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A KRASNOSEL'SKII-TYPE THEOREM INVOLVING p-ARCS

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ABSTRACT. Let p be a point in E_2. A convex arc joining a pair of distinct points x and y in E_2 is called a p-arc if it is contained in the simplex with vertices x, y, and p. In this paper, we prove the following Krasnosel'skii-type theorem: *Let S be a compact simply connected set in E_2 and let p be a point not in S. If for each three points x_1, x_2, and x_3 of S there exists at least one point $y \in S$ such that y and x_i ($i = 1,2,3$) can be joined by p-arcs in S, then there exists a point $k \in S$ such that every point $x \in S$ can be joined to k by some p-arc in S.*

1. Introduction. The well-known theorem of Krasnosel'skii [**2**] includes the following result: If for each three points x_1, x_2, and x_3 of a compact connected set S in a Euclidean plane E_2 there exists at least one point $y \in S$ such that the line segments $x_iy \subseteq S$ ($i = 1,2,3$), then S is starshaped. A comprehensive discussion of Krasnosel'skii's theorem is included in Valentine [**5**]. In this paper, we prove a Krasnosel'skii-type theorem involving p-arcs relative to a point p in E_2 as conjectured in Stanek [**4**].

2. The main theorem. The concept of a p-arc and notations follow Stanek [**4**]. Briefly, let p be a point in E_2. A convex arc $C(x,y)$ joining a pair of distinct points x and y in E_2 is called a p-arc if it is contained in the simplex $\Delta(p,x,y)$ with vertices p, x, and y. In what follows, $C(x,y) = C(y,x)$, and it will always denote a p-arc. We note that a subarc of a p-arc is a p-arc. $L(p,x)$ denotes the line through p and x, px is the closed line segment joining p and x, and $\operatorname{intv} px$ denotes the set $px - \{p,x\}$. For a set $S \subseteq E_2$, $\operatorname{compl} S$, $\operatorname{bd} S$, $\operatorname{int} S$, and $\operatorname{conv} S$ denote the complement of S, the boundary of S, the interior of S, and the convex hull of S in E_2, respectively.

THEOREM 1. *Let S be a simply connected compact set in E_2 and let p be a point not in S. If for each three points x_1, x_2, and x_3 of S there exists at least one point $y \in X$ such that there exist p-arcs in S joining x_i and $y \in S$ ($i = 1,2,3$), then there exists a point $k \in S$ such that every point $x \in S$ can be joined to k by some p-arc in S.*

The proof of Theorem 1 makes use of the following theorem of Molnar [**3**]. A proof of Molnar's theorem is included in Buchman [**1**].

THEOREM 2 (MOLNAR). *A family of three or more simply connected compact sets in E_2 has a nonempty intersection if every two of its members have an*

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arcwise connected intersection and if every three of its members have a nonempty intersection.

The scheme of the proof of Theorem 1 is as follows. Assume that the set S is not a singleton and note that our hypotheses imply that S is connected. For each point $x \in S$, let $S(x)$ be the subset of S consisting of the point x and every point $t \in S$ that can be joined to x by a p-arc in S. We will show by several lemmas that the family $F = \{S(x)\colon x \in S\}$ satisfies the hypotheses of Molnar's theorem. First, we apply Blaschke's convergence theorem [**5**] to conclude that each member of F is compact in Lemma 1, then we show that each member of F is simply connected by topological arguments in Lemma 2. The hypotheses of Theorem 1 imply that every three members of F have a nonempty intersection. Next, using the fact that S is simply connected, we show that every two members of F have an arcwise connected intersection in Lemma 4. Finally, we apply Molnar's theorem to complete the proof of Theorem 1.

3. Proofs. In what follows, we use the notation in §2 and assume the hypotheses of Theorem 1.

LEMMA 1. *For each $x \in S$, $S(x)$ is compact.*

PROOF. Let x be an arbitrary point of S and suppose $\{t_n\}$ is an infinite sequence of points in $S(x)$. Let $C_n = C(t_n, x)$ denote a p-arc joining t_n to x in S for $n = 1, 2, 3, \ldots$. It follows from Blaschske's convergence theorem [**5**] that there exists a subsequence of p-arcs $C_{n_i} = C(t_{n_i}, x)$ of $\{C_n\}$ converging to $C_0(t_0, x)$, which is a p-arc joining t_0 and x or a single point $t_0 = x$. Since S is closed, this limit lies in S. In either case, the sequence $\{t_{n_i}\}$ converges to t_0 lying in $S(x)$. Hence, $S(x)$ is compact.

LEMMA 2. *For each $x \in S$, $S(x)$ is simply connected.*

PROOF. Let $x \in S$ be an arbitrary point. We will show that $\operatorname{compl} S(x)$ has no nonempty bounded components. Suppose $H \neq \varnothing$ is a bounded component in $\operatorname{compl} S(x)$. We note that $\operatorname{bd} H \subseteq S(x)$ because H is a component of $\operatorname{compl} S(x)$. We claim that H is a subset of S. To prove this, let $h \in H$ and assume $h \notin S$. Since $h \in \operatorname{compl} S \subseteq \operatorname{compl} S(x)$, the component of K of $\operatorname{compl} S$ containing h is a subset of H. Thus, K is a nonempty bounded component of $\operatorname{compl} S$. This contradicts the fact that S is simply connected. Therefore, $H \subseteq S$ holds.

Since $\operatorname{compl} S(x)$ is open by Lemma 1 and since a component of an open set is open, H is open. Choose a point $y \in H$ and a line segment sw on $L(p, y)$ such that $y \in \operatorname{intv} sw \subseteq H$ with $s \in \operatorname{bd} H$ and $w \in \operatorname{bd} H$. Since s and w are in $\operatorname{bd} H \subseteq S$ and $\operatorname{intv} sw \subseteq H \subseteq S$, we have $sw \subseteq S$. If x is collinear with p, s, and w, then the points s and w in $\operatorname{bd} H \subseteq S(x)$ can be joined to x by p-arcs $C(s, x) = sx$ and $C(w, x) = wx$ in S, respectively. It follows that $y \in S(x)$, contradicting the fact $y \in H \subseteq \operatorname{compl} S(x)$. Therefore, we assume that x is noncollinear with p, s, and w. We note that $p \notin sw$ since $p \notin S$ and $sw \subseteq S$. Without loss, we can assume that $s \in \operatorname{intv} pw$. Let $C_1(s, x)$ be a p-arc in S joining s and x. Then the set $ys \cup C_1(s, x)$ is a p-arc in S joining y and x. This shows that $y \in S(x)$, contradicting the fact $y \in H \subseteq \operatorname{compl} S(x)$. Hence, H must be empty and $S(x)$ is simply connected.

REMARK (1). If two distinct points x and y in S can be joined by a p-arc in S, then there exists a unique minimal p-arc $C_0(x, y)$ joining x and y in S in the sense that $\operatorname{conv} C_0(x,y) \subseteq \operatorname{conv} C(x,y)$ for any other p-arc $C(x,y)$ joining x and y in S. To see this, first assume $xy \not\subseteq S$. Let $\{C(x,y)\}$ be the collection of all p-arcs joining x and y in S. This collection is nonempty. Now set $C_0(x,y) = \operatorname{bd}[\bigcap \operatorname{conv} C(x,y)] - \operatorname{intv} xy$ where the intersection is taken over all members of the collection. Since S is compact, we have $C_0(x,y) \subseteq S$. Clearly, $C_0(x,y)$ is the unique minimal p-arc joining x and y in S. If $xy \subseteq S$, then xy is the unique miminal p-arc required.

REMARK (2). If u and v are points on the minimal p-arc $C(x,y)$, then we see that the unique minimal p-arc joining u and v in S is the subarc of $C(x,y)$ joining u and v. For simplicity, we will use the notation $C(u,v) \subseteq C(x,y)$ to indicate that $C(u,v)$ is the subarc of $C(x,y)$ that connects u and v.

REMARK (3). If x, y, and z are distinct points in S and if $C(x,y)$ and $C(z,y)$ are minimal p-arcs in S, then either $C(x,y) \cap C(z,y) = \{y\}$ or there exists a point v such that $C(x,y) \cap C(z,y) = C(y,v)$ where $C(y,v)$ is the minimal p-arc in S joining y and v. To see this, we assume $C(x,y) \cap C(z,y)$ contains more than one point, for otherwise $C(x,y) \cap C(z,y) = \{y\}$ holds. Let $t \in [C(x,y) \cap C(z,y)] - \{y\}$. By Remark (2) above, a minimal p-arc $C(t,y)$ exists in S and $C(t,y) \subseteq C(x,y) \cap C(z,y)$. Since t is an arbitrary point in $[C(x,y) \cap C(z,y)] - \{y\}$, the set $C(x,y) \cap C(z,y)$ is connected. Hence, there exists a point v such that $C(y,v) = C(x,y) \cap C(z,y)$ where $C(y,v)$ is the minimal p-arc in S joining y and v. Note that we may have $C(v,y) = C(v,y') \cup yy'$ where $C(v,y') \subseteq C(x,y)$ and $yy' \subseteq py$ (see Figure 1).

DEFINITION. Let x, y, and z be distinct points of S with minimal p-arcs $C(x,y)$, $C(y,z)$, and $C(x,z)$ in S joining the indicated pairs of points. The compact set bounded by $C(x,y) \cup C(y,z) \cup C(x,z)$ is called the geodesic triangle, denoted by $T(x,y,z)$, determined by x, y, and z relative to S.

DEFINITION. Let $A \subseteq E_2$ be a compact set not containing p, and let $x \in A$. Let b_x be the point on $L(p,x) \cap A$ closest to x. The set $\{b_x : x \in A\}$ is called the inner boundary of A relative to p, or simply the inner boundary of A.

The next two lemmas will establish the fact that any two members of the family F have an arcwise connected intersection.

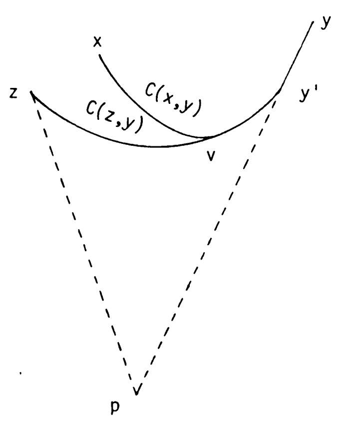

FIGURE 1

LEMMA 3. *Let x, y, and z be distinct points in S and let $C(x,y)$, $C(y,z)$ and $C(x,z)$ be the minimal p-arcs in S joining the indicated pairs of points. Then the inner boundary of the geodesic triangle $T(x,y,z)$ is a p-arc. Furthermore, if $t \in C(x,y)$, then there exists a p-arc $C(t,z)$ in S joining t and z.*

PROOF. As mentioned in Remark (3), there exist points u, v, and w such that $C(x,y) \cap C(x,z) = C(x,u)$, $C(x,y) \cap C(y,z) = C(y,v)$, and $C(x,z) \cap C(y,z) = C(z,w)$ where $C(x,u)$, $C(y,v)$, and $C(z,w)$ are minimal p-arcs in S (see Figure 2). Claim $T(u,v,w) \subseteq \Delta(u,v,w)$. If not, then at least one of the p-arcs in $\operatorname{bd} T(u,v,w)$, say $C(u,w)$, lies outside of $\Delta(u,v,w)$ except for the endpoints u and w. Thus the minimal p-arc $C(u,w)$ is not uw, and $uw \not\subseteq S$. Since S is simply connected, $T(u,v,w) \subseteq S$, so $uw \not\subseteq T(u,v,w)$. It follows that $C(u,v) \cup C(v,w)$ must cross $\operatorname{intv} uw$ and intersect $\operatorname{int}\operatorname{conv} C(u,w)$. Let

$$K = \operatorname{conv}\{\operatorname{conv} C(u,w) \cap [C(u,v) \cup C(v,w)]\}$$

(see Figure 2). Since u, v, and w are the only points on more than one of the boundary arcs of $T(u,v,w)$, K is a proper subset of $\operatorname{conv} C(u,w)$. Set $C_1(u,w) = \operatorname{bd} K - \operatorname{intv} uw$. From the construction, $C_1(u,w)$ is clearly a p-arc. The existence of $C_1(u,w)$ contradicts the minimality of $C(u,w)$. Hence, $T(u,v,w) \subseteq \Delta(u,v,w)$.

Let B_0 denote the inner boundary of $T(u,v,w)$. Since u, v, and w are the only points on more than one of the boundary arcs of $T(u,v,w)$, B_0 consists of one or two of the boundary arcs. Since $T(u,v,w) \subseteq \Delta(u,v,w)$ and since the boundary arcs are p-arcs, B_0 consists of one or two line segments. For example, $B_0 = uw$ or $B_0 = uw \cup vw$.

In case $B_0 = uw$, the inner boundary of $T(x,y,z)$ is $C(x',z') = C(x',u) \cup uw \cup C(w,z')$ where $x' \in px$, $z' \in pz$, $C(x',u) \subseteq C(x,y)$, and $C(w,z') \subseteq C(y,z)$. Note that xx', zz', $C(x',u)$ or $C(w,z')$ may reduce to a singhle point. Clearly $C(x',z')$ is a p-arc (see Figure 3).

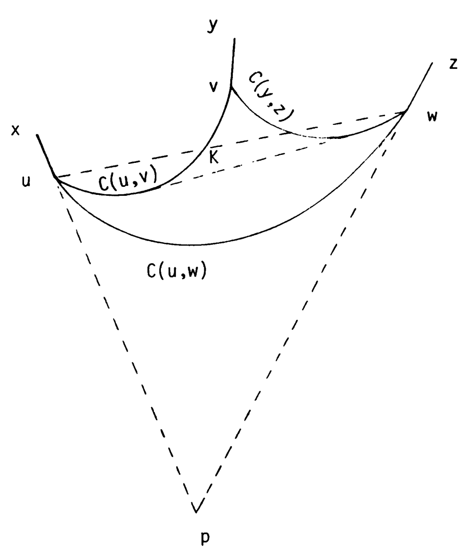

FIGURE 2

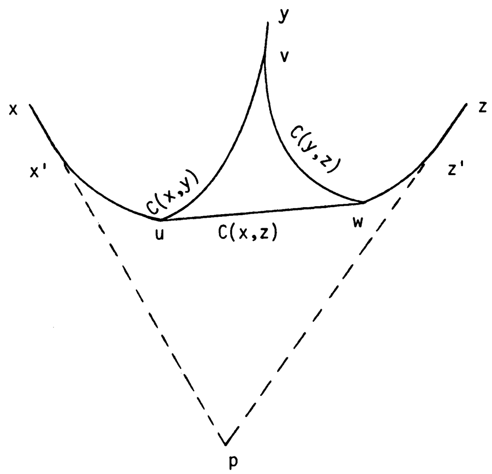

FIGURE 3

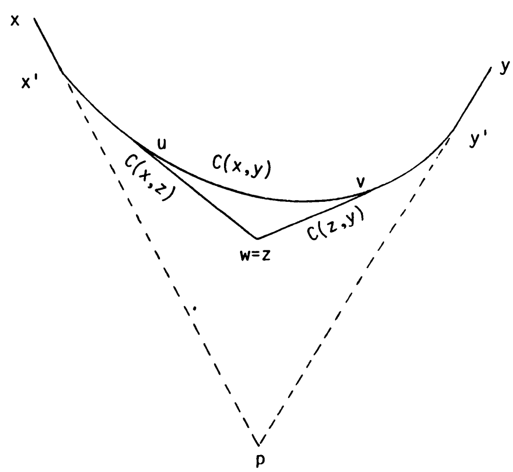

FIGURE 4

In case $B_0 = uw \cup vw$, then $w = z$. The inner boundary of $T(x,y,z)$ is $C(x',z) \cup C(z,y')$ where $x' \in px$, $y' \in py$, $u \in C(x',z) \subseteq C(x,z)$, and $v \in C(z,y') \subseteq C(z,y)$. Clearly, the inner boundary of $T(x,y,z)$ is a p-arc (see Figure 4).

Since S is a simply connected set and since the p-arcs $C(x,y)$, $C(y,z)$, and $C(x,z)$ lie in S, the region R bounded by these three p-arcs is a subset of S. Let $D = R \cup C(x,y) \cup C(y,z) \cup C(x,z)$. Clearly $D \subseteq S$. Let B denote the inner boundary of $T(x,y,z)$. Since S is closed, $B \subseteq S$. Now, suppose $t \in C(x,y)$. Let $\{b\} = pt \cap B$, and let $C(t,z) = tb \cup C(b,z') \cup zz'$ where tb may reduce to a single point, $C(b,z')$ is a subarc of the p-arc B, $zz' \subseteq pz \cap C(x,z)$, and zz' may reduce to a single point z. Since $C(u,v)$, $C(v,w)$, and $C(u,w)$ are p-arcs, $tb \subseteq D$. Clearly, $C(t,z)$ is a p-arc lying in $D \subseteq S$ joining t and z as required. (See Figure 5 for a typical configuration.)

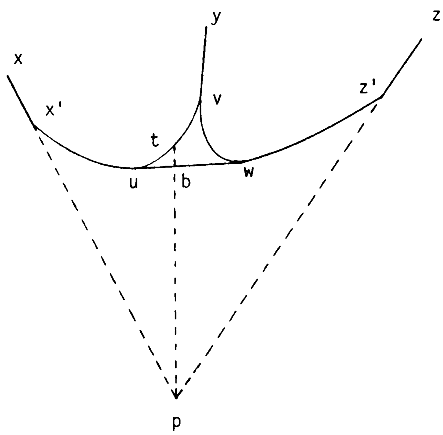

FIGURE 5

LEMMA 4. *For distinct points x and y in $\mathcal{S}$, the set $S(x) \cap S(y)$ is arcwise connected.*

PROOF. The hypotheses of Theorem 1 imply that $S(x) \cap S(y) \neq \emptyset$. If $S(x) \cap S(y)$ is a singleton, it is trivially arcwise connected. Hence, assume $S(x) \cap S(y)$ is not a singleton and let a and b be two arbitrary distinct points in $S(x) \cap S(y)$. Let $C(x,a)$, $C(x,b)$, $C(y,a)$, and $C(y,b)$ be p-arcs in S and assume these p-arcs to be minimal. If at least one of $a = x$, $a = y$, $b = x$, and $b = y$ holds, then $S(x) \cap S(y)$ is arcwise connected. For example, if $a = x$, then $C(x,a) = x = a$ and $C(x,b) = C(a,b) \subseteq S(x)$. Moreover, $C(a,b) \subseteq S(y)$ by Lemma 3. Hence, $C(a,b) \subseteq S(x) \cap S(y)$ showing that $S(x) \cap S(y)$ is arcwise connected. Therefore, we assume that the points a, b, x, and y are all distinct.

Without loss of generality, we may assume that four p-arcs $C(x,a)$, $C(y,a)$, $C(x,b)$, and $C(y,b)$ intersect in common endpoints only. Furthermore, we assume that

$$C(x,a) \cap \operatorname{bd} \Delta(p,x,a) = \{x,a\}, \quad C(y,a) \cap \operatorname{bd} \Delta(p,y,a) = \{y,a\},$$

$$C(x,b) \cap \operatorname{bd} \Delta(p,x,b) = \{x,b\}, \quad C(y,b) \cap \operatorname{bd} \Delta(p,y,b) = \{y,b\}.$$

If $C(x,a) \cap C(y,b) \neq \emptyset$ or $C(x,b) \cap C(y,a) \neq \emptyset$, then it follows easily from Lemma 3 that $S(x) \cap S(y)$ is arcwise connected. For the rest of this proof, we assume that $C(x,a) \cap C(y,b) = \emptyset$ and $C(x,b) \cap C(y,a) = \emptyset$. We shall refer to these two conditions as the nonintersection conditions. Let G denote the compact set bounded by these four p-arcs and let B denote the inner boundary of G relative to p. Since S is simply connected, $G \subseteq S$. Note that $p \notin G$ since $p \notin S$. Because of the nonintersection conditions and the mimimality of the p-arcs, it is easy to see that at least one of the points a and b belongs to B. For definiteness, assume $a \in B$. The remaining points b, x, and y may or may not lie in B. There are the following cases:

(1) $b \in B, x \in B, y \in B$; (2) $b \in B, x \in B, y \notin B$; (3) $b \in B, x \notin B, y \in B$;
(4) $b \notin B, x \in B, y \in B$; (5) $b \in B, x \notin B, y \notin B$; (6) $b \notin B, x \in B, y \notin B$;
(7) $b \notin B, x \notin B, y \in B$; (8) $b \notin B, x \notin B, y \notin B$.

Cases (5) and (6) cannot happen due to the nonintersection conditions. Since case (2) is similar to case (3), and case (6) is similar to case (7), we need only consider cases (1), (2), (4), and (6). In each case, we will find an arc C joining a and b such that $C \subseteq S(x) \cap S(y)$.

Case (1). The points a, b, x, and y all lie in B. Without loss, assume $B = C(x,a) \cup C(x,b) \cup C(y,b)$. Let $C = C(a,b_1) \cup bb_1$ where $\{b_1\} = L(p,b) \cap C(a,y)$ and $C(a,b_1) \subseteq C(y,a)$. Note that $b_1 \neq y$ since $C(y,b) \subseteq B$. Clearly $C \subseteq S$. Next, we show $C \subseteq S(x) \cap S(y)$. Since $C(a,b_1) \subseteq C(y,a)$, we have $C(a,b_1) \subseteq S(y)$. Also, $bb_1 \subseteq S(y)$ by Lemma 3. Therefore, $C \subseteq S(y)$. For any point $t \in C$, let $\{t_0\} = L(p,t) \cap B$. The arc $tt_0 \cup C(x,t_0)$ is a p-arc joining t and x in $G \subseteq S$. Hence, $C \subseteq S(x)$. Thus, we have $C \subseteq S(x) \cap S(y)$. (See Figure 6 for a typical configuration.)

Case (2). $a \in B, b \in B, x \in B, y \notin B$. Because of the nonintersection conditions, we conclude that a and b lie on distinct half-planes determined by $L(p,x)$. In this

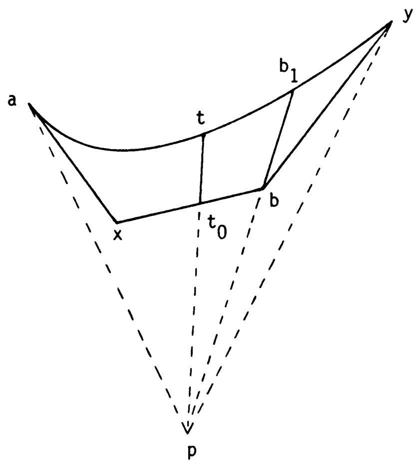

FIGURE 6

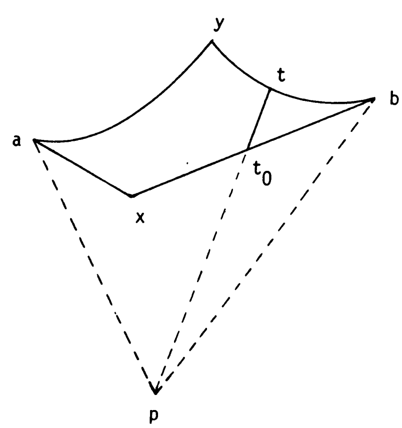

FIGURE 7

case, $B = C(x,a) \cup C(x,b)$. Let $C = C(y,a) \cup C(y,b)$. Clearly, $C \subseteq S(y)$. To show $C \subseteq S(x)$, let $t \in C$. Let $\{t_0\} = pt \cap B$. The arc $tt_0 \cup C(x,t_0)$ is a p-arc in $G \subset S$ joining t and x where either $C(x,t_0) \subseteq C(x,a)$ or $C(x,t_0) \subseteq C(x,b)$. Hence, $C \subseteq S(x)$ and $C \subseteq S(x) \cap S(y)$. (See Figure 7 for a typical configuration.)

Case (4). $b \notin B$, $a \in B$, $x \in B$, $y \in B$. Note that x and y must lie on distinct half-planes determined by $L(p,a)$ because of the nonintersection conditions. In this case, $B = C(x,a) \cup C(y,a)$. Let $C = C(a_1,b) \cup aa_1$ where $\{a_1\} = L(p,a) \cap [C(x,b) \cup C(y,b)]$ and $C(a_1,b) \subseteq C(x,b)$ or $C(a_1,b) \subseteq C(y,b)$. Clearly, $C \subseteq S$. If $C(a_1,b) \subseteq C(x,b)$, then $C(a_1,b) \subseteq S(x)$. By Lemma 3, $aa_1 \subseteq S(x)$. Hence, $C \subseteq S(x)$. For $t \in C$, let $t_0 = L(p,t) \cap C(a,y)$. The set $tt_0 \cup C(t_0,y)$ is a p-arc in S joining t and y where $C(t_0,y) \subseteq C(a,y)$. Hence, $C \subseteq S(y)$ and $C \subseteq S(x) \cap S(y)$. The proof is similar if $C(a_1,b) \subseteq C(y,b)$. (See Figure 8 for a typical configuration.)

Case (6). $a \in B$, $x \in B$, $b \notin B$, $y \notin B$. Note that $B = C(x,a)$. The typical configurations are shown in Figures 9, 10, and 11.

For type (i), let $C = C(x,a) \cup C(x,b)$. We show $C \subseteq S(y)$. If w is an arbitrary point in C and $w \in C(x,a)$, then $yy_0 \cup C(w,y_0)$ is a p-arc in S joining w and y where $L(p,y) \cap C(x,a) = \{y_0\}$ and $C(w,y_0) \subseteq C(x,a)$. Hence, $w \in S(y)$ and $C(x,a) \subseteq S(y)$ (see Figure 9). If t is an arbitrary point in C and $t \in C(x,b)$, let $\{t_0\} = L(p,t) \cap C(x,a)$. The arc $tt_0 \cup C(y_0,t_0) \cup yy_0$ is a p-arc in S joining y and t where $C(y_0,t_0) \subseteq C(x,a)$. Therefore, $t \in S(y)$ and $C(x,b) \subseteq S(y)$. Thus, $C \subseteq S(y)$. Clearly, $C \subseteq S(x)$. Therefore, $C \subseteq S(x) \cap S(y)$ (see Figure 9).

For type (ii), let $C = C(y_1,b) \cup y_1y_0 \cup C(y_0,a)$ where $L(p,y) \cap C(x,b) = \{y_1\}$, $L(p,y) \cap C(x,a) = \{y_0\}$, $C(y_1,b) \subseteq C(x,b)$, and $C(y_0,a) \subseteq C(x,a)$. If t is an arbitrary point in C and $t \in C(y_1,b)$, let $\{t_0\} = L(p,t) \cap C(y,b)$. The arc $tt_0 \cup C(y,t_0)$ is a p-arc in S joining y and t where $C(y,t_0) \subseteq C(y,b)$. Therefore, $t \in S(y)$ (see Figure 10). If w is an arbitrary point in C and $w \in C(y_0,a)$, then the arc $yy_0 \cup C(y_0,w)$ is a p-arc in S joining y and w where $C(y_0,w) \subseteq C(x,a)$. Hence, $w \in S(y)$. Clearly, $y_1y_0 \subseteq S(y)$. In short, $C \subseteq S(y)$ and $C \subseteq S(x) \cap S(y)$ (see Figure 10).

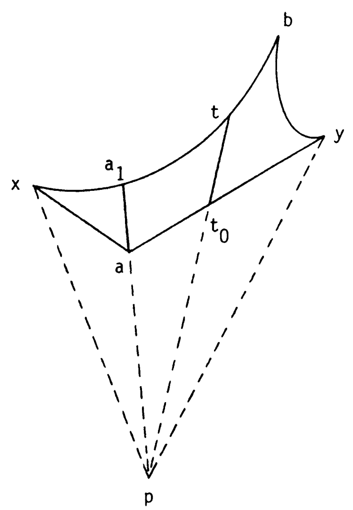

FIGURE 8

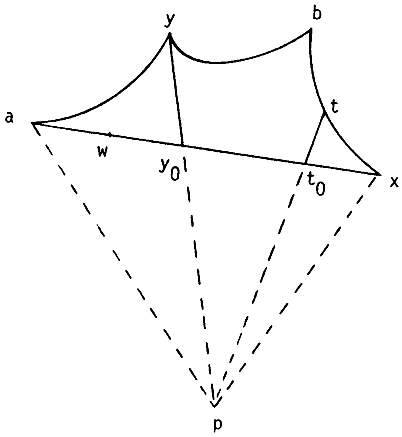

FIGURE 9

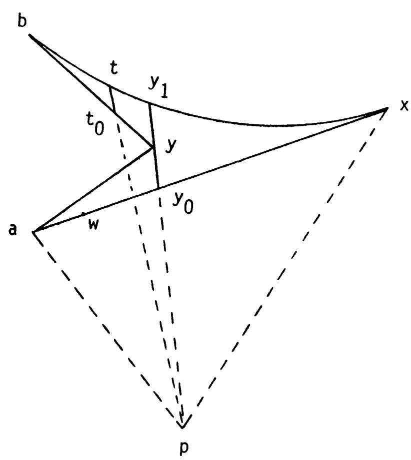

FIGURE 10

For type (iii), note that $bb_0 \cup C(b_0, a)$ is a p-arc in G joining a and b where $L(p, b) \cap C(x, a) = \{b_0\}$ and $C(b_0, a) \subseteq C(x, a)$. Let C be the minimal p-arc in G joining a and b. By Lemma 3, $C \subseteq S(y)$. To show $C \subseteq S(x)$, let $t \in C$. The arc $tt_0 \cup C(t_0, x)$ is a p-arc in S joining t and x where $\{t_0\} = L(p, t) \cap C(x, a)$. Hence $C \subseteq S(x)$ and $C \subseteq S(x) \cap S(y)$ (see Figure 11).

We can now give a proof of Theorem 1.

PROOF. Assume S is not a singleton and note that the hypotheses imply that the set S is connected. For each point $x \in S$, let $S(x)$ denote the subset of S consisting of the point x and every point $t \in S$ that can be joined to x by a p-arc in S. Consider the family $F = \{S(x) \colon x \in S\}$. Each member of F is compact and simply connected by Lemma 1 and Lemma 2, respectively. By Lemma 4, every two members of F have an arcwise connected intersection. The hypotheses imply that every three members of F have a nonempty intersection. By Molnar's theorem (Theorem 2), the members of F have a nonempty intersection K. Let $k \in K$.

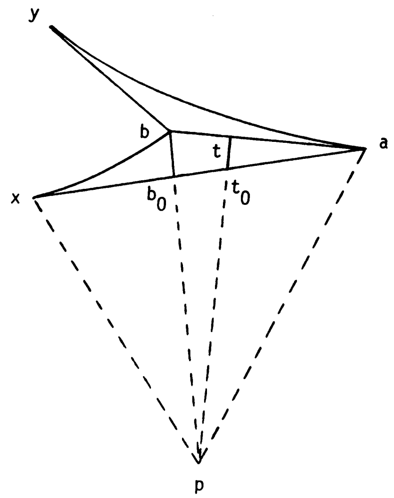

FIGURE 11

Since $k \in S(x)$ for each $x \in S$, there is a p-arc $C(x, k)$ joining x and k in S. This completes the proof of Theorem 1.

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