PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 97, Number 3, July 1986

ESSENTIAL MAPS EXIST FROM BU TO coker J

MARK FESHBACH[1]

ABSTRACT. We show that $[\mathrm{BU}, \operatorname{coker} J] \neq 0$ but that there are no infinite loop maps from BU to coker J. The proofs involve the Segal conjecture.

It is known that $[\operatorname{coker} J, \mathrm{BU}] = 0$ [**HS**]. A conjecture dating from the early 70's is that [BU, coker J] $= 0$ also. This conjecture is attributed by some people to Sullivan. It seems quite reasonable since BU and coker J should have little to do with each other. The purpose of this note is to show that despite this intuition, [BU, coker J] $\neq 0$. Indeed it is uncountable. However, there are no infinite loop maps from BU to coker J. So the conjecture is true on the level of infinite loop maps. Both these statements follow from the Segal conjecture, in particular from the compact Lie group version of the Segal conjecture proven by this author. We give a proof which does not use the main theorem in [**F**], the compact Lie group version; the full power of this generalization is not needed. Although this result is interesting in its own right, what is particularly striking is how inaccessible such a calculation seemed just a few years ago. The author wishes to thank Ib Madsen for bringing this conjecture to his attention and for his encouragement. Conversations with S. Priddy were also helpful.

LEMMA 1. $[\mathrm{BU}, Q_0S^0]$ *is torsion free. Hence* $[\mathrm{BU}, \operatorname{Im} J]$ *is torsion free.*

PROOF. The second statement follows from the first, since $Q_0S^0 \sim \operatorname{Im} J \times \operatorname{coker} J$ [**MM**, p. 110].

We show that $\pi_s^0(\mathrm{BU})$ is detected on finite subgroups; the result follows by the Segal conjecture. $[\mathrm{BU}, Q_0S^0] = \pi_s^0\mathrm{BU}$, the reduced stable cohomotopy in dimension 0 of BU. Since BU equals $\varinjlim \mathrm{BU}(n)$, there is a Milnor short exact sequence

$$0 \to \lim{}^1 \pi_s^{-1}\mathrm{BU}(n) \to \pi_s^0\mathrm{BU} \to \varprojlim \pi_s^0\mathrm{BU}(n) \to 1.$$

$\pi_s^{-1}\mathrm{BU}(n)$ is compact for all n by the Atiyah-Hirzebruch spectral sequence for finite skeleta. Hence, $\pi_s^0\mathrm{BU} \approx \varprojlim \pi_s^0\mathrm{BU}(n)$. $\pi_s^0\mathrm{BU}(n)$ is detected on finite subgroups of $\mathrm{U}(n)$ by Corollary 2.14 [**F**]. Since the stable cohomotopy in dimension 0 of a finite group is torsion free by the Segal conjecture for finite groups [**C**], it follows that $\pi_s^0\mathrm{BU}$ is torsion free.

LEMMA 2. $[\mathrm{BU}, \operatorname{Im} J] = 0$. *Hence* $[\mathrm{BU}, \operatorname{coker} J] = [\mathrm{BU}, Q_0S^0]$.

Received by the editors February 28, 1985.
1980 *Mathematics Subject Classification*. Primary 55N20; Secondary 55P42, 55Q55.
Key words and phrases. Infinite loop maps, Segal conjecture, stable cohomotopy, compact Lie group.
[1]Partially supported by a grant from the NSF.

0002-9939/86 $1.00 + $.25 per page

PROOF. At various primes depending on k, $\operatorname{Im} J$ is the fiber of $\Psi^k - 1: \mathrm{BSO} \to \mathrm{BSO}$ [**MM**, p. 110]. Hence we have an exact sequence

$$[\mathrm{BU}, \mathrm{SO}] \to [\mathrm{BU}, \operatorname{Im} J] \to [\mathrm{BU}, \mathrm{BSO}].$$

$[\mathrm{BU}, \mathrm{SO}]$ is all 2-torsion since $[\mathrm{BU}, \mathrm{U}] = \varprojlim [\mathrm{BU}(n), U] = 0$ ($\lim^1 = 0$) and complexification followed by realification is multiplication by 2. Hence the first map is 0. For the second map note that $[\mathrm{BU}, \operatorname{Im} J] = \varprojlim [L_k, \operatorname{Im} J]$ where L_k is the kth skeleton of $\mathrm{BU}(2k)$. Consider the following:

$$\begin{array}{ccc} [\mathrm{BU}, \operatorname{Im} J] & \approx & \varprojlim [L_k, \operatorname{Im} J] \\ \downarrow & & \downarrow \\ [\mathrm{BU}, \mathrm{BSO}] & \approx & \varprojlim [L_k, \mathrm{BSO}] \\ \downarrow & & \downarrow \\ [\mathrm{BU}, \mathrm{BU}] & \approx & \varprojlim [L_k, \mathrm{BU}] \end{array}$$

(The various $\lim^1$ terms are 0.)

First note that the image of $[L_k, \mathrm{BU}]$ in $[L_{k-1}, \mathrm{BU}]$ is torsion free by the Atiyah-Hirzebruch spectral sequence. Hence the image of $[L_k, \operatorname{Im} J]$ in $[L_{k-1}, \mathrm{BU}]$ must be torsion free and hence 0, since $[L_k, \operatorname{Im} J]$ is finite. Hence,

$$2 \operatorname{im}([L_k, \operatorname{Im} J] \to [L_{k-1}, \mathrm{BSO}]) = 0.$$

So $2 \operatorname{im}(\varprojlim [L, \operatorname{Im} J] \to [\mathrm{BU}, \mathrm{BSO}]) = 0$. But $[\mathrm{BU}, \mathrm{BSO}]$ is torsion free, so the map from $\varprojlim [L, \operatorname{Im} J]$ to $[\mathrm{BU}, \mathrm{BSO}]$ is 0. Hence by exactness, $[\mathrm{BU}, \operatorname{Im} J] = 0$.

We now recall some facts relating to the Segal conjecture. First, the Segal conjecture for finite groups G gives a natural isomorphism between $\hat{I}(G)$, the completion of the augmentation ideal of the Burnside ring of G with respect to itself, and $\pi_s^0(BG)$, the 0th reduced stable cohomotopy of the classifying space of G. Corollary 3.2 of [**F**] shows that $\pi_s^0 \mathrm{BU}(n) \approx \varprojlim_k (\hat{I}(\Sigma_n \wr \mathbf{Z}/a_k))^S$ where the inverse limit is over any sequence of integers a_k s.t. $\forall k a_k | a_{k+1}$ and for each $m \in \mathbf{N}$ $\exists k$ s.t. $m | a_k$. S denotes the stable elements. Hence

$$\pi_s^0(\mathrm{BU}) \approx \varprojlim_n \varprojlim_k (\hat{I}(\Sigma_n \wr \mathbf{Z}/a_k))^S.$$

To show that this is nonzero we find an element X in the inverse limit which projects to a nonzero element in $\hat{I}(\Sigma_2 \wr \mathbf{Z}/2)$. This element is obtained by considering the Burnside rings of the compact Lie groups $\mathrm{U}(n)$, denoted by $A(\mathrm{U}(n))$, as defined by tom Dieck [**D**]. Let $N(n) = \Sigma_n \wr \mathrm{U}(1)$ be the normalizer of the maximal torus $\mathrm{U}(1)^n$ in $\mathrm{U}(n)$. Then the $\mathrm{U}(n)$ space $\mathrm{U}(n)/N(n)$ represents an element in $A(\mathrm{U}(n))$. Since $\chi(\mathrm{U}(n)/N(n)) = 1$ we have $[\mathrm{U}(n)/N(n)] - 1$, denoted by X_n, is an element in the augmentation ideal $I(\mathrm{U}(n))$ and hence represents an element in $\hat{I}(\mathrm{U}(n))$ which we also denote by X_n. Suppose H is a closed subgroup of a compact Lie group G. Let $\rho(H, G): A(G) \to A(H)$ be the restriction map given by considering a G space to be an H-space.

LEMMA 3. *The elements* $\{X_n\}$ *represent an element in* $\varprojlim I(\mathrm{U}(n))$ *and hence in* $\varprojlim \hat{I}(\mathrm{U}(n))$. *That is,* $\rho(\mathrm{U}(n-1), \mathrm{U}(n))(X_n) = X_{n-1}$.

PROOF. This follows from the double coset theorem for the transfer [**D**]. Recall that a transfer $t\colon A(H) \to A(G)$ is defined by $t[M] = [M \times_H G]$ where M is an H-space. The double coset theorem says that

$$\rho(K,G) \circ t(H,G) = \sum \chi^{\#}(M_i) t(H^g \cap K, K) \rho(H^g \cap K, H^g) C_g$$

where the sum is over all orbit type manifold components $\{M\}$ of the double coset space $K\backslash G/H$, considered as a K-orbit space. $\chi^{\#}(M) = \chi(\overline{M}) - \chi(\overline{M} - M)$. Here g is any representative in G of M, $H^g = gHg^{-1}$, and C_g is the conjugation isomorphism $A(H) \to A(H^g)$. We apply this to the case where $K = \mathrm{U}(n-1), H = N(n)$ and $G = \mathrm{U}(n)$, as in the following diagram:

$$\begin{array}{ccc} & & A(N(n)) \\ & & \downarrow t \\ A(\mathrm{U}(n-1)) & \xleftarrow[\rho]{} & A(\mathrm{U}(n)) \end{array}$$

Now $t(1) = [G/H] = X_n + 1$. We wish to show that $\rho(K,G)t(H,G)(1) = X_{n-1} + 1$. By the double coset formula,

$$\begin{aligned} \rho(K,G)t(1) &= \sum \chi^{\#}(M) t(H^g \cap K, K) \rho(H^g \cap K, H^g) C_g(1) \\ &= \sum \chi^{\#}(M) t(H^g \cap K, K)(1) = \sum \chi^{\#}(M) [K/H^g \cap K]. \end{aligned}$$

Now $[K/H^g \cap K] = 0$ if $H^g \cap K$ is not of finite index in its normalizer in K [**D**]. Using this we shall see that the sum reduces to one term. If $\alpha \in \mathrm{U}(1)$, then αI_{n-1}, denoted $\bar{\alpha}$, is in the center of $\mathrm{U}(n-1)$. We also note that each right coset gH of $\mathrm{U}(n)$ contains an element with bottom row of the following kind. First all entries are real and secondly they are nondecreasing from left to right. This follows since elements of H act by permuting the columns of g and multiplying entire columns by a complex number of norm one. We assume that g is of the form.

Assume $\bar{\alpha}gH = gH$, i.e. $\bar{\alpha} \in H^g \cap K$. Suppose $g \notin \mathrm{U}(n-1)$. Then the bottom row of g has more than one nonzero entry. It follows that each of these nonzero entries lie in a column with more than one nonzero entry. Consider one such column c, say with bottom entry r. Since $\bar{\alpha}gH = gH$, we have $\bar{\alpha}gh = g$ for some $h \in H$. The effect of multiplication by $\bar{\alpha}$ on c is to multiply the first $(n-1)$ entries by α and leave the last entry r alone. The column remains in the same position. Since elements of H act by permuting columns and multiplying whole columns by a constant of norm one, the only way $\bar{\alpha}gh = g$ is if h permutes the columns with entry r in the bottom row. Hence $\alpha^{n!} = 1$. Thus only a finite number of $\{\bar{\alpha} | \alpha \in \mathrm{U}(1)\}$ are in $H^g \cap K$. It follows that $H^g \cap K$ is not of finite index in its normalizer in $\mathrm{U}(n-1)$. Hence unless $g \in \mathrm{U}(n-1)$, $[K/H^g \cap K] = 0$.

Thus the sum simplifies to $[K/H \cap K]$. The coefficient $\chi^{\#}(M) = 1$ since M is a single orbit. It follows that $\rho(X_n) = X_{n-1}$, as claimed.

REMARK. The argument reducing the double coset formula in this specific case to a single term also applies to the double coset formula for cohomology theories and underlies the proof of Theorem 4 of [**F2**], which in turn is used in [**Sn**] to calculate the stable homotopy type of $\mathrm{BU}(n)$.

The system $\{X_n\}$ in $\varprojlim \hat{I}(\mathrm{U}(n))$ gives rise by restriction to an element in $\varprojlim_n \varprojlim_k \hat{I}(\Sigma_n \wr \mathbf{Z}/k)$. To show that this is nontrivial we need only show that $\rho(\Sigma_2 \wr \mathbf{Z}/2, \mathrm{U}(2))(X_2)$ is not 0 in $\hat{I}(\Sigma_2 \wr \mathbf{Z}/2)$. The advantage of looking at the image in $\hat{I}(\Sigma_2 \wr \mathbf{Z}/2)$ is that the I-adic completion of $I(G)$ equals the p-adic completion if G is a p-group [**L**]. Hence it suffices to show that the image of X_2 in $I(\Sigma_2 \wr \mathbf{Z}/2)$ is nonzero.

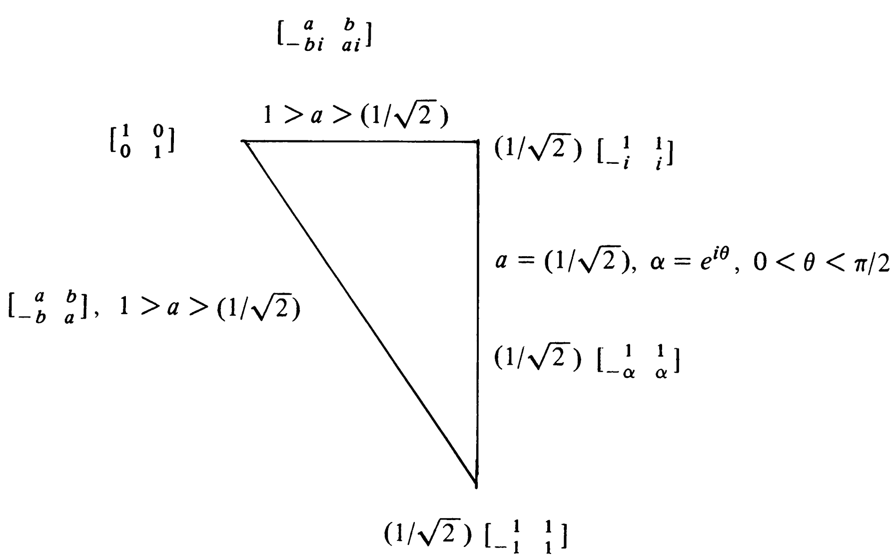
We now calculate $\rho(\Sigma_2 \wr \mathbf{Z}/2, \mathrm{U}(2))(X_2)$. Recall that $X_2 = [\mathrm{U}(2)/\Sigma_2 \wr \mathrm{U}(1)] - 1$. The calculation amounts to analyzing $\mathrm{U}(2)/\Sigma_2 \wr \mathrm{U}(1)$ as a $\Sigma_2 \wr \mathbf{Z}/2$ space, i.e. to understanding the double coset space $\Sigma_2 \wr \mathbf{Z}/2 \backslash \mathrm{U}(2)/\Sigma_2 \wr \mathrm{U}(1)$.

It is easiest to think of the matrix representations of these groups. $\Sigma_2 \wr \mathbf{Z}/2$ consists of signed 2×2 permutation matrices (and is isomorphic to the dihedral group of order 8). It sits inside of $\Sigma_2 \wr \mathrm{U}(1) = N$, the normalizer of the diagonal maximal torus $\mathrm{U}(1) \times \mathrm{U}(1)$ in $\mathrm{U}(2)$. Since $-I_2$ is in the center of $\mathrm{U}(2)$, $-I_2$ is contained in any isotropy subgroup. There are three subgroups of order 4 containing $-I_2$, namely $\langle h = \left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right]\rangle = H$, $\langle k = \left[\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right], \left[\begin{smallmatrix} -1 & 0 \\ 0 & -1 \end{smallmatrix}\right]\rangle = K$ and $\langle \left[\begin{smallmatrix} -1 & 0 \\ 0 & -1 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right] = g\rangle = G$; $D = \Sigma_2 \wr \mathbf{Z}/2$ and $C = \langle -I_2 \rangle$ are the other two groups that can occur as isotropy subgroups.

PROPOSITION 4. *$\rho(D, \mathrm{U}(2))[\mathrm{U}(2)/N] = 3 - [D/G] - [D/H] - [D/K] + [D/C]$. Hence, $\rho(D, U(2))(X_2) \neq 0$ in $\hat{I}(D)$.*

PROOF. Each right coset of N in $\mathrm{U}(2)$ has a representative with top row real nonnegative and nonincreasing from left to right, since the elements of N act on $\mathrm{U}(2)$ on the right by switching columns and multiplying whole columns by complex numbers of norm 1.

Let $u \in \mathrm{U}(2)$ be such a representative with top row $ab, 1 \geq a \geq b \geq 0$. A calculation shows the orbit space $D\backslash U/N$ has the structure of a triangle with representatives of each double coset described as in the following diagram:



The interior of the triangle consists of orbits of type D/C.

The calculation consists of checking which right cosets uN are fixed by the various subgroups H, G, and K. As pointed out earlier, since C is obtained in N and is in the center of U(2), every right coset is fixed by C.

First we calculate the cosets fixed by $H = \langle \left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right] = h \rangle$. A general element u of U(2) with top row $ab, 1 \geq a \geq b \geq 0$, has the form $\left[\begin{smallmatrix} a & b \\ -b\eta & a\eta \end{smallmatrix}\right]$ for some η of norm 1.

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ -b\eta & a\eta \end{bmatrix} = \begin{bmatrix} -b\eta & a\eta \\ -a & -b \end{bmatrix} = hu.$$

There are two cases to consider.

Case 1. $a > b$. Then any representative of huN with top row ab must be obtained from hu by switching columns and multiplying each column by an appropriate element of norm 1. Thus,

$$hu \sim \begin{bmatrix} a\eta & -b\eta \\ -b & -a \end{bmatrix} \sim \begin{bmatrix} a & b \\ -b\bar{\eta} & a\bar{\eta} \end{bmatrix}.$$

If $b \neq 0$ this is the unique representative of huN with top row ab. (If $b = 0$, then $I_2 \in uN$.) So $uN = huN$ iff $-b\eta = -b\bar{\eta}$ and $a\eta = a\bar{\eta}$, i.e. $\eta = \bar{\eta}$ or $\eta = \pm 1$. Since $\left[\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right]\left[\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}\right] = \left[\begin{smallmatrix} a & b \\ b & -a \end{smallmatrix}\right]$, both $\left[\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} a & b \\ b & -a \end{smallmatrix}\right]$ represent the same orbit. This describes the hypotenuse of the triangle.

If $a = b$, then there is a second representative in huN with top row ab. Namely, do not switch the columns of hu, but multiply each column by a suitable element of norm 1. This gives $hu \sim \left[\begin{smallmatrix} a & a \\ a\bar{\eta} & -b\bar{\eta} \end{smallmatrix}\right]$. This representative of huN equals u iff $\eta = -\bar{\eta}$, i.e. iff $\eta = \pm i$. As in the preceding case we need only consider the case $\eta = i$. This gives rise to the vertex of the right angle.

In a similar manner one analyzes the cosets fixed by G and K.

One checks that the cosets fixed by K have the form $(1/\sqrt{2})\left[\begin{smallmatrix} 1 & 1 \\ -\alpha & \alpha \end{smallmatrix}\right]N$. Let $\left[\begin{smallmatrix} -1 & 0 \\ 0 & 1 \end{smallmatrix}\right] = k$. Then $k\left[\begin{smallmatrix} a & b \\ -b\eta & a\eta \end{smallmatrix}\right] = \left[\begin{smallmatrix} -a & -b \\ -b\eta & a\eta \end{smallmatrix}\right]$. If $a > b$ and $b \neq 0$ the only representative of kuN with top row ab is $\left[\begin{smallmatrix} a & b \\ b\eta & -a\eta \end{smallmatrix}\right]$. But this is never equal to u. So the only possibilities for cosets fixed by K are where $a = b$ or $b = 0$. The latter corresponds to the identity coset. If $a = b$, then

$$ku = \begin{bmatrix} -a & -a \\ -a\eta & a\eta \end{bmatrix} \sim \begin{bmatrix} -a & -a \\ a\eta & -a\eta \end{bmatrix} \sim \begin{bmatrix} a & a \\ -a\eta & a\eta \end{bmatrix} = u.$$

Since the double cosets of $\left[\begin{smallmatrix} a & a \\ -a\eta & a\eta \end{smallmatrix}\right], \left[\begin{smallmatrix} a & a \\ a\eta & -a\eta \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]\left[\begin{smallmatrix} a & a \\ -a\eta & a\eta \end{smallmatrix}\right]\left[\begin{smallmatrix} -\bar{\eta} & \\ & \bar{\eta} \end{smallmatrix}\right] = \frac{1}{\sqrt{2}}\left[\begin{smallmatrix} 1 & 1 \\ -\bar{\eta} & \bar{\eta} \end{smallmatrix}\right]$ are the same we can assume $\eta = e^{i\theta}$ where $0 \leq \theta \leq \pi/2$. Hence the orbits with isotropy subgroup containing K are the vertical edge of the triangle and the opposite vertex.

Finally suppose uN is fixed by $\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right] = g$. Then

$$gu = g\begin{bmatrix} a & b \\ -b\eta & a\eta \end{bmatrix} = \begin{bmatrix} -b\eta & a\eta \\ a & b \end{bmatrix}.$$

There are two cases as usual. First suppose $a > b$. Then $gu \sim \left[\begin{smallmatrix} a\eta & -b\eta \\ b & a \end{smallmatrix}\right] \sim \left[\begin{smallmatrix} a & b \\ b\bar{\eta} & -a\bar{\eta} \end{smallmatrix}\right]$. If $b \neq 0$, then $\eta = -\bar{\eta}$, i.e. $\eta = \pm i$. As usual, we can assume $\eta = i$. This gives the top edge of the triangle. If $b = a$, then in addition to the case $\left[\begin{smallmatrix} a & a \\ -ai & ai \end{smallmatrix}\right]$ we get the opposite vertex $\left[\begin{smallmatrix} a & a \\ -a & a \end{smallmatrix}\right]$. For $gu \sim \left[\begin{smallmatrix} a & a \\ -a\bar{\eta} & a\bar{\eta} \end{smallmatrix}\right]$. If this equals u, then $\bar{\eta} = \eta$, so $\eta = \pm 1$. As usual, we can assume $\eta = 1$.

The remaining double cosets correspond to orbits with isotropy subgroup C. They have representatives $\left[\begin{smallmatrix} a & b \\ -b\eta & a\eta \end{smallmatrix}\right]$ with $a > b, \eta \neq \pm 1, \pm i$. Since the double cosets containing $\left[\begin{smallmatrix} a & b \\ -b\eta & a\eta \end{smallmatrix}\right]$, $\left[\begin{smallmatrix} a & b \\ b\eta & -a\eta \end{smallmatrix}\right]$, and $\left[\begin{smallmatrix} a & b \\ b\bar{\eta} & -a\bar{\eta} \end{smallmatrix}\right]$ are equal we can assume $\eta = e^{i\theta}$ where $0 < \theta < \pi/2$. This gives rise to the interior of the triangle.

Hence the three vertices have isotropy subgroup D. The interior of the top edge has isotropy subgroup G, the interior of the vertical edge has isotropy subgroup K and the interior of the hypotenuse has isotropy subgroup H. The interior of the triangle has isotropy subgroup C. The formula follows by noting that $\chi^{\#}$ of a point is 1, $\chi^{\#}$ of an interior of a line segment is -1, and $\chi^{\#}$ of an interior of a 2-simplex is 1. ($\chi^{\#}(M) = \chi(\bar{M}) - \chi(\bar{M} - M)$, where $\bar{M}$ is the closure of M in the double coset space.)

This finishes the proof of the proposition. This also gives our main result.

THEOREM 5. $[\mathrm{BU}, \operatorname{coker} J] \neq 0$. *Indeed, there is an element* $X \in [\mathrm{BU}, \operatorname{coker} J]$ *which restricts to the element in* $\pi_s^0(B\Sigma_2 \wr \mathbf{Z}/2)$ *corresponding to* $2 - [D/G] - [D/H] - [D/K] + [D/C]$ *in* $\hat{I}(\Sigma_2 \wr \mathbf{Z}/2)$ *under the Segal conjecture isomorphism* (*see Proposition* 4). *Since the image of* $[\mathrm{BU}, \operatorname{coker} J]$ *in* $\hat{I}(\Sigma_2 \wr \Sigma_2)$ *is a* $\hat{\mathbf{Z}}_2$*-submodule,* $[\mathrm{BU}, \operatorname{coker} J]$ *is uncountable.*

REMARK. Other calculations indicate that $[\mathrm{BU}, \operatorname{coker} J]$ is a very complicated object. As pointed out above it is isomorphic to $\varprojlim_{n,k} (\hat{I}(\Sigma_n \wr \mathbf{Z}/k))^S$. At odd primes this is equivalent to $\varprojlim C(\mathrm{U}(n))$ by the main theorem in [**F**]. Here C denotes a double completion of A. At the prime 2 $[\mathrm{BU}, \operatorname{coker} J]$ surjects onto $\varprojlim C(\mathrm{U}(n))$ but may not be equal to it.

The situation is vastly simplified if one only considers infinite loop maps from BU to coker J.

THEOREM 6. *There are no infinite loop maps from* BU *to* coker J.

PROOF. Segal has shown that every infinite loop map from BU factors through $\mathrm{BU}(1) = \mathbf{C}P^{\infty}$ [**S**]. The result follows by noting that $\pi_s^0(\mathrm{BU}(1)) \approx \hat{I}(\mathrm{U}(1)) = 0$. This can be obtained from [**R** or **F**].

REFERENCES

[**AS**] M. Atiyah and G. Segal, *Equivariant K-theory and completion*, J. Differential Geom. **3** (1969), 1–18.

[**C**] G. Carlsson, *Equivariant stable homotopy and Segal's Burnside ring conjecture*, Ann. of Math. (2) **120** (1984), 189–224.

[**D**] T. tom Dieck, *Transformation groups and representation theory*, Lecture Notes in Math., vol. 766, Springer-Verlag, Berlin, 1979.

[**F**] M. Feshbach, *The Segal conjecture for compact Lie groups*, Topology (to appear).

[**F2**] ——, *The transfer and compact Lie groups*, Bull. Amer. Math. Soc. **83** (1977), 372–374.

[**HS**] L. Hodgkin and V. Snaith, *Topics in K-theory, two independent contributions*, Lecture Notes in Math., vol. 496, Springer-Verlag, Berlin, 1975.

[**L**] E. Laitinen, *On the Burnside ring and stable cohomotopy of a finite group*, Math. Scand. **44** (1979), 37–72.

[**MM**] I. Madsen and R. J. Milgram, *The classifying spaces for surgery and cobordism of manifolds*, Ann. of Math. Studies, no. 92, Princeton Univ. Press, Princeton, N.J., 1979.

[**R**] D. Ravenal, *The Segal conjecture for cyclic groups and its consequences*, Amer. J. Math. **106** (1984), 415–446.

[**S**] G. Segal, *The stable homotopy of complex projective space*, Quart. J. Math. Oxford Ser. (2) **24** (1973), 1–5.

[**Sn**] V. Snaith, *Algebraic cobordism and K-theory*, Mem. Amer. Math. Soc. No. 221 (1979).

SCHOOL OF MATHEMATICS, UNIVERSITY OF MINNESOTA, MINNEAPOLIS, MINNESOTA 55455