

MAPPINGS OF THE SIERPIŃSKI CURVE ONTO ITSELF

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ABSTRACT. Given two points p and q of the Sierpiński universal plane curve S , necessary and/or sufficient conditions are discussed in the paper under which there is a mapping f of S onto itself such that $f(p) = q$ and f belongs to one of the following: homeomorphisms, local homeomorphisms, local homeomorphisms in the large sense, open, simple or monotone mappings.

1. Introduction. The paper is devoted to the problem of homogeneity of the Sierpiński universal plane curve from one point to another with respect to various classes of continuous mappings. The Krasinkiewicz result for homeomorphisms [4] is extended to local homeomorphisms and also the problem is completely solved for local homeomorphisms in the large sense. It is also shown that the Sierpiński curve is homogeneous with respect to simple mappings and with respect to monotone ones. Furthermore, the Whyburn result [10] on an extension of a homeomorphism between boundaries of two complementary domains to one between the whole Sierpiński curves is generalized to open mappings. Some unresolved problems are posed in the final part of the paper.

Thanks are due to W. J. Charatonik and J. Nikiel for fruitful discussions on the topic of this paper.

2. Preliminaries. All mappings considered in the paper are assumed to be continuous. A *curve* means a one-dimensional metric continuum. By the *standard Sierpiński curve* we mean the well-known geometric realization of a plane locally connected curve (see e.g. [6, §51, I, Example 5, p. 275 and Figure 8, p. 276]) which is located in the unit square I^2 with opposite vertices $(0, 0)$ and $(1, 1)$, and which is known to be universal in the class of all plane curves (see e.g. [1, Theorem 12.11, p. 433]). Any homeomorphic image of this continuum is called the *Sierpiński curve*, and is denoted by S . The union of all boundaries of complementary domains of S in the plane is called the *rational part* of S and is denoted by R . The remaining part, $S \setminus R$, of the curve is called its *irrational part* (cf. [3, p. 188; 4, p. 255]).

Let a class M of mappings be given. A space X is said to be *homogeneous with respect to M from a point $p \in X$ to a point $q \in X$* provided there exists a mapping f of X onto itself such that $f \in M$ and $f(p) = q$. This is a generalization of a concept of a space being homogeneous between points p and q (see [4, p. 255]). A space X is said to be *homogeneous with respect to M* provided that it is homogeneous with respect to M from p to q for each pair of points $p, q \in X$. This generalizes the concept of a *homogeneous space* (i.e. homogeneous with respect to homeomorphisms).

Received by the editors August 8, 1983.

1980 *Mathematics Subject Classification.* Primary 54C10; Secondary 54F20, 54F50.

Key words and phrases. Universal plane curve, rational part, local homeomorphism, open, simple, monotone mappings.

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0002-9939/84 \$1.00 + \$.25 per page

The Sierpiński curve is not homogeneous [9, p. 137]. A detailed study of this fact is given in [4], where the following result is proved [4, p. 255].

THEOREM A (KRASINKIEWICZ). *The Sierpiński curve is homogeneous from one point to another if and only if both these points belong either to the rational part or to the irrational part of the Sierpiński curve.*

The Sierpiński curve has been characterized by Whyburn [10, Corollary, p. 323] as an *S-curve*, i.e., a plane locally connected curve such that the boundary of each complementary domain of the curve is a simple closed curve and no two of these complementary domain boundaries intersect [10, p. 321]. From the proof of a theorem stating that any two *S-curves* are homeomorphic [10, 3, pp. 322, 323], the following result can be extracted (cf. [4, p. 255]):

THEOREM B (WHYBURN). *If K_1, K_2 are *S-curves* and C_1, C_2 are boundaries of unbounded components of complements of K_1, K_2 in the plane, respectively, then each homeomorphism of C_1 onto C_2 can be extended to a homeomorphism of K_1 onto K_2 .*

In the present paper we prove results similar to Theorems A and B for some other classes of mappings which are wider than the class of homeomorphisms.

A mapping $f: X \rightarrow Y$ is called *open* if images under f of open subsets of X are open in Y . It is said to be a *local homeomorphism in the large sense* if every point x in X has an open neighborhood U such that the partial mapping $f|U: U \rightarrow f(U)$ is a homeomorphism (see [7, p. 51]). If, moreover, $f(U)$ is an open subset of Y , then f is said to be a *local homeomorphism* [11, p. 199]. Thus a mapping f is a local homeomorphism if and only if it is a local homeomorphism in the large sense and simultaneously an open mapping [7, Theorem 1, p. 54]. Further, it is known that a mapping $f: X \rightarrow Y$ of X onto a continuum Y is a local homeomorphism if and only if f is open and there is a natural number n such that $\text{card } f^{-1}(y) = n$ for each $y \in Y$ [8, Proposition, p. 64]. If this last condition holds, we say that f is of *degree n* (cf. [11, p. 199]) or, equivalently, that it is an *n -to-one mapping*. A mapping f for which the inequality $\text{card } f^{-1}(y) \leq 2$ holds for each $y \in Y$ is called *simple* (see [2, p. 84, 3, p. 186]).

A mapping is called *monotone* if its point-inverses are connected (for equivalent conditions see [11, p. 127 and (2.2), p. 137; 6, §46, II, Theorem 9, p. 131]).

A class M of mappings is said to be *admissible* if it contains all homeomorphisms and if for each mapping in M , its composite with a homeomorphism is also in M . Many well-known classes of mappings are admissible, e.g. open, monotone, simple, local homeomorphisms, etc. As an example of a class of mappings that is not admissible, one can take the class of ε -mappings (i.e. having all point-inverses of diameter less than ε).

The following statement is obvious.

STATEMENT 1. Let a continuum X be the union of two sets A and B such that if both p, q are in A or if both p, q are in B , then X is homogeneous (with respect to homeomorphisms) from p to q . Let a class M of mappings of X onto itself be admissible. If there are points $p_0 \in A$ and $q_0 \in B$ and a mapping $g \in M$ such that $g(p_0) = q_0$, then X is homogeneous with respect to M from each point of A to each point of B .

The next statement is a consequence of Statement 1 and Theorem A.

STATEMENT 2. Let a class M of mappings of S onto itself be admissible. Let $p_0 \in R$ and $q_0 \in S \setminus R$ (or $p_0 \in S \setminus R$ and $q_0 \in R$). If there is a mapping $g \in M$ with $g(p_0) = q_0$, then S is homogeneous with respect to M from each point p of R to each point q of $S \setminus R$ (or from each point p of $S \setminus R$ to each point q of R , respectively).

In fact, by Theorem A there are homeomorphisms h_1 and h_2 of S onto S with $h_1(p) = p_0$ and $h_2(q_0) = q$. Thus h_2fh_1 is the required mapping.

A point p is said to be *accessible from a set Z* provided there are $z \in Z$ and an arc zp from z to p lying in $Z \cup \{p\}$ (see [11, p. 111]).

The following lemma is evident by Theorem A.

LEMMA 1. *A point p of the Sierpiński curve S belongs to the rational part of S if and only if p is accessible from the complement of S in the plane.*

As a consequence of Lemma 1 we get

LEMMA 2. *If a Sierpiński curve T lies in a Sierpiński curve S and if a point p is in the irrational part of T , then p is in the irrational part of S .*

Now we prove

LEMMA 3. *A point p of the Sierpiński curve S belongs to the irrational part of S if and only if for each positive number ε there exists an open connected ε -neighborhood of p whose boundary is a simple closed curve.*

PROOF. If $p \in R$, then p belongs to a simple closed curve C being the boundary of a complementary domain of S in the plane. Thus the boundary of no open connected neighborhood of p having diameter less than $\text{diam } C$ is a simple closed curve.

If $p \in S \setminus R$, consider S as the standard Sierpiński curve in the unit square. Then for each natural number n the curve S is the union of 8^n copies of S located in 8^n squares (of the n th step of the construction of S), whose boundaries are contained in S . Then p belongs either to one, two, or (at most) three copies of S . Choose n large enough such that the union U of (at most three) copies of S to which p belongs has diameter less than ε . Then the interior of U is an open connected ε -neighborhood of p whose boundary is a simple closed curve.

3. Local homeomorphism. We start with the following

PROPOSITION 1. *Each local homeomorphism in the large sense of S onto S maps the irrational part $S \setminus R$ of S into itself.*

PROOF. Let a local homeomorphism in the large sense, $f: S \rightarrow S$, map a point $p \in S \setminus R$ to a point $q \in S$. Let U be an open connected neighborhood of p such that \bar{U} is homeomorphic to S and $f|_{\bar{U}}: \bar{U} \rightarrow f(\bar{U}) \subset S$ is a homeomorphism. Since $p \in S \setminus R$ and U is a neighborhood of p , we conclude that p is in the irrational part of the Sierpiński curve \bar{U} . Thus by Theorem A the point q , as the image of p under the homeomorphism $f|_{\bar{U}}$, is in the irrational part of the Sierpiński curve $f(\bar{U}) \subset S$. Applying Lemma 2 we conclude that $q \in S \setminus R$.

COROLLARY 1. *The Sierpiński curve S is not homogeneous with respect to local homeomorphisms from any point of the irrational part of S to any point of the rational part of S .*

PROPOSITION 2. *Each local homeomorphism of S onto S maps the rational part R of S into itself.*

PROOF. Let a local homeomorphism $f: S \rightarrow S$ map a point $p \in R$ to a point $q \in S$, and let U be an open ε -neighborhood of p such that $f|U: U \rightarrow f(U)$ is a homeomorphism and $f(U)$ is an open neighborhood of q . If q were in $S \setminus R$, then by Lemma 3 an open connected neighborhood $V \subset \bar{V} \subset f(U)$ would exist, having a simple closed curve C as its boundary. Therefore $(f|U)^{-1}(V) = U \cap f^{-1}(V)$ would be an open connected ε -neighborhood of p with the simple closed curve $(f|U)^{-1}(C)$ as its boundary, a contradiction to the assumption $p \in R$ according to Lemma 3.

As a consequence of Theorem A, Corollary 1 and Proposition 2 we have

THEOREM 1. *The Sierpiński curve is homogeneous with respect to local homeomorphisms from one point to another if and only if both these points belong either to the rational part or to the irrational part of the Sierpiński curve.*

Observe that openness of the mapping is essential in Proposition 2, i.e., we cannot replace a local homeomorphism by a local homeomorphism in the large sense in that proposition. Moreover, we have

PROPOSITION 3. *For each pair of points p, q of S with $p \in R$ and $q \in S \setminus R$ there is a simple local homeomorphism in the large sense, $f: S \rightarrow S$, such that $f(p) = q$.*

PROOF. Note that the class of all simple local homeomorphisms in the large sense of S onto itself is admissible. Thus Statement 2 can be applied. We do this as follows. In the standard Sierpiński curve S identify the points $(0, y)$ and $(1, y)$, where $y \in [0, 1]$, of two opposite sides E_0 and E_1 of the unit square. Let g' be the identification mapping. Thus g' is a simple local homeomorphism in the large sense. The resulting space $g'(S)$ is homeomorphic to the Sierpiński curve by the Whyburn characterization theorem [10, Theorem 4, p. 323]). Let h' be a homeomorphism from $g'(S)$ onto S and put $g = h'g'$. Thus $g: S \rightarrow S$ maps all points $(0, y)$ and $(1, y)$ of $E_0 \cup E_1$ with $0 < y < 1$ (belonging to R) into points $g(0, y) = g(1, y)$ which lie in the irrational part of S (note that $g|S \setminus (E_0 \cup E_1)$ is a homeomorphism, so $g(0, y)$ and $g(1, y)$ do not belong to the boundary of a complementary domain of S in the plane). Hence we can put $p_0 = (0, \frac{1}{2}) \in R$ and $q_0 = g(p_0) \in S \setminus R$ (see Statement 2), and the proof is complete.

Propositions 1 and 3 and Theorem A imply

THEOREM 2. *The Sierpiński curve S is homogeneous with respect to either simple or arbitrary local homeomorphisms in the large sense from a point p to a point q if and only if either p is in the rational part of S and q is arbitrary or both p and q are in the irrational part of S .*

There are some curves having the property that each local homeomorphism on them is a homeomorphism (see [11, Corollary, p. 199; 8, Theorem, p. 64 and Corollary, p. 67]). This is not the case for the Sierpiński curve.

PROPOSITION 4. *For each natural number n there exists a local homeomorphism of degree n from the Sierpiński curve onto itself.*

PROOF. In the euclidean plane equipped with the polar coordinate system (ρ, φ) let us consider the annulus A bounded by the circles $\rho = \frac{1}{2}$ and $\rho = 1$. Divide A into n congruent closed sets A_0, A_1, \dots, A_{n-1} with n straight line segments L_0, L_1, \dots, L_{n-1} defined by $L_k = \{(\rho, \varphi_k) : \frac{1}{2} \leq \rho \leq 1\}$, where $\varphi_k = 2k\pi/n$ for $k \in \{0, 1, \dots, n-1\}$. Thus, $A_k = \{(\rho, \varphi) : \frac{1}{2} \leq \rho \leq 1 \text{ and } \varphi_k \leq \varphi \leq \varphi_{k+1}\}$ (we assume $\varphi_n = \varphi_0$) for $k \in \{0, 1, \dots, n-1\}$. Next locate in every A_k a homeomorphic copy S_k of the standard Sierpiński curve in such a way that the consecutive sides of the unit square are mapped onto the arcs $\{(\frac{1}{2}, \varphi) : \varphi_k \leq \varphi \leq \varphi_{k+1}\}$, $L_k, \{(1, \varphi) : \varphi_k \leq \varphi \leq \varphi_{k+1}\}$ and L_{k+1} , respectively, and any two copies S_k are congruent by a rotation. Note that $S' = \bigcup\{S_k : k \in \{0, 1, \dots, n-1\}\}$ is homeomorphic to S by the Whyburn characterization theorem [10, Theorem 4, p. 323]). It is easy to verify that the mapping $f_n : S' \rightarrow A$ defined by $f_n(\rho, \varphi) = (\rho, n\varphi)$ is a local homeomorphism of degree n , and that the resulting space $f_n(S') \subset A$ is again homeomorphic to S . The proof is complete.

Recall that two mappings $f_1 : X_1 \rightarrow Y_1$ and $f_2 : X_2 \rightarrow Y_2$ are *topologically equivalent* (see [11, footnote, p. 127]) provided there exist homeomorphisms $h' : X_1 \rightarrow X_2$ and $h'' : Y_2 \rightarrow Y_1$ such that $f_1 = h''f_2h'$. For example, if $f_n : S' \rightarrow f_n(S') \subset A$ is the mapping defined at the end of the proof of Proposition 4, then for each n the partial mapping $f_n|_{S_0} : S_0 \rightarrow f_n(S_0) = f_n(S') \subset A$ is equivalent to the mapping $f : S \rightarrow S$ defined in the proof of Proposition 3.

The next theorem is related to Theorem B above and the Whyburn extension theorem in [11, (3.3), p. 215].

THEOREM 3. *If K_1, K_2 are S -curves and C_1, C_2 are simple closed curves in the rational parts of K_1 (resp. K_2), then each open mapping of C_1 onto C_2 can be extended to a local homeomorphism of K_1 onto K_2 .*

PROOF. First, note that for $i \in \{1, 2\}$ there is a homeomorphism $h_i : K_i \rightarrow h_i(K_i)$ such that $h_i(K_i)$ lies in the plane, $h_i|_{C_i} : C_i \rightarrow C_i$ is the identity and $h_i(C_i) = C_i$ is the boundary of the unbounded complementary domain of $h_i(K_i)$ in the plane. A construction of such homeomorphisms h_1, h_2 has been shown in [4, p. 256 (case (i))]. Second, note that each open mapping from one simple closed curve onto another is topologically equivalent to the mapping $z \rightarrow z^n$ (for some fixed natural n) on the unit circle $|z| = 1$, where z is a complex number (see [11, Theorem 1.1, p. 182]). Observe that the unit circle $C_0 = \{(1, \varphi) : 0 \leq \varphi \leq 2\pi\} \subset A$ (we apply here the notation of the proof of Proposition 4) is the boundary of the unbounded complementary domain of S' and—simultaneously—of $f_n(S')$, and that the mapping $z \rightarrow z^n$ is nothing else but $f_n|_{C_0} : C_0 \rightarrow C_0$. Therefore if $g : C_1 \rightarrow C_2$ is a given open mapping of $C_1 = h_1(C_1)$ onto $C_2 = h_2(C_2)$, then there are homeomorphisms $h' : C_1 \rightarrow C_0$ and $h'' : C_0 \rightarrow C_2$ such that $g = h''(f_n|_{C_0})h'$. Applying now Theorem B to two pairs of S -curves: first $h_1(K_1)$ and $S' \subset A$, and second $f_n(S') \subset A$ and $h_2(K_2)$, and to two pairs of the boundaries of the unbounded components of their complements in the plane: first $h_1(C_1) \subset h_1(K_1)$ and $C_0 \subset S'$, and second $C_0 \subset f_n(S')$ and $h_2(C_2) \subset h_2(K_2)$, we extend the homeomorphisms h' and h'' to homeomorphisms $h'_* : h_1(K_1) \rightarrow S'$ and $h''_* : f_n(S') \rightarrow h_2(K_2)$ respectively. It

can be easily verified that the mapping $f = h_2 h'_* f_n h'_* h_1$ is a local homeomorphism of K_1 onto K_2 such that $f|_{C_1} = g$.

4. Simple and monotone mappings. It is known that each locally connected continuum is homogeneous for the class of all continuous mappings (see e.g. [5, Theorem 1, p. 347]). When applied to the Sierpiński curve, this result can be generalized in two different ways: for the classes of simple and of monotone mappings of S onto itself. We begin with simple mappings.

PROPOSITION 5. *For each pair of points p, q of S with $p \in S \setminus R$ and $q \in R$ there is a simple open mapping f of S onto itself such that $f(p) = q$ and $f(R) \subset R$.*

PROOF. The class of all simple open mappings of S onto S is admissible.

Thus, to apply Statement 2, we have to find points $p_0 \in S \setminus R$ and $q_0 \in R$ and a simple open surjective mapping $g: S \rightarrow S$ with $g(p_0) = q_0$ and $g(R) \subset R$. To this end, let D be the diagonal of the unit square I^2 that joins the opposite vertices $(0, 0)$ and $(1, 1)$ of the square, and let S denote the standard Sierpiński curve. Further, let g' be an identification mapping on S which identifies the points (x, y) and (y, x) . In other words, we may consider $g': S \rightarrow g'(S) \subset S$ as the mapping defined by

$$g'(x, y) = \begin{cases} (x, y) & \text{if } x \geq y, \\ (y, x) & \text{if } x < y. \end{cases}$$

So g' is the identity on the lower right half of S and the symmetry with respect to the diagonal D on the upper left half of S . Note that g' is simple and open. Further, $g'(S) \subset S$ is homeomorphic to S by the Whyburn characterization theorem [10, Theorem 4, p. 323]). Let h' be a homeomorphism from $g'(S)$ onto S and put $g = h'g'$. Thus $g: S \rightarrow S$ is simple, open and onto. Moreover, by its definition the mapping g' maps R into the rational part of $g'(S)$, whence it follows from Theorem A that $g(R) \subset R$. Denote by K the union of all simple closed curves $C \subset R$ such that $C \cap D \neq \emptyset$. Observe $g'(\overline{K})$ is a simple closed curve, namely the boundary of the unbounded component of $g'(S)$ in the plane. Thus, according to Theorem A, we have $g(\overline{K}) = h'g'(\overline{K}) \subset R$. Finally, note that the set $(D \cap S) \setminus K \subset S \setminus R$ is nonempty (it is uncountable even), and choose a point $p_0 \in (D \cap S) \setminus K$. Therefore $p_0 \in D \cap \overline{K}$, so we have $g'(p_0) = p_0$ (since p_0 lies in D) and $g(p_0) \in R$ (since $p_0 \in \overline{K}$ and $g(\overline{K}) \subset R$). Thus putting $q_0 = g(p_0)$ we complete the proof.

Proposition 5 and Theorem A imply

COROLLARY 2. *If either p is in the irrational part of S and q is arbitrary, or both p and q are in the rational part of S , then S is homogeneous with respect to simple open mappings from p to q .*

PROBLEM 1. Is the condition mentioned in Corollary 2 not only sufficient but also necessary for homogeneity of S with respect to simple open mappings from p to q ?

Corollary 2 and Proposition 3 imply

THEOREM 4. *The Sierpiński curve is homogeneous with respect to simple mappings.*

Note that we cannot replace the Sierpiński curve by an arbitrary locally connected continuum in the above result: there is no simple mapping of a simple triod

carrying its center to an endpoint. On the other hand an arc, or—more generally—an n -dimensional cube (where $n = 1, 2, \dots, \aleph_0$), is homogeneous with respect to simple mappings. Thus the following problem seems to be natural.

PROBLEM 2. Characterize locally connected continua which are homogeneous with respect to simple mappings.

Now we come to consider monotone mappings.

PROPOSITION 6. *For each $\varepsilon > 0$ there exists a mapping f of S onto S such that:*

- (1) *all point-inverses of f are singletons except one which is the boundary C of a complementary domain of S in the plane (thus f is monotone);*
- (2) *$f(C)$ is a singleton in the irrational part of S (thus f maps the set $C \subset R$ into a point in $S \setminus R$);*
- (3) *for each open set $U \subset S$ the interior of $f(U)$ is nonempty;*
- (4) *f is an ε -mapping.*

PROOF. Given $\varepsilon > 0$, choose a simple closed curve $C \subset R$ of diameter less than ε and consider a decomposition of S into the continuum C and single points of $S \setminus C$. This decomposition is upper semicontinuous and induces a natural mapping $g: S \rightarrow g(S)$ from S onto the decomposition space $g(S)$. It is evident that this space is homeomorphic to S . Let $h: g(S) \rightarrow S$ be a homeomorphism. Put $f = hg$ and note that (1) and (4) hold by the definition. Since the only nondegenerate point-inverse is a closed boundary set, we have (3). Finally $f|S \setminus C$ is a homeomorphism from $S \setminus C$ onto $S \setminus f(C)$, where $f(C)$ is a singleton. Thus $f(C)$ does not belong to the boundary of a complementary domain of S in the plane, whence (2) follows. The proof is complete.

PROPOSITION 7. *For each $\varepsilon > 0$ there exists a mapping f of S onto S such that:*

- (1) *all point-inverses of f are singletons except one which is an arc L (thus f is monotone);*
- (2) *the arc L lies entirely in $S \setminus R$ except one of its endpoints that is in R (thus f maps the other endpoint of L from $S \setminus R$ to a point in R);*
- (3) *for each open set $U \subset S$ the interior of $f(U)$ is nonempty;*
- (4) *f is an ε -mapping.*

PROOF. The proof is similar to that of Proposition 6. Namely given $\varepsilon \in (0, 1/3)$ let S be the standard Sierpiński curve and $L = [1/3 - \varepsilon/2, 1/3] \times \{1/3\}$. One can easily verify that the natural projection f of S onto the Sierpiński curve S/L satisfies all the required conditions. The proof is complete.

In the sequel we need conditions (1) and (2) only of Propositions 6 and 7. Note that the class of monotone mappings of S onto itself is admissible. Therefore Statement 2 and Proposition 6 imply that S is homogeneous with respect to monotone mappings from each point of R to each point of $S \setminus R$, and similarly Statement 2 and Proposition 7 give the result in the opposite direction. Thus by Theorem A we obtain

THEOREM 5. *The Sierpiński curve is homogeneous with respect to monotone mappings.*

5. Final remarks. Let us note that a very important class of mappings, namely the open ones, is not separately discussed in the paper. The reader can find below two questions concerning this class of mappings. The questions indicate some directions of further study in this domain. The first of them is a more general version of Problem 1 and is also related to Proposition 2.

PROBLEM 3. Is it true that for an arbitrary open mapping $f: S \rightarrow S$ of S onto itself we have $f(R) \subset R$?

The next question is not related directly to the Sierpiński curve. However, it is connected with homogeneity with respect to a class of mappings, a concept discussed extensively in this paper.

PROBLEM 4. Does there exist a locally connected plane continuum which is homogeneous with respect to open mappings and which is different from a simple closed curve?

REFERENCES

1. L. M. Blumenthal and K. Menger, *Studies in geometry*, Freeman, San Francisco, Calif., 1970.
2. K. Borsuk and R. Molski, *On a class of continuous mappings*, Fund. Math. **45** (1958), 84–98.
3. B. Knaster and A. Lelek, *Coutures et tapis*, Fund. Math. **45** (1958), 186–199.
4. J. Krasinkiewicz, *On homeomorphisms of the Sierpiński curve*, Comment. Math. Prace Mat. **12** (1969), 255–257.
5. P. Krupski, *Continua which are homogeneous with respect to continuity*, Houston J. Math. **5** (1979), 345–356.
6. K. Kuratowski, *Topology*, Vol. II, Academic Press, New York; PWN, Warsaw, 1968.
7. A. Lelek, *Sur l'unicohérence, les homéomorphies locales et les continus irréductibles*, Fund. Math. **45** (1958), 51–63.
8. T. Maćkowiak, *Local homeomorphisms onto tree-like continua*, Colloq. Math. **38** (1977), 63–68.
9. S. Mazurkiewicz, *Sur les continus homogènes*, Fund. Math. **5** (1924), 137–146.
10. G. T. Whyburn, *Topological characterization of the Sierpiński curve*, Fund. Math. **45** (1958), 320–324.
11. ———, *Analytic topology*, Amer. Math. Soc. Colloq. Publ., vol. 28, Amer. Math. Soc., Providence, R.I., 1963.

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