MINIMAL DEGREES OF FAITHFUL CHARACTERS OF FINITE GROUPS WITH A T.I. SYLOW $p$-SUBGROUP

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Abstract. Using the classification of the finite simple groups we show in this article that a faithful complex character $\chi$ of a finite group $G$ with a nonnormal T.I. Sylow $p$-subgroup $P$ has degree $\chi(1) > \sqrt{|P|} - 1$. This result verifies a conjecture of H. S. Leonard [10].

Introduction. Let $p$ be a fixed prime, and let $G$ be a finite group with a T.I. Sylow $p$-subgroup $P$. That is, two different conjugates of $P$ have only the identity element in common. In [10] H. S. Leonard conjectured that if $G$ has a faithful complex character $\chi$ with degree $\chi(1) \leq \sqrt{|P|} - 1$, then $P$ is normal in $G$. Using the classification of the finite simple groups we prove Leonard’s conjecture in this note (Theorem 3.2).

In §1 this theorem is first proved for $p$-solvable groups $G$ (Proposition 1.3). Then we determine the composition series of a minimal counterexample $G$ to Leonard’s conjecture (Proposition 1.4). Since by Sibley’s theorem [12] the main result of this article is known if $P$ is cyclic, we give in §2 a complete list of all finite simple groups $G$ having a noncyclic T.I. Sylow $p$-subgroup for some prime $p$ (Proposition 2.3). Here for odd $p$ we use Gorenstein and Lyons’ theorem [4] classifying all finite groups $G$ with $O_p(G) = 1$, $p$-rank $m_p(G) > 1$, and containing a strongly $p$-embedded subgroup. If $p = 2$, then Proposition 2.3 is only a restatement of Suzuki’s theorem [13]. After these preparations Leonard’s conjecture is proved in §3. In Remark 3.3 we show that the bound of Theorem 3.2 cannot be replaced by $\frac{1}{2}(|P| - 1)$, which is the bound of Sibley’s theorem [12].

For notation and terminology we refer to the books by Feit [1], Gorenstein [2, 3], Huppert [5], Huppert and Blackburn [6], and Landrock [9]. All character tables of finite simple groups used here are contained in the CAS-system [11] of J. Neubüser, H. Pahlings, and W. Plesken (TH. Aachen, Federal Republic of Germany).

1. Reduction to almost simple groups. In this section we determine the structure of a finite group $G$ of minimal order among the groups $H$ without a normal Sylow $p$-subgroup, but satisfying the hypothesis of Leonard’s conjecture.

The following lemma due to Feit [1, p. 123] is our basic tool.

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Lemma 1.1. Let \( S \) be a splitting field of characteristic zero for the finite group \( G \) with a T.I. Sylow \( p \)-subgroup \( P \). Let \( \chi \) be a character of \( SG \) such that \( \chi(1)^2 \leq |P| \). Let \( H \) be a subgroup of \( G \) containing \( N_G(P) \). Then \( \langle \chi, \chi \rangle_C = \langle \chi, \chi \rangle_H \).

For a short proof of this result we refer to [9, p. 129].

Lemma 1.2. Let \( G \) be a finite group with a T.I. Sylow \( p \)-subgroup \( P \). Then the following assertions hold.

(a) Every subgroup \( U \) of \( G \) with \( p \nmid |U| \) has a T.I. Sylow \( p \)-subgroup.
(b) \( G/N \) has a T.I. Sylow \( p \)-subgroup for every normal subgroup \( N \) of \( G \) with \((p, |N|) = 1\).
(c) \( C_G(x) \) has a normal Sylow \( p \)-subgroup for every \( 1 \neq x \in P \).

Proof. See Suzuki [13, p. 59].

Proposition 1.3. Let \( G \) be a \( p \)-solvable group with T.I. Sylow \( p \)-subgroup \( P \) and a faithful complex character \( \chi \) such that \( \chi(1)^2 \leq |P| \). Then \( P \) is a normal subgroup of \( G \).

Proof. Let \( G \) be a minimal counterexample and let \((F = R/\pi, R, S = \text{quot}(R))\) be a splitting \( p \)-modular system for \( G \) (see [9, p. 47]). Then \( O_p(G) = 1 \), and so \( Q = O_p(G) \neq 1 \). Let \( H = QN_G(P) \). If \( H \neq G \), then \( P < H \) by induction. Hence

\[
O_{p', p}(H) = O_{p', p}(H) \times P \geq O_{p', p}(G) > Q,
\]

because \( G \) is \( p \)-solvable. This forces \( O_{p}(G) \neq 1 \), a contradiction. Therefore, \( G = H \).

But \( G = O_{p'}(G) \) by minimality, whence \( G = QP \). In particular, \( G \) is \( p \)-nilpotent, and every \( p \)-block \( B \) of \( G \) contains only one modular character by Theorem 14.9 of [6]. Since \( P \) is a T.I. Sylow \( p \)-subgroup, and \( \chi(1) < |P| \) it follows from Theorem 14.8 of [6] that (the possibly reducible) \( \chi \) contains an irreducible constituent \( \mu \) belonging to a nonprincipal \( p \)-block \( B \) with defect group \( \delta(B) = _GP \), because \( \chi \) is faithful. As \( G \) is \( p \)-nilpotent, by Theorem 2.1 of [1, p. 419], we also may assume that \( \mu \) remains irreducible under restriction modulo \( \pi \). Let \( \bar{\mu} \) be a module over \( F \) affording \( \mu \) modulo \( \pi \).

Let \( b \) be the block of \( U = N_G(P) \) associated with \( B \) by the Brauer correspondence. Since \( \mu(1)^2 < |P| \), Lemma 1.1 asserts that \( \bar{\mu} \upharpoonright_U \) is irreducible. Because \( P \) is a T.I. set, Green’s correspondence theorem implies that \( \bar{\mu} \upharpoonright_U \) is an indecomposable module in \( b \). Notice that \( b \) contains only one modular irreducible character, so that all composition factors of \( \bar{\mu} \upharpoonright_U \) are isomorphic. In particular, if \( \bar{\mu} \upharpoonright_U \) is not irreducible, it has a nonzero nilpotent \( FU \)-endomorphism \( \tau \), namely any nonzero map from the head to the socle of \( \bar{\mu} \upharpoonright_U \). As \( P \) is a T.I. set, Corollary 5.8 of [9, p. 122], implies that \( \tau \) is a projective endomorphism of \( \bar{\mu} \upharpoonright_U \). Therefore, \( \mu(1)^2 > |P| \) by Corollary 6.11 of [9, p. 128], a contradiction. We now know that \( \bar{\mu} \upharpoonright_U \) is an irreducible \( FU \)-module of \( b \) and that \( P \) acts trivially on \( \bar{\mu} \upharpoonright_U \). Let \( A = \ker \bar{\mu} \) in \( G \). Now \( A \neq G \) since \( B \) is not the principal block. We have

\[
P \leq \ker \bar{\mu} = A < G.
\]

Therefore, \( Q \leq A \) as \( G = QP \).

By Lemma 1.2 \( P \) is a T.I. Sylow \( p \)-subgroup of \( A \), and \( \chi_A \) is a faithful complex
character of $A$ with $\chi_A(1)^2 < |P|$. So $P \triangleleft A$ by induction, which implies $P \triangleleft G$. This contradiction completes the proof.

The center of the group $G$ is denoted by $Z(G)$.

**Proposition 1.4.** Let $G$ be a minimal counterexample to Leonard’s conjecture. Let $P$ be a T.I. Sylow $p$-subgroup of $G$, and let $\chi$ be a faithful complex character of degree $\chi(1) \leq \sqrt{|P|} - 1$. Then:

(a) $Z(G) = O_p(G) < O^{p'}(G) = H, G = O^{p'}(G)$.
(b) $H/Z(G)$ is a nonabelian simple group with a T.I. Sylow $p$-subgroup.
(c) $\chi$ may be assumed to be irreducible.

**Proof.** As $G$ is a minimal counterexample, $G = O^{p'}(G)$. Let $H = O^{p'}(G)$, and let $H/N \neq 1$ be a chief factor of $G$.

Suppose that $H/N$ is a $p'$-group. By Proposition 1.3 $G$ is not $p$-solvable. Thus $P_0 = P \cap N \neq 1$, and $L = N_G(P_0) < G$. Since $P \cap N \triangleleft P$, $P \triangleleft L$. As $G = O^p(G)$, and as $G = NL$ by the Frattini argument, we obtain $G = NP$, and so $N = H$, a contradiction.

Therefore $H/N$ is a direct product of isomorphic nonabelian simple groups $A$ with $|p||A|$. Hence $NN_G(P) < G$, which implies $P \triangleleft NN_G(P)$ by induction. Since $P$ is a T.I. set in $G$, we now get $O_p(N) = N \cap P = 1$. So $N$ is a $p'$-group commuting with $P$. Hence $C_G(N) \supseteq \langle P^g | g \in G \rangle = O^{p'}(G) = G$,

and so $Z(G) = N \leq O_p(G)$.

As $(p, |A|) = 1$, Lemma 1.2 asserts that $H/N$ has a T.I. Sylow $p$-subgroup. Thus by Lemma 1.2(c) $H/N$ is simple. Since $O_p(G) \subset O^p(G) = H$, it follows that $N = O_p(G)$.

Finally, we may replace $\chi$ by an irreducible constituent, which does not have $H$ in its kernel. This completes the proof.

**2. Simple groups with a noncyclic T.I. Sylow $p$-subgroup.** In this section we list the simple groups with a noncyclic T.I. Sylow $p$-subgroup. In [13] Suzuki classified the simple groups with such a Sylow 2-subgroup. For odd primes $p$ our subsidiary result follows from Gorenstein and Lyons’ classification [4] of the finite groups $G$ with $O_p(G) = 1$, $p$-rank $m_p(G) > 1$, and containing a strongly $p$-embedded subgroup.

Here $m_p(G)$ denotes the maximum rank of an elementary abelian subgroup of a Sylow $p$-subgroup of the finite group $G$.

**Definition [3].** Let $P$ be a Sylow $p$-subgroup of the finite group $G$, and let $k$ be a positive integer. The $k$-generated $p$-core of $G$ is $\Gamma_{p,k}(G) = \langle N_G(Q) | Q \leq P, m_p(Q) \geq k \rangle$.

The proper subgroup $M$ of $G$ is called strongly $p$-embedded in $G$ if $\Gamma_{p,1}(G) \leq M$.

**Remark 2.1.** If the finite group $G$ contains a nonnormal T.I. Sylow $p$-subgroup $P$, then $M = N_G(P)$ is strongly $p$-embedded in $G$, as is easily seen.

A finite group $G$ is quasi-simple if $G = G'$ and $G/Z(G)$ is simple. The layer $L(G)$ of $G$ is the product of all subnormal quasi-simple subgroups of $G$, where $L(G) = 1$ if no such subnormal subgroup exists. The generalized Fitting subgroup of the finite group $G$ is defined as $F^*(G) = F(G)L(G)$, where $F(G)$ denotes the Fitting subgroup of $G$ (see [3, p. 44]).
In view of the classification theorem of the finite simple groups we now can restate Theorems (24.1), (24.2), and (24.9) of Gorenstein and Lyons [4, pp. 307, 311, and 318, respectively], as

**Proposition 2.2.** Let \( p \) be an odd prime, \( M \) a strongly \( p \)-embedded subgroup of the finite group \( G \) with \( O_p(G) = 1 \) and \( m_p(G) > 1 \). Let \( V = O_p^*(G) \) and let \( P \) be a Sylow \( p \)-subgroup of \( M \). Then \( F^*(G) = L(V) \) is simple and one of the following holds.

1. \( V \cong PSL_2(p^n) \) or \( PSU_3(p^n) \), and \( M = N_G(P) \).
2. \( V \cong A_{2p} \) and \( F^*(M) \cong A_p \times A_p \).
3. \( p = 3, V \cong C_2G_2(3^{2m+1}) \), and \( M = N_G(P) \), where \( m \geq 0 \).
4. \( p = 3, V \cong M_{11} \) or \( PSL_4(4) \), and \( M = N_G(P) \).
5. \( p = 5, V \cong M(22) \), and \( V \cap M = \text{Aut}(D_4(2)) \).
6. \( p = 5, V \cong F_4(2)^\prime \), \( \text{Aut}(2B_2(2^5)) \) or \( M_22 \), and \( M = N_G(P) \).
7. \( p = 11, V \cong J_4 \), and \( M = N_G(P) \).

**Proof.** By hypothesis, \( N_{p-1}(G) \leq M \neq G \) and \( P \leq V \cap M \). Thus \( O_p(G) = 1 = F(G) \), because otherwise \( G = N_G(O_p(G)) \leq N_{p-1}(G) \leq M \neq G \), a contradiction.

Let \( K \) be a normal subgroup of \( G \). As \( O_p(G) = 1 \), \( P_0 = P \cap K \neq 1 \). The Frattini argument asserts that \( G = N_G(P_0)K \). Hence \( K \neq N_{p-1}(G) \). It follows that every quasi-simple subnormal subgroup \( L \) of \( G \) is simple and \( L \neq N_{p-1}(L) \), where \( P_1 \in \text{Syl}_p(L) \).

Thus \( F^*(G) = L(G) = L(V) \) is a direct product of simple groups \( E_i \), \( 1 \leq i \leq k \), each of which contains a strongly \( p \)-embedded subgroup.

Let \( E \in \{ E_i \mid 1 \leq i \leq k \} \), \( P^* = P \cap L(V) \), and \( X = EP^* \). Then \( O_p(X) = 1 = O_p(P) \) and \( \Gamma_{p-1}(X) \neq X \), because \( P^* \subseteq \Gamma_{p-1}(G) \), but \( E \neq \Gamma_{p-1}(G) \). Applying now Theorem (24.9)(4) of Gorenstein and Lyons [4, p. 318], we obtain that \( O_{p^*}(P^*) \leq E \) or \( E \in \{ G_2(3)^\prime, 2B_2(2^5) \} \). Since \( P^* \in \text{Syl}_p(L(V)) \) it follows that \( P^* \leq E \). As \( O_p(L(V)) = 1 \) we get \( F^*(G) = L(G) = L(V) = E \). Hence \( F^*(G) \) is simple. Now Theorems (24.1) and (24.2) of Gorenstein and Lyons [4, pp. 307, 311] complete the proof.

Combining this result with Suzuki’s theorem [13] we obtain

**Proposition 2.3.** Let \( G \) be a nonabelian simple group with a noncyclic T.I. Sylow \( p \)-subgroup \( P \). Then \( G \) is isomorphic to one of the following groups.

- (a) \( PSL_2(q) \) or \( PSU_3(q) \), where \( q = p^n \) and \( n \geq 2 \) or \( n \geq 1 \), respectively.
- (b) \( p = 2 \) and \( G \cong B_2(2^{2m+1}) \).
- (c) \( p = 3 \) and \( G \cong G_2(3^{2m+1}) \), where \( m \geq 1 \).
- (d) \( p = 3 \) and \( G \cong PSL_3(4) \) or \( M_{11} \).
- (e) \( p = 5 \) and \( G \cong F_4(2)^\prime \) or \( M_{22} \).
- (f) \( p = 11 \) and \( G \cong J_4 \).

**Proof.** If \( p = 2 \), then (a) and (b) follow from Theorem 1 of [13].

Let \( p \) be odd. By Remark 2.1 \( G \) can only be one of the simple \( L(V) \) occurring in the list of Proposition 2.2. Since \( A_p \times A_p \) is a subgroup of \( A_{2p} \), Lemma 1.2 asserts that \( G \neq A_{2p} \). A group \( H \) with a T.I. Sylow \( p \)-subgroup has only \( p \)-blocks of defect zero and of highest defect. By the character table system CAS [11] \( M(22) \) has a 5-block of defect one. Thus \( G \neq M(22) \). Since \( \text{Aut}(2B_2(2^5)) \) is not simple, \( G \neq \text{Aut}(2B_2(2^5)) \).
Now $\text{PSL}_3(p^n)$ and $\text{PSU}_3(p^n)$ have T.I. Sylow $p$-subgroups (see [5, pp. 191, 242]). By Ward [14] $2G_2(3^{2m+1})$ has a T.I. Sylow 3-subgroup. As can be seen from the character table of $\text{PSL}_3(4)$ the Sylow 3-subgroup $P$ equals $C_G(x)$ for every $1 \neq x \in P$. Hence $P$ is a T.I. set.

It is well known and easy to check that the Sylow 3-subgroups of $M_{11}$ and the Sylow 5-subgroups of the Tits group $2F_4(2)'$ and the McLaughlin group $Mc$ are T.I. By Propositions 22 and 26 of Janko [7] the Sylow 11-subgroups of $J_4$ are T.I. This completes the proof.

3. Proof of the main result. In this section, Leonard’s conjecture is proved by means of the results mentioned above.

Let $G$ be a finite group with a Sylow $p$-subgroup $P$. If $C_G(P) = C_G(x)$ for every $1 \neq x \in P$, then $P$ is called weakly self-centralizing. The following lemma is well known.

**Lemma 3.1.** Let $G$ be a finite group with a cyclic Sylow $p$-subgroup $P$. Then $P$ is a T.I. set if and only if $P$ is weakly self-centralizing.

**Theorem 3.2.** Let $G$ be a finite group with a T.I. Sylow $p$-subgroup $P$. If $G$ has a faithful complex character $\chi$ with degree $\chi(1) \leq \sqrt{|P|} - 1$, then $P$ is a normal subgroup of $G$.

**Proof.** If $P$ is cyclic, then $P$ is weakly self-centralizing. As $\sqrt{|P|} - 1 < \frac{1}{2}(|P| - 1)$ for every prime $p > 0$, it follows from Sibley’s theorem [12] that $P < G$.

Now let $G$ be a counterexample of minimal order. Then $P$ is not cyclic, $G = O_p^p(G)$, and by Proposition 1.4 $Z = Z(G) = O_p(G) < O_p^p(G) = H$. Furthermore, $H/Z$ is a nonabelian simple group with a T.I. Sylow $p$-subgroup, and we may assume that $\chi$ is irreducible. We also can assume that $G$ does not have a proper abelian direct factor.

Suppose that $p$ is odd. Then $m_p(G) > 1$, because $P$ is not cyclic. By Remark 2.1 $N_G(P)$ is strongly $p$-embedded in $G$. Therefore it follows from Propositions 2.2 and $p = 3$ and $G \cong 2G_2(3)$ or 2.3 that $H = O_p^p(G) = G$ except when $p = 3$ and $G \cong 2G_2(3)$ or $p = 5$ and $G/Z \cong \text{Aut}(2B_2(2^5))$. Now remembering that $p \nmid |Z|$ and using Gorenstein’s table [3, Table 4.1, p. 302] of the Schur multipliers of the finite simple groups, the structure of $G$ can be described as in the following table.

<table>
<thead>
<tr>
<th>prime $p$</th>
<th>$G/Z$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p</td>
<td>q$</td>
<td>$\text{PSL}_2(q)$ or $\text{PSU}_3(q)$</td>
</tr>
<tr>
<td>$p = 3$</td>
<td>$2G_2(3^{2m+1})$</td>
<td>$Z = 1$</td>
</tr>
<tr>
<td>$p = 3$</td>
<td>$\text{PSL}<em>3(4)$ or $M</em>{11}$</td>
<td>$</td>
</tr>
<tr>
<td>$p = 5$</td>
<td>$2F_4(2)'$</td>
<td>$Z = 1$</td>
</tr>
<tr>
<td>$p = 5$</td>
<td>$\text{Aut}(2B_2(2^5))$</td>
<td>$Z = 1$</td>
</tr>
<tr>
<td>$p = 5$</td>
<td>$Mc$</td>
<td>$</td>
</tr>
<tr>
<td>$p = 11$</td>
<td>$J_4$</td>
<td>$Z = 1$</td>
</tr>
</tbody>
</table>
Applying the theorem of Landazuri and Seitz [8] on the minimal degrees of the nontrivial complex projective representations \( \pi \) of \( \text{PSL}_2(q) \), \( \text{PSU}_3(q) \), or \( \text{PSU}_3(3^{2m+1}) \) we see that \( \pi(1) \geq \frac{1}{2}(q - 1), \pi(1) \geq q(q - 1), \) and \( \pi(1) \geq 3^{2m+1}(3^{2m+1} - 1), \) respectively. In any case \( \pi(1) > \sqrt{|P|} - 1, \) a contradiction. If \( p = 3 \) and \( G/Z \in \{ \text{PSL}_3(4), M_{24} \} \), then \( |P| = 9. \) But another contradiction is obtained since the nontrivial irreducible projective characters of these simple groups have minimal degrees

\[
\chi(1) = \begin{cases} 
4, & \text{if } G/Z = \text{PSL}_3(4), \\
10, & \text{if } G = M_{24}.
\end{cases}
\]

If \( p = 5 \) and \( G = \text{PSU}_3(2^4) \) then every nontrivial irreducible character \( \chi \) of \( G \) has degree \( \chi(1) \geq 26 \). However, \( |P| = 25 \), a contradiction.

If \( p = 5 \) and \( G = \text{PSU}_3(2^5) \), then every faithful irreducible character \( \chi \) of \( G \) has degree \( \chi(1) \geq \pi(1) \), where \( \pi \) is a nontrivial irreducible character of the Suzuki group \( ^2B_2(q), q = 2^5 \), of minimal degree. Now by Landazuri and Seitz [8, p. 419], \( \pi(1) = 4 \cdot 31 = 124 \). Since \( |P| = 125 \), we obtain \( \chi(1) > \sqrt{|P|} - 1, \) a contradiction.

If \( p = 5 \) and \( G/Z = M_{24} \), then every irreducible nontrivial character \( \chi \) of \( G \) has degree \( \chi(1) \geq 22 \) by the character table of \( M_{24} \) (see \([11]\)). As \( |P| = 125 \), \( G \) cannot be a minimal counterexample. If \( G/Z \cong M_{24} \) and \( |Z| = 3 \), we again use the character table of \( G \) (see \([11]\)) and find that the nontrivial projective irreducible character \( \chi \) of minimal degree has degree \( \chi(1) = 126 \geq \sqrt{125} - 1 \), another contradiction.

If \( p = 11 \) and \( G = J_4 \), then \( \chi(1) \geq 1333 \) by \([11]\). Since \( |P| = 11^3 \), \( \chi(1) > \sqrt{|P|} - 1 \), which is impossible by hypothesis.

Therefore \( p = 2 \). Hence by Theorem 2 of Suzuki \([13]\) and Proposition 1.4 we get \( G/Z \in \{ \text{PSL}_2(q), \text{PSU}_3(q), ^2B_2(q) \} \), where \( q \) is a power of 2. Using the theorem of Landazuri and Seitz \([8]\) as above, we obtain our final contradiction. This completes the proof.

**Remark 3.3.** It is not possible to replace the bound \( \sqrt{|P|} - 1 \) by the bound \( \frac{1}{2}(|P| - 1) \) of Sibley’s theorem \([12]\). Let \( G = M_{24}, p = 5, \) and \( \chi \) be the irreducible character of \( G \) with degree \( \chi(1) = 22 \). The Sylow 5-subgroup \( P \) of \( G \) is T.I. and has order \( |P| = 5^3 = 125 \). Hence \( \chi(1) = 22 < 62 = \frac{1}{2}(|P| - 1) \). However, \( P \) is not normal. In particular, Sibley’s condition that \( P \) be weakly self-centralizing cannot be weakened to \( P \) being a T.I. set.

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