $f$ be a homeomorphism of a connected topological manifold $M$ with fixed point set $F$. Then either (1) $f$ is invariant on each (connected) component of $M - F$ or (2) there are exactly two components and $f$ interchanges them.

Proof. Suppose $f$ is not invariant on a component $U$ of $M - F$. Let $g$ be the involution on $M$ defined by

$$
g(x) = \begin{cases} f(x), & x \in U, \\ f^{-1}(x), & x \in f(U), \\ x, & x \in M - (U \cup f(U)). \end{cases}$$

It is an elementary exercise to verify that $g$ is continuous. By a well-known theorem of M. H. A. Newman [3] (cf. also [1, p. 157]) $g$ cannot be the identity on a nonempty open set. Hence $U$ and $f(U)$ are the only components of $M - F$ and $f(f(U)) = U$, thus proving the Proposition.

In the case of (2) the above argument shows that $F$ cannot contain an open set, hence $\dim F \leq (\dim M) - 1$, and since $F$ separates $M$ we have $\dim F = (\dim M) - 1$. In a refinement of Smith Theory, G. Bredon [2] has shown that if $M$ is also orientable then any involution with an odd codimensional fixed point set must reverse the orientation; hence we obtain

Corollary. Let $f$ be an orientation-preserving homeomorphism of an orientable manifold $M$; then $f$ is invariant on each component of $M - F$.

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REFERENCES


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