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ON A CHARACTERIZATION OF W-SETS AND THE DIMENSION OF HYPERSPACES

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ABSTRACT. A subcontinuum A of a continuum X is a W-set if for each mapping $f\colon Y \twoheadrightarrow X$ of an arbitrary continuum Y onto X there is a continuum in Y which is mapped by f onto A. We characterize W-sets in terms of accessibility by small continua. We localize several known results on continua all of whose subcontinua are W-sets. Finally, we extend a result of J. T. Rogers by proving that if X is an atriodic continuum whose first Čech cohomology group is finitely generated then the hyperspace $C(X)$ of subcontinua of X is two dimensional.

1. Introduction. By a *continuum* we mean a compact connected, metric space and by a *mapping* we mean a continuous function. A subcontinuum K of a continuum Y is said to be a *W-set in Y* provided that for every mapping $f\colon Y \twoheadrightarrow Y$ (where X is any continuum) onto Y there exists a subcontinuum L of X which is mapped by f onto K. A continuum Y is said to belong to *Class* (W) provided that every subcontinuum of Y is a W-set. In [**3**–**5**] there were given some necessary and some sufficient conditions for a continuum to be in Class (W). In §2 we give several analogous characterizations of W-sets.

By $H^n(X)$ we denote the nth Čech cohomology group with integer coefficients of the space X. A continuum X is said to be a *triod* if there exists a subcontinuum M of X such that $X\backslash M$ has at least three components. If a continuum does not contain any triod it is said to be *atriodic.*

If A is a subset of a space X we let $\mathrm{Cl}(A)$ and $\mathrm{Bd}(A)$ denote the closure and boundary, respectively, of A in X. For $\varepsilon > 0$ we let $S(A, \varepsilon)$ denote the open ε-ball about A with respect to a fixed but arbitrary metric on X.

If X is a continuum we let $C(X)$ denote the hyperspace of subcontinua of X. We let $\mu\colon C(X) \twoheadrightarrow [0,1]$ denote a Whitney map from $C(X)$ onto $[0,1]$ (i.e. if $A, B \in C(X)$ with $A \not\subseteq B$, then $\mu(A) < \mu(B)$ and $\mu(\{x\}) = 0$ for each $x \in X$). The reader may consult [**11**] for properties of hyperspaces.

In §3 we prove that the property of being an atriodic cohomologically trivial continuum is inherited by all Whitney levels $\mu^{-1}(t)$ of the hyperspace. We also prove that if X is an atriodic continuum with finitely generated $H^1(X)$, then $\dim C(X) \leq 2$. These results generalize results of Oversteegen and Tymchatyn [**13**] and Rogers [**15**].

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2. Some characterizations of W-sets.

2.1 THEOREM. *Let A be a proper subcontinuum of a continuum X. Then A is not a W-set in X if and only if there exists some $\varepsilon > 0$ and a neighborhood G of A such that*

(1) *for each $x \in G$ there exists a continuum B from x to* $\mathrm{Bd}(G)$ *such that $A \not\subset S(B, \varepsilon)$, and*

(2) *for each decomposition of* $\mathrm{Bd}(G) = R \cup S$ *into disjoint closed sets R and S, there exists a continuum K from R to S with $A \not\subset S(K, \varepsilon)$.*

PROOF. Suppose that ε is a positive number and G is a neighborhood of A satisfying (1), (2), and $S(A, \varepsilon) \subset G$. Since A is totally bounded, there exist an integer n and a set $\{x_1, \dots, x_n\} \subset A$ such that if $x \in G$ then there exists a continuum B from x to $\mathrm{Bd}(G)$ such that $B \cap S(x_i, \varepsilon/2) = \varnothing$ for some $i \in \{1, \dots, n\}$, and for each decomposition of $\mathrm{Bd}(G)$ into disjoint closed sets R and S there exists a continuum B from R to S which misses $S(x_i, \varepsilon/2)$ for some $i \in \{1, \dots, n\}$.

For each $i \in \{1, \dots, n\}$ let

$$F_i = \{L \in C(X) | L \cap \mathrm{Bd}(G) \neq \varnothing \text{ and } L \cap S(x_i, \varepsilon/2) = \varnothing\}.$$

For each i, F_i is a compact subset of $C(X)$. Let $X_i = \bigcup_{i=1}^{n} F_i$. By [**9**, p. 23], X_i is compact. By (1) and (2), we have that $X = \bigcup_{i=1}^{n} X_i$. Put $Y_i = X_i \times \{i\}$ and let $Z = \bigcup_{i=1}^{n} Y_i$. Let $f \colon Z \to X$ be the natural projection. Define an equivalence relation $\sim$ on Z by setting $x \sim y$ if and only if either $x = y$ or $f(x) = f(y)$ and $f(x) \in X \backslash G$. Then $\sim$ is upper semicontinuous. Let $Y = Z/\sim$ be the quotient space and let $\phi \colon Z \to Y$ be the quotient map. Let $g \colon Y \to X$ be the unique mapping which causes the diagram

$$\begin{tikzcd} Z \arrow[r, "f"] \arrow[d, "\phi"'] & X \\ Y \arrow[ur, dashed, "g"'] & \end{tikzcd}$$

to commute. Since f and ϕ are onto, g is also onto. The set $g^{-1}(A)$ is the disjoint union of the closed subsets

$$\phi\left[f^{-1}(A) \cap Y_1\right], \dots, \phi\left[f^{-1}(A) \cap Y_n\right].$$

None of these sets maps onto A. Thus, no subcontinuum of Y is mapped by g onto A.

It remains to prove that Y is a continuum. Clearly, each component of Y meets $g^{-1}[\mathrm{Bd}(G)]$. By (2) and the definition of Y, no decomposition of $g^{-1}[\mathrm{Bd}(G)]$ into disjoint closed sets extends to a decomposition of Y into disjoint closed sets. Hence, Y is connected. This completes the proof that A is not a W-set in X.

Now, suppose that A is not a W-set in X. Let $f \colon Z \twoheadrightarrow X$ be a mapping of a continuum Z onto X such that no subcontinuum of Z is mapped by f onto A. Let U be an open neighborhood of A such that no component of $f^{-1}[\mathrm{Cl}(U)]$ maps onto a set containing A. Let $\varepsilon > 0$ so that $S(A, \varepsilon) \subset U$ and if K is a component of $f^{-1}[\mathrm{Cl}(U)]$, then $f(K) \cap S(y, \varepsilon) = \varnothing$ for some $y \in A$. If $x \in U$ and R is a component of $f^{-1}[\mathrm{Cl}(U)]$ which meets $f^{-1}(x)$, then $R \cap f^{-1}[\mathrm{Bd}(U)] \neq \varnothing$ by the Boundary Bumping Theorem (see [**11**, p. 625]). Hence, $f(R)$ is a continuum from x to $\mathrm{Bd}(U)$ in X such that $f(R) \cap S(y, \varepsilon) = \varnothing$ for some $y \in A$. So (1) holds for U. By taking an appropriate quotient space Z, we may suppose that f maps $Z\backslash f^{-1}(U)$ homeomorphically onto $X\backslash U$.

Let $\mathrm{Bd}(U) = R \cup S$, where R and S are nonempty disjoint closed sets. Then $f^{-1}[\mathrm{Bd}(U)] = f^{-1}(R) \cup f^{-1}(S)$. If a component K of $f^{-1}[\mathrm{Cl}(U)]$ meets both $f^{-1}(R)$ and $f^{-1}(S)$, then $f(K)$ is a continuum in $\mathrm{Cl}(U)$ from R to S so that $f(K) \cap S(y, \varepsilon) = \varnothing$ for some $y \in A$ by the choice of U and ε. If no component of $f^{-1}[\mathrm{Cl}(U)]$ meets both $f^{-1}(R)$ and $f^{-1}(S)$, then some component K of $f^{-1}(X\backslash U)$ meets both $f^{-1}(R)$ and $f^{-1}(S)$, since Z is a continuum. Then $f(K)$ is a continuum in $X\backslash S(A, \varepsilon)$ from R to S. So U also satisfies (2). This completes the proof of the theorem.

From 2.1 we get the following intrinsic characterization of Class (W).

2.2 COROLLARY. *A continuum X is in Class (W) if and only if for every subcontinuum A of X, for each $\varepsilon > 0$ and each neighborhood U of A we have that either*

(1) *there exists $x \in U$ such that for every continuum B from x to* $\mathrm{Bd}(U)$ *in* $\mathrm{Cl}(U)$ *we have that $A \subset S(B, \varepsilon)$, or*

(2) *there exists a decomposition of* $\mathrm{Bd}(U) = R \cup S$ *into disjoint nonempty closed sets R and S such that for every subcontinuum K of X from R to S in* $\mathrm{Cl}(U)$ *we have that $A \subset S(K, \varepsilon)$.*

The following is a local version of Theorem 3.2 in [**5**].

2.3 THEOREM. *Let X be a continuum and A a subcontinuum of X. Then the following are equivalent:*

(i) *A is a W-set in X,*

(ii) *for every Whitney map $\mu\colon C(X) \to [0, \infty)$ and for every subcontinuum Λ of $\mu^{-1}(\mu(A))$ such that $\bigcup \Lambda = X$ we have $A \in \Lambda$,*

(iii) *if $X \subset Q$, the Hilbert cube, and $(Y_i)_{i=1}^{\infty}$ is a sequence of arcs in Q such that $X = \lim Y_i$, then $A \in \liminf C(Y_i)$.*

PROOF. The proof that (i) implies (ii) is very similar to the proof in [**5**, Theorem 3.2] that (a) implies (b).

Suppose A is a subcontinuum of X satisfying (ii). Let $\{Y_i\}_{i=1}^{\infty}$ be a sequence of arcs in Q such that $X = \lim_i Y_i$. Let Λ be a Whitney level of $C(X)$ such that $A \in \Lambda$. By a slight modification of [**17**, Theorem 1] there is a Whitney map $\mu\colon C(Q) \to [0, \infty)$ such that $\Lambda \subset \mu^{-1}(t_0)$ for some t_0. Then Λ is the Whitney level of X with respect to $\mu|C(X)$. Let $\Lambda_i = \mu(t_0) \cap C(Y_i)$ for each i. Since $C(Q)$ is compact we may assume after passing to a subsequence if necessary that $\lim_i \Lambda_i = \Lambda' \subset \Lambda$, where Λ' is a continuum. Then $\bigcup \Lambda' = \lim_i \bigcup \Lambda_i = \lim_i Y_i = X$. Hence, $A \in \Lambda' \subset \lim_i \inf C(Y_i)$. We have proved that (ii) implies (iii).

The proof that (iii) implies (i) is similar to the proof in [**5**, Theorem 3.2] that (c) implies (a).

3. Dimension of hyperspaces. It was proved by Kelley [**9**] that if X is a hereditarily indecomposable continuum of dimension at least 2, then $\dim C(X) = \infty$. Bing proved that each continuum X of dimension ≥ 3 contains a hereditarily indecomposable continuum of dimension ≥ 2. It remains an open question whether each continuum X of dimension ≥ 2 has $\dim C(X) = \infty$. It is well known that if

$\dim C(X) < k$, then X does not contain a k-od [**11**]. It would be very useful to know if Kelley's theorem is true for atriodic continua.

3.1 LEMMA. *Let X be an atriodic continuum, $x \in X$, and $\mu\colon C(X) \twoheadrightarrow [0,1]$ a Whitney map. Then $C_t(x) = \{K \in \mu^{-1}(t) | x \in K\}$ is either an arc or a point.*

PROOF. By Sorgenfrey [**16**, Theorem 1.8] if A_1, A_2, and A_3 are three continua in X with a point in common, then $A_1 \cup A_2 \cup A_3 \subset A_i \cup A_j$ for some $i, j \in \{1,2,3\}$.

Since $C_t(x)$ is compact it follows by a simple maximality argument that there exist $A, B \in C_t(x)$ such that $\bigcup C_t(x) \subset A \cup B$. If $A = B$ there is nothing to prove so we suppose $A \neq B$.

Suppose first that $A \cap B$ is connected and let $C \in C_t(x)\backslash\{A,B\}$. Then $C \subset A \cup B$ and, since A and B are closed, $C\backslash A$ and $C\backslash B$ are separated sets. Since $A, B, C \in \mu^{-1}(t)$ and μ is a Whitney map, $C \not\subset B$ and $C \not\subset A$ so $C\backslash A$ and $C\backslash B$ are nonempty sets. If $A \cap B \not\subset C$, then $C\backslash(A \cap B)$ is connected by [**3**, Proposition 4.4] since $C \not\subset A \cap B$. This is a contradiction. Hence, $A \cap B \subset C$ for each $C \in C_t(x)$. If $C, D \in C_t(x)$ then either $C\backslash B \subset D$ or $D\backslash B \subset C$ by [**16**, Theorem 1.8] since $C \cup D \cup B$ does not contain a triod. Hence, the set $\{C \cap A | C \in C_t(x)\}$ is a chain in $C(X)$. Similarly, $\{C \cap B | C \in C_t(x)\}$ is a chain. It is well known that $C_t(x)$ is compact and arcwise connected. The lemma follows in this case.

If $A \cap B$ is not connected then $A \cap B$ consists of exactly two components K and L (by [**3**, Lemma 4.1]). We may choose A and B so that the component K of x in $A \cap B$ is minimal. By [**3**, Proposition 4.2] A is a minimal continuum with respect to containing K and L. In particular, if R is a continuum in A which meets both K and L, then $A\backslash B \subset R$. If $C \in C_t(x)\backslash\{A,B\}$ and $C \cap L = \varnothing$, then K separates C and, hence, $K \subset C$ by [**3**, Proposition 4.4]. If $C \in C_t(x)\backslash\{A,B\}$ such that C meets L, then $B \cup C = A \cup B$ and C is unicoherent by [**3**, Proposition 4.2]. Since $C \not\subset A$, $K \cup L$ separates C. Since C is unicoherent, either K separates C or L separates C. If K separates C, then $K \subset C$ by [**3**, Proposition 4.4]. If K does not separate C, then L separates C and $L \subset C$ as above. Since $A \not\subset C$ is follows that $K \not\subset C$. Hence, C and B are continua such that $C \cup B = A \cup B$ and the component of $C \cap B$ which contains x is a proper subset of K which contradicts the minimality of K. Hence, if $C \in C_t(x)$, then $K \subset C$. The proof now proceeds as in the case where $A \cap B$ is connected.

3.2 THEOREM. *If X is an atriodic continuum with $\check{H}^1(X) = 0$ and μ is a Whitney map for X, then for each $t \in [0,1]$, $\mu^{-1}(t)$ is an atriodic one-dimensional continuum with $\check{H}^1(\mu^{-1}(t)) = 0$.*

PROOF. If Λ is a subcontinuum of $\mu^{-1}(t)$, then $\bigcup \Lambda$ is a subcontinuum of X. By [**6**, Theorem 4.4] X is one-dimensional and, hence, $\check{H}^1(\bigcup \Lambda) = 0$. Also, $\bigcup \Lambda$ is atriodic and one-dimensional. By a theorem of Davis [**1**] $\bigcup \Lambda$ is in Class (W). By [**5**, Theorem 3.2] Λ is a Whitney level of $\bigcup \Lambda$. By [**11**, 14.73.3] Λ is an irreducible continuum. In particular, Λ is not a triod. Hence, $\mu^{-1}(t)$ is atriodic.

To see that $\check{H}^1(\mu^{-1}(t)) = 0$ let

$$K = \{(A,x) | x \in A \in \mu^{-1}(t)\} \subset \mu^{-1}(t) \times X.$$

Let $\pi_1\colon K \to \mu^{-1}(t)$ and $\pi_2\colon k \to X$ be the coordinate projections. By Lemma 3.1 $\pi_2^{-1}(x)$ is an arc or a point for each $x \in X$. By the Hurewicz Theorem, $\dim(K) \leq 2$.

Since $\check{H}^1(X) = 0$, $\check{H}^1(K) = 0$ by the Vietoris Mapping Theorem. Since π_1 is monotone, the homomorphism $\pi_1^*\colon \check{H}^1(\mu^{-1}(t)) \to \check{H}^1(K)$ is a monomorphism by [**2**]. Thus, $\check{H}^1(\mu^{-1}(t)) = 0$. By [**6**] $\mu^{-1}(t)$ is one-dimensional.

3.3 PROPOSITION. *If X is an atriodic continuum and $\check{H}^1(X) = 0$, then* $\dim(C(X)) \leq 2$.

PROOF. For each $t \in [0,1]$, $\mu^{-1}(t)$ is a continuum of dimension no more than 1. The proposition follows from the Hurewicz Theorem.

Proposition 3.3 had been proved in [**13**] for tree-like continua. The theorem that follows generalizes Proposition 3.3. It also generalizes a result of Rogers [**15**] for hereditarily indecomposable continua.

3.4 THEOREM. *If X is an atriodic continuum such that $\check{H}^1(X)$ has finite rank, then* $\dim(C(X)) \leq 2$.

PROOF. By [**6**] $\dim X \leq 1$. Let $\mu\colon C(X) \to [0,1]$ be a Whitney map. By the Hurewicz Theorem it suffices to prove that $\dim(\mu^{-1}(t)) \leq 1$ for each $t \in [0,1]$. This follows by Theorem 3.2 in case $\check{H}^1(X) = 0$.

Suppose the above is true for all atriodic continua X such that $\check{H}^1(X)$ has rank $\leq n$. Let X be an atriodic continuum such that $\check{H}^1(X)$ has rank $n+1$, let $\mu\colon C(X) \to [0,1]$ be a Whitney map, and $t \in [0,1]$.

Suppose first that for each proper subcontinuum K of X, $\operatorname{rank}(\check{H}^1(K)) \leq n$. Let $\{U_i\}_{i=1}^{\infty}$ be a basis of open sets for X such that $\lim_i \operatorname{diameter}(U_i) = 0$. Then $\{X\backslash U_i\}_{i=1}^{\infty}$ is a sequence of closed proper subsets of X such that

$$C(X) = \bigcup_{i=1}^{\infty} C(X\backslash U_i) \cup \{X\}.$$

For each i each component K of $X\backslash U_i$ is atriodic and $\operatorname{rank}(\check{H}^1(K)) \leq n$. By induction $\dim(\mu^{-1}(t) \cap C(K) \leq 1$. Hence, $\dim(\mu^{-1}(t) \cap C(X\backslash U_i)) \leq 1$ since the components of $\mu^{-1}(t) \cap C(X\backslash U_i)$ are exactly the sets $\mu^{-1}(t) \cap C(K)$, where K is a component of $X\backslash U_i$. By the Sum Theorem for Dimension

$$\dim \mu^{-1}(t) = \dim \left[\bigcup_{i=1}^{\infty} (\mu^{-1}(t) \cap C(X\backslash U_i)) \cup \{X\} \right] = 1.$$

Let $f\colon X \to S^1$ be an essential map and let A be a subcontinuum of X such that f is irreducibly essential on A. Let E be a proper subcontinuum of A. To prove that $\dim(\mu^{-1}(t) \cap C(A)) \leq 1$ it suffices to prove by the above paragraph that $\check{H}^1(E)$ has rank $\leq n$. Just suppose that there exist linearly independent maps $g_1, \ldots, g_{n+1}\colon E \to S^1$. By [**6**, Corollary 3.3] there exists for each $i = 1, \ldots, n+1$ a continuous extension $\hat{g}_i\colon X \to S^1$ of g_i. Then $\{\hat{g}_1, \ldots, \hat{g}_{n+1}, f\}$ is linearly dependent. There exist integers $\alpha_1, \ldots, \alpha_{n+2}$ not all zero such that

$$g = \hat{g}_1^{\alpha_1} \cdot \ldots \cdot \hat{g}_{n+1}^{\alpha_{n+1}} \cdot f^{\alpha_{n+2}}$$

is inessential. Thus,

$$g|E = g_1^{\alpha_1} \cdot \ldots \cdot g_{n+1}^{\alpha_{n+1}} \cdot (f|E)^{\alpha_{n+2}}$$

is inessential. Also, $f|E$ is inessential so $g_1^{\alpha_1} \cdot \ldots \cdot g_{n+1}^{\alpha_{n+1}}$ is inessential. Since

$\{g_1, \ldots, g_{n+1}\}$ is linearly independent $0 = \alpha_1 = \cdots = \alpha_{n+1}$ and, hence, $\alpha_{n+2} \neq 0$. Since $\check{H}^1(X)$ is torsion free and f is essential, $f^{\alpha_{n+2}}$ is essential. This is a contradiction.

Now, let D be a continuum in $X \backslash A$. By [**6**] we may suppose that f is an extension of $f|A$ to X such that $f|D$ is inessential. By the same proof as above $\check{H}^1(D)$ has rank $\leq n$.

By [**12**, Lemma 8] there exist two points p, $q \in A$ such that if $B \in C(X)$ with $B \neq B \cap A \neq \varnothing$, then $\{p, q\} \cap B \neq \varnothing$. Hence,

$$\mu^{-1}(t) = [\mu^{-1}(t) \cap C(X \backslash A)] \cup \{B \in \mu^{-1}(t) | \{p, q\} \cap B \neq \varnothing\} \\ \cup (\mu^{-1}(t) \cap C(A)).$$

By Lemma 3.1 $\{B \in \mu^{-1}(t) | \{p, q\} \cap B \neq \varnothing\}$ is compact and contained in the union of two arcs and, so, is at most one-dimensional. For each positive integer i let $X_i = X \backslash S(A, i^{-1})$. Then,

$$\mu^{-1}(t) \cap C(X \backslash A) = \mu^{-1}(t) \cap \bigcup_{i=l}^{\infty} C(X_i).$$

If K is a component of X_i then rank $\check{H}^1(K) \leq n$ so $\dim(\mu^{-1}(t) \cap C(K)) \leq 1$. Each component of $\mu^{-1}(t) \cap C(X_i)$ has the form $\mu^{-1}(t) \cap C(K)$, where K is a component of X_i. Hence, $\dim(\mu^{-1}(t) \cap C(X_i)) \leq 1$. By the Sum Theorem for Dimension $\dim(\mu^{-1}(t)) \leq 1$.

Problem. Let X be an atriodic continuum embeddable in the plane. Is $\dim(C(X)) \leq 2$? Krasinkiewicz [**10**] has given a positive solution in case X is hereditarily indecomposable.

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