We recall that given an uncountable cardinal $\kappa$, $\Box_\kappa$ asserts the existence of a family $s_\alpha \subseteq \alpha$, $\alpha < \kappa$, such that the set $\{\alpha < \kappa : A \cap \alpha = s_\alpha\}$ is stationary in $\kappa$ for all $A \subseteq \kappa$.

It occurred to us that the implication (iv)$\rightarrow$(i) in Proposition 4 of [4] needs not hold for $\kappa > \omega_1$. In that case, a slight modification of the original argument yields that (i)$\rightarrow$(iii) in the proposition are indeed equivalent, and that they are equivalent to this stronger form of (iv): There exists a family $Z_\alpha \in (\kappa)^2$, $\alpha < \kappa$, such that the diagonal intersection $\Delta\{Z_\alpha(h(\alpha)) : \alpha < \kappa\}$ is stationary for every $h \in 2^\kappa$.

The incorrect proof relied on the claim, p. 39 of [1], that assuming $\kappa$ is regular, $0^K$ follows from the existence of a sequence $s_\alpha \subseteq \alpha$, $\alpha < \kappa$, such that whenever $A \subseteq k$, there is an infinite $\alpha$ with $A \cap a = s_\alpha$. That this is indeed the case when $\kappa = \omega_1$ was shown by Devlin in [2]. However, this cannot be true in general, as it would imply that $0^K$ holds whenever $\kappa$ is regular and $\Box_\lambda$ holds for some uncountable cardinal $\lambda < \kappa$. It is nevertheless possible to generalize Devlin's result as follows:

**PROPOSITION.** Let $\lambda, \kappa$ be infinite cardinals with $2^\lambda \geq \kappa$. Assume there are $P_\alpha$, $\alpha < \kappa$, such that each $P_\alpha$ is a collection of size $\leq |\alpha|$ of subsets of $\alpha$, and that for every $A \subseteq \kappa$, there is an $\alpha \geq \lambda$ with $A \cap \alpha \in P_\alpha$. Then $\Box_\kappa$ holds.

**PROOF.** Let $P_\alpha$, $\alpha < \kappa$, be as in the statement of the proposition. By a well-known result of Kunen [3], it is enough to show that $\kappa$ is regular and there exist $Q_\alpha$, $\alpha < \kappa$, such that each $Q_\alpha$ is a collection of size $\leq |\alpha|$ of subsets of $\alpha$, and the set $\{\alpha : B \cap \alpha \in Q_\alpha\}$ is stationary in $\kappa$ for all $B \subseteq \kappa$. Define functions $i,j$ from $\kappa$ to $\kappa$ by letting $i(\alpha) = \lambda \alpha$ and $j(\alpha) = 2\alpha$. Set $N = \kappa - j[\kappa]$. For each $\alpha \in [\lambda, \kappa)$, denote by $Q_\alpha$ the collection of all those subsets $D$ of $\alpha$ such that there are $\beta < \lambda$ and $H \in P_{\alpha+\beta}$ with $D = j^{-1}[H \cap \alpha]$. Fix $B \subseteq \kappa$, and let $C$ be a closed unbounded subset of $\kappa$. Let $c_\beta$, $0 < \beta < \rho$, be the increasing enumeration of the set $C \cap i[\kappa - 2]$, and put $c_0 = 0$. Choose $E_\beta \subseteq N$, $\beta < \rho$, with the following properties: $E_\beta \subseteq N \cap [c_\beta, c_\beta + \lambda)$, and $E_\beta \neq H \cap N \cap [c_\beta, c_\beta + \lambda)$ whenever $H \in P_\alpha$ with $\alpha \in [c_\beta + \lambda, c_\beta + 1)$. Then set $A = j[B] \cup \left(\bigcup_{\beta < \rho} E_\beta\right)$. Select $\alpha \geq \lambda$ with $A \cap \alpha \in P_\alpha$. It is easily verified that $B \cap c_\beta \in Q_{c_\beta}$, where $\beta$ is such that $\alpha \in [c_\beta, c_\beta + \lambda)$. It only remains to show that $\kappa$ is regular. First note that $2^\mu = \kappa$ holds for every cardinal $\mu \in [\lambda, \kappa)$, as the set $\bigcup_{\alpha \geq \lambda} Q_\alpha$ has size $\kappa$. Thus $\text{cf} \kappa = \text{cf}(2^\mu) > \mu$ for all $\mu \in [\lambda, \kappa)$, and consequently $\text{cf} \kappa = \kappa$.

We remark that Theorem 4 (where (b) should read $\Diamond_{\kappa^+}(\lambda^+ - \lambda)$) of [5] is the special case of our result when $\kappa = \lambda^+$. Also, a straightforward modification of the proof of the proposition yields the implication c)$\rightarrow$a) of Theorem 3 of [5].

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Finally, we would like to point out that in [4], Proposition 3 easily follows from Proposition 1, by the following remark: given a cardinal \( \kappa \) and a family \( A_\alpha \in [\kappa]^\kappa \), \( \alpha < \kappa \), with the property that \( A_\alpha \subseteq A_\beta \) whenever \( \beta < \alpha \), there is a \( B \in [\kappa]^\kappa \) such that \( |B - A_\alpha| < \kappa \) for all \( \alpha \).

REFERENCES

2. _, Variations on \( \diamond \), J. Symbolic Logic 44 (1979), 51–58.
3. R. B. Jensen and K. Kunen, Some combinatorial properties of \( L \) and \( V \), unpublished manuscript.