PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 98, Number 3, November 1986

A JC-ALGEBRA WHICH IS NOT THE RANGE OF A POSITIVE PROJECTION ON A C^*-ALGEBRA

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ABSTRACT. An example is given of a JC-algebra which is not the range of any positive projection on the selfadjoint part of its generated C^*-algebra.

JC-algebras arise naturally in the study of positive projections on operator algbras [**1**, Theorem 1.4]. Effros and Størmer showed in [**1**, Theorem 2.1] that if A is a simple JC-algebra and B is the C^*-algebra generated by A, then there exists a positive projection P on B such that $P(B_{sa}) = A$. Furthermore they expressed the belief that this result may fail for general JC-algebras [**1**, Remark 2.7]. The purpose of this note is to provide an explicit example showing that this is indeed the case.

The example is constructed as follows. Let F be the CAR-algebra, acting irreducibly on a Hilbert space H. By [**2**, Theorem 6.2.2], F is the C^*-algebra generated by an infinite-dimensional spin factor V. If K denotes the algebra of compact operators on H, then $F \cap K = \{0\}$ [**3**, Theorem 6.5.7]. Let $A = K_{sa} + V$ and let B be the C^*-algebra generated by A. Then A is a JC-algebra and $B = K + F$. These statements follow easily from [**3**, Corollary 1.5.8] and its Jordan-algebraic analogue [**2**, 3.4].

THEOREM. *There is no positive projection P on B such that $P(B_{sa}) = A$.*

PROOF. Suppose that such a P exists. We show that this implies that V is *reversible* in the sense that it is closed under symmetric products of the form $a_1a_2\cdots a_n + a_n\cdots a_2a_1$ [**2**, 2.3.2]. This will contradict [**2**, Theorem 6.2.5], which asserts that V is not reversible.

Given $a_1, \ldots, a_n \in V$, let $a = a_1a_2\cdots a_n + a_n\cdots a_2a_1$. If $b \in K_{sa}$ then $a \circ b = \frac{1}{2}(ab + ba) \in K_{sa}$. Therefore $a \circ b = P(a \circ b) = P(a) \circ b$, where the last equality follows from [**1**, Lemma 1.1] together with the fact that the range of P is assumed to be a Jordan algebra. Now the identity operator is a strong limit of elements of K_{sa}. Hence $a = P(a) \in A$. It follows that $a = k + x$, where $k \in K_{sa}$ and $x \in V$. Since it is obvious from its definition that $a \in F$, we have

$$a - x = k \in F \cap K = \{0\}.$$

Therefore $a = x \in V$. This shows that V is reversible, contradicting [**2**, Theorem 6.2.5].

REMARK. The argument above actually shows that A is not the range of a positive projection on any reversible JC-algebra containing A as a JC-subalgebra.

Received by the editors July 16, 1985.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 46L05; Secondary 17C65.

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