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WEIGHTED NORM INEQUALITIES FOR BOCHNER-RIESZ SPHERICAL SUMMATION MULTIPLIERS

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ABSTRACT. Sufficient conditions to be satisfied by nonnegative weight functions $\omega(|x|)$ are given in order that the Bochner-Riesz spherical summation multiplier operators restricted to radial functions of R^n be bounded on $L^p(R^n;\omega(|x|)dx)$. For a certain class of weights these conditions are also necessary.

1. Introduction. Let $\hat{f}(\xi)$ denote the Fourier transform of f and let $B = \{\xi\colon |\xi| \le 1\}$ be the unit ball in R^n. For $\lambda \ge 0$ the Bochner-Riesz spherical summation multiplier operator $T_\lambda = T_{n,\lambda}$ is defined by

$$(\widehat{T_\lambda f})(\xi) = (1-|\xi|^2)^\lambda \chi_B(\xi)\hat{f}(\xi)$$

and then [**14**] for suitable f

$$(T_\lambda f)(x) = c_{n,\lambda}\int_{R^n} f(y)\frac{J_{n/2+\lambda}(|x-y|)}{|x-y|^{n/2+\lambda}}\,dy \tag{1.1}$$

where J_ν is the Bessel function of the first kind of order ν.

$L^p(R^n;\omega(x)dx)$, abbreviated $L^p(R^n)$ when $\omega(x) = 1$, denotes the weighted Lebesgue space of measurable functions f on R^n for which $\int_{R^n}|f(x)|^p\omega(x)\,dx < \infty$. The boundedness of T_λ on L^p spaces has been widely studied because of its intimate connection with the inversion of the Fourier transform by Bochner-Riesz means.

If λ exceeds the *critical index* $\lambda_c = (n-1)/2$, (1.1) shows that T_λ is a covolution operator with kernel in $L^1(R^n)$ so T_λ is bounded on $L^p(R^n)$ for all p, $1 \le p \le \infty$; moreover, since in this case $T_\lambda f$ is dominated by a multiple of the Hardy-Littlewood maximal function of f [**15**, Theorem 2, p. 62], a result of B. Muckenhoupt [**13**] shows that T_λ is bounded on $L^p(R^n;\omega(x)\,dx)$, $1 < p < \infty$, if ω satisfies

$$\left(\int_Q \omega(x)\,dx\right)^{1/p}\left(\int_Q \omega(x)^{-p'/p}\,dx\right)^{1/p'} \le C\int_Q dx \tag{$A_p(R^n)$}$$

for all cubes $Q \subset R^n$ with sides parallel to the coordinate axis.

When $n = 1$, T_0 reduces essentially to the Hilbert transform and hence T_0 is bounded on $L^p(R^1;\omega(x)\,dx)$ if (and only if) ω satisfies $A_p(R^1)$, (see [**10**]) while for $n > 1$ C. Fefferman [**6**] has shown that T_0 is bounded on $L^p(R^n)$ only if $p = 2$.

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For $0 < \lambda \leq \lambda_c$, the boundedness properties of T_λ are not yet fully understood. Herz [**8**] has shown that T_λ is unbounded if $p \leq p_0$ or $p \geq p_0'$ where $p_0 = p_0(\lambda, n) = 2n/(n+1+2\lambda)$ and it has been conjectured that T_λ is bounded for $p_0 < p < p_0'$. The conjecture has been verified for $n = 2$ by L. Carleson and P. Sjölin [**3**] but for $n \geq 3$ it has been verified only for $\lambda > (n-1)/2(n+1)$, see [**5**, **7**, **17**].

The restriction of T_λ to radial functions in R^n is bounded on L^p for $p_0 < p < p_0'$, $0 \leq \lambda \leq \lambda_c$, see [**8**, **19**, **4**]. S. Chanillo and B. Muckenhoupt [**4**] proved weak type (p_0, p_0) estimates for the restricted operator when $0 < \lambda \leq \lambda_c$ but as C. Kenig and P. Tomas [**11**] have shown, these do not extend to the case $\lambda = 0$ when $n > 1$.

I. Hirschman, Jr. [**9**] obtained the boundedness of T_λ, $0 \leq \lambda < \lambda_c$, on the power weighted space $L^2(R^n; |x|^\alpha\, dx)$ for $|\alpha| < 1 + 2\lambda$.

The main result of this paper gives sufficient conditions on the weight function ω which ensure that T_λ restricted to radial functions is bounded on $L^p(R^n; \omega(|x|)\, dx)$. For a certain class of weights of the form $\omega(r) = r^\alpha(1+r)^{\beta-\alpha}$ these conditions are also seen to be necessary. These results generalize those of [**1**] where the case $\lambda = 0$ was treated.

THEOREM 1. *Let $1 < p < \infty$, $1/p + 1/p' = 1$, $\lambda > 0$ and suppose $\omega(|x|)$ is nonnegative on R^n and satisfies*

$$\left(\int_a^b \frac{\omega(r) r^{n-1}}{(1+r)^{(n-1-2\lambda)p/2}}\, dr\right)^{1/p} \left(\int_a^b \frac{\omega(r)^{-p'/p} r^{n-1}}{(1+r)^{(n-1-2\lambda)p'/2}}\, dr\right)^{1/p'} \tag{1.2}$$
$$\leq K \int_a^b \frac{r^{n-1}}{(1+r)^{n-1-2\lambda}}\, dr$$

for some constant $K = K_{n,\lambda,p,\omega}$ and all $0 \leq a < b < \infty$. Then there is a constant $C = C_{n,\lambda,p,K}$ such that

$$\int_{R^n} |(T_{n,\lambda} f)(x)|^p \omega(|x|)\, dx \leq C \int_{R^n} |f(x)|^p \omega(|x|)\, dx \tag{1.3}$$

for all radial functions f in $L^p(R^n; \omega(|x|)\, dx)$.

COROLLARY. *If $-n < \alpha < n(p-1)$ then (1.3) holds with $\omega(r) = r^\alpha(1+r)^{\beta-\alpha}$ if and only if*

$$-n + (n-1-2\lambda)p/2 < \beta < -n + (n+1+2\lambda)p/2. \tag{1.4}$$

The corollary may be proved as follows. It is easily verified that (1.2) holds with $\omega(r) = r^\alpha(1+r)^{\beta-\alpha}$ if $-n < \alpha < n(p-1)$ and β satisfies (1.4) so the sufficiency follows directly from the theorem. Since $|(T_\lambda \chi_B)(x)| \sim c_{n,\lambda}|x|^{-(n+1+2\lambda)/2}$ as $|x| \to \infty$, and $\chi_B \in L^p(R^n; \omega(|x|)dx)$ for $\alpha > -n$, (1.3) with $f = \chi_B$ shows the necessity of the upper bound for β in (1.4). On the other hand, $\chi_B \in L^{p'}(R^n; \omega(|x|)^{-p'/p} dx)$ for $\alpha < n(p-1)$ so a standard duality argument then shows the necessity of the lower bound for β in (1.4).

Note that with $\alpha = \beta = 0$ the corollary recovers the $L^p(R^n)$ boundedness result for $p_0 < p < p_0'$ cited above while with $p = 2$ and $\alpha = \beta$ (1.4) coincides with the range obtained by Hirschman for the unrestricted operator.

Let $\Omega(n,\lambda,p)$ denote the class of ω which satisfy (1.2). The following properties are easily verified:

$$n_1 < n_2 \quad\Rightarrow\quad \Omega(n_1,\lambda,2) \subset \Omega(n_2,\lambda,2), \tag{1.5}$$

$$\lambda_1 < \lambda_2 \quad\Rightarrow\quad \Omega(n,\lambda_1,p) \subset \Omega(n,\lambda_2,p), \tag{1.6}$$

$$p_1 < p_2 \quad\Rightarrow\quad \Omega(n,\lambda_c,p_1) \subset \Omega(n,\lambda_c,p_2). \tag{1.7}$$

The examples provided by the corollary show that the analogues of (1.5) for $p \neq 2$ and of (1.7) for $\lambda \neq \lambda_c$ do not hold. Note also that if $\omega(r) = r^\alpha$ then $\omega \in \Omega(n,\lambda_c,p)$ if and only if $\omega(|x|)$ satisfies $A_p(R^n)$.

Theorem 1 may be generalized somewhat as follows. For radial functions $f(x) = f_0(|x|)$ on R^n, $(T_{n,\lambda}f)(x) = (R_{n,\lambda}f_0)(|x|)$ is also radial and if $P_k(x)$ is a harmonic homogeneous polynomial of degree k in R^n, then

$$(T_{n,\lambda}fP_k)(x) = (-1)^k(R_{n+2k,\lambda}f_0)(|x|)P_k(x),$$

see [**16**, Chapter IV, §§2, 3]. This with Theorem 1 yields Theorem 2.

THEOREM 2. *Let $1 < p < \infty$, $1/p + 1/p' = 1$, $\lambda > 0$, $k \geq 0$ an integer and suppose $\omega(|x|)$ is nonnegative on R^n and satisfies*

$$\left(\int_a^b \frac{\omega(r)r^{kp+n-1}}{(1+r)^{(n+2k-1-2\lambda)p/2}}\,dr\right)^{1/p}\left(\int_a^b \frac{\omega(r)^{-p'/p}r^{kp'+n-1}}{(1+r)^{(n+2k-1-2\lambda)p'/2}}\,dr\right)^{1/p'}$$
$$\leq K\int_a^b \frac{r^{n+2k-1}}{(1+r)^{n+2k-1-2\lambda}}\,dr$$

for some constant $K = K_{n,\lambda,p,\omega}$ and all $0 \leq a < b < \infty$. Then there is a constant $C = C_{n,\lambda,p,k,K}$ such that

$$\int_{R^n} |(T_{n,\lambda}fP_k)(x)|^p\omega(|x|)\,dx \leq C\int_{R^n} |f(x)P_k(x)|^p\omega(|x|)\,dx$$

for all radial functions f and all harmonic homogeneous polynomials P_k of degree k in R^n with fP_k in $L^p(R^n;\omega(|x|)\,dx)$.

2. Proof of Theorem 1. We follow the usual practice that C will denote an absolute constant, not necessarily the same from line to line. The proof requires the following lemmas. Define the operators $P = P_{n,\lambda}$, $Q = Q_{n,\lambda}$, and M for $s > 0$ by

$$(P\phi)(s) = (1+s)^{-(n+1+2\lambda)/2}\int_0^s (1+r)^{-(n-1)/2}\phi(r)r^{n-1}\,dr,$$
$$(Q\phi)(s) = (1+s)^{-(n-1)/2}\int_s^\infty (1+r)^{-(n+1+2\lambda)/2}\phi(r)r^{n-1}\,dr,$$
$$(M\phi)(s) = \sup_{0<h<s}\frac{1}{2h}\int_{s-h}^{s+h}|\phi(r)|\,dr.$$

LEMMA 1. *If $\lambda > 0$ and $f(x) = f_0(|x|)$ is a simple radial function on R^n then there is a constant $C = C_{n,\lambda}$ such that*

$$|(T_\lambda f)(x)| \leq C\{([P+Q]|f_0|)(|x|) + (Mf_0\chi_{[|x|/2,2|x|]})(|x|)\}. \tag{2.1}$$

PROOF. Set $|x| = s$, $|y| = r$ and consider separately the cases $s \leq 4$ and $s > 4$.

If $s \le 4$, (1.1) and the estimates [**18**, pp. 49, 199]

$$|J_\nu(t)| \le C_\nu t^\nu, \qquad |J_\nu(t)| \le C_\nu (1+t)^{-1/2}, \quad t > 0, \tag{2.2}$$

show that $|(T_\lambda f)(x)|$ does not exceed

$$C_{n,\lambda}\left\{\int_0^8 |f_0(r)| r^{n-1}\,dr + \int_8^\infty (1+r)^{-(n+1+2\lambda)/2}|f_0(r)| r^{n-1}\,dr\right\}$$

since $s \le 4$ and $r \ge 8$ implies $|x-y| \ge r - s \ge r/2 \ge (1+r)/4$. Thus $|(T_\lambda f)(x)| \le C_{n,\lambda}\{([P+Q]|f_0|)(s)\}$ for $s \le 4$.

Now let $s > 4$ and set $f = f_1 + f_2$, where $f_1(y) = (f_0\chi_{(0,2)})(|y|)$. Then (1.1) and (2.2) show that

$$|(T_\lambda f_1)(x)| \le C_{n,\lambda}\int_0^2 (1+s)^{-(n+1+2\lambda)/2}|f_0(r)| r^{n-1}\,dr \le C_{n,\lambda}(P|f_0|)(s)$$

since in this case $|x-y| \ge s - r \ge s/2 \ge (1+s)/4$. The estimates (3.3), (3.4) and (3.5) of [**4**, p. 701] show that $|(T_\lambda f_2)(x)|$ does not exceed a constant multiple of

$$\begin{aligned} s^{-(n+1+2\lambda)/2}&\int_2^{s/2} (1+r)^{-(n-1)/2}|f_0(r)| r^{n-1}\,dr \\ &+ s^{-(n-1)/2}\int_{2s}^\infty (1+r)^{-(n+1+2\lambda)/2}|f_0(r)| r^{n-1}\,dr \\ &+ (Mf_0\chi_{[s/2,2s]})(s) \end{aligned}$$

and this is bounded by

$$C_{n,\lambda}\{([P+Q]|f_0|)(s) + (Mf_0\chi_{[s/2,2s]})(s)\}.$$

Thus, $|(T_\lambda f)(x)| \le |(T_\lambda f_1)(x)| + |(T_\lambda f_2)(x)|$ shows that (2.1) holds for $s > 4$ also. This proves Lemma 1.

LEMMA 2. *Let $1 < p < \infty$, $1/p + 1/p' = 1$ and suppose $\omega \ge 0$ on $(0,\infty)$. The following statements are equivalent:*

(a) *There is a constant $C = C_{n,\lambda,p,\omega}$ such that*

$$\int_0^\infty |(P\phi)(s)|^p \omega(s) s^{n-1}\,ds \le C\int_0^\infty |\phi(r)|^p \omega(r) r^{n-1}\,dr.$$

(b) *There is a constant $C = C_{n,\lambda,p,\omega}$ such that*

$$\int_0^\infty |(Q\psi)(s)|^{p'} \omega(s)^{-p'/p} s^{n-1}\,ds \le C\int_0^\infty |\psi(r)|^{p'} \omega(r)^{-p'/p} r^{n-1}\,dr.$$

(c) *There is a constant $C = C_{n,\lambda,p,\omega}$ such that for all $a > 0$*

$$\left(\int_a^\infty \frac{\omega(r) r^{n-1}}{(1+r)^{(n+1+2\lambda)p/2}}\,dr\right)^{1/p}\left(\int_0^a \frac{\omega(r)^{-p'/p} r^{n-1}}{(1+r)^{(n-1)p'/2}}\,dr\right)^{1/p'} \le C.$$

PROOF. Since $\int_0^\infty \psi(s)(P\phi)(s)s^{n-1}\,ds = \int_0^\infty \phi(r)(Q\psi)(r)r^{n-1}\,dr$ for $\phi, \psi \ge 0$, a standard duality argument shows that (a) and (b) are equivalent. Theorem 1 of [**12**] yields the equivalence of (a) and (c).

LEMMA 3. *Let $1 < p < \infty$, $a_0 \geq 0$, and $V(r) \geq 0$. If for some $\varepsilon > 0$ there is a constant $C = C_{p,a_0,\varepsilon,V}$ such that for all $a > a_0$*

$$\left(\int_a^\infty \left(\frac{a}{r}\right)^\epsilon \frac{V(r)}{r^p}\,dr\right)^{1/p}\left(\int_0^a V(r)^{-p'/p}\,dr\right)^{1/p'} \leq C$$

then there is a constant $C = C_{p,a_0,\varepsilon,V}$ such that for all $a > a_0$

$$\left(\int_a^\infty \frac{V(r)}{r^p}\,dr\right)^{1/p}\left(\int_0^a V(r)^{-p'/p}\,dr\right)^{1/p'} \leq C.$$

PROOF. This is proved in [**2**, Lemma 2] for the case $a_0 = 0$; the general case is entirely similar and is therefore omitted.

Returning to the proof of Theorem 1, observe first that (1.2) shows ω and $\omega^{-p'/p}$ are locally integrable on $[0,\infty)$. Hence the set of simple functions is dense in both $L^p(R^n;\omega(|x|)dx)$ and $L^{p'}(R^n;\omega(|x|)^{-p'/p}dx)$ and therefore by Lemma 1 it suffices to show that

$$\int_0^\infty |(Pf_0)(s)|^p\omega(s)s^{n-1}\,ds, \tag{2.3}$$

$$\int_0^\infty |(Qf_0)(s)|^p\omega(s)s^{n-1}\,ds \tag{2.4}$$

and

$$\int_0^\infty |(Mf_0\chi_{[s/2,2s]})(s)|^p\omega(s)s^{n-1}\,ds \tag{2.5}$$

are each bounded by a constant multiple of $\int_0^\infty |f_0(r)|^p\omega(r)r^{n-1}\,dr$.

Consider (2.3) first. For $t > a > 0$, (1.2) implies there is a constant $C = C_{n,\lambda,p,K}$ such that

$$\left(\int_a^t \frac{\omega(r)r^{n-1}}{(1+r)^{(n-1-2\lambda)p/2}}\,dr\right)\left(\int_0^a \frac{\omega(r)^{-p'/p}r^{n-1}}{(1+r)^{(n-1-2\lambda)p'/2}}\,dr\right)^{p/p'}$$
$$\leq K^p\left(\int_0^t \frac{r^{n-1}}{(1+r)^{n-1-2\lambda}}\,dr\right)^p \leq C(1+t)^{(2\lambda+1)p}.$$

Multiplying this by $(1+t)^{-(2\lambda+1)p-2}$, integrating the result over $a < t < \infty$ and using Fubini's theorem on the left shows that

$$\left(\int_a^\infty \left(\frac{1+a}{1+r}\right)\frac{\omega(r)r^{n-1}}{(1+r)^{(n+1+2\lambda)p/2}}\,dr\right)\left(\int_0^a \frac{\omega(r)^{-p'/p}r^{n-1}}{(1+r)^{(n-1-2\lambda)p'/2}}\,dr\right)^{p/p'}$$

is bounded by a constant C. Thus, Lemma 3 shows that

$$\left(\int_a^\infty \frac{\omega(r)r^{n-1}}{(1+r)^{(n+1+2\lambda)p/2}}\,dr\right)^{1/p}\left(\int_0^a \frac{\omega(r)^{-p'/p}r^{n-1}}{(1+r)^{(n-1-2\lambda)p'/2}}\,dr\right)^{1/p'} \tag{2.6}$$

is bounded by C for all $a \geq 1$. If $a < 1$, the first factor of (2.6) is bounded by a constant times the sum of

$$\left(\frac{1}{(1+a)^{(1+2\lambda)p}}\int_a^1 \frac{\omega(r)r^{n-1}}{(1+r)^{(n-1-2\lambda)p/2}}\,dr\right)^{1/p}$$

and

$$\left(\int_1^\infty \frac{\omega(r)r^{n-1}}{(1+r)^{(n+1+2\lambda)p/2}}\,dr\right)^{1/p}.$$

Thus, for $a < 1$, (2.6) is bounded by the sum of two terms, the first of which is bounded because of (1.2) while the second is bounded because of (2.6) with $a = 1$. Thus for $a > 0$ we have

$$\left(\int_a^\infty \frac{\omega(r)r^{n-1}}{(1+r)^{(n+1+2\lambda)p/2}}\,dr\right)^{1/p}\left(\int_0^a \frac{\omega(r)^{-p'/p}r^{n-1}}{(1+r)^{(n-1-2\lambda)p'/2}}\,dr\right)^{1/p'} \le C$$

so parts (a) and (c) of Lemma 2 show that (2.3) has the required bound.

That (2.4) has the required bound now follows easily from the duality expressed by parts (a) and (b) of Lemma 2 since (1.2) is self-dual in the sense that ω satisfies (1.2) for a given p if and only if (1.2) is satisfied with ω and p replaced by $\omega^{-p'/p}$ and p'.

Finally, to show that (2.5) has the required bound, observe first that (1.2) shows there is a constant $C = C_{n,\lambda,K}$ independent of the integer m such that

$$\left(\int_a^b \omega(r)\,dr\right)^{1/p}\left(\int_a^b \omega(r)^{-p'/p}\,dr\right)^{1/p'} \le C(b-a)$$

whenever $2^{m-1} \le a < b \le 2^{m+2}$. Thus Muckenhoupt's well-known result [**13**] for the Hardy-Littlewood maximal function shows that

$$\int_{2^{m-1}}^{2^{m+2}} |(M\phi)(s)|^p\omega(s)\,ds \le C\int_{2^{m-1}}^{2^{m+2}} |\phi(r)|^p\omega(r)\,dr$$

for a constant $C = C_{n,\lambda,K}$ independent of m. Hence (2.5) is bounded by

$$\begin{aligned}
&\sum_{m=-\infty}^{\infty}\int_{2^m}^{2^{m+1}} |(M\phi\chi_{[2^{m-1},2^{m+2}]})(s)|^p\omega(s)s^{n-1}\,ds \\
&\quad\le C\sum_{m=-\infty}^{\infty} 2^{(m+1)(n-1)}\int_{2^{m-1}}^{2^{m+2}} |\phi(r)|^p\omega(r)\,dr \\
&\quad\le C\int_0^\infty |\phi(r)|^p\omega(r)r^{n-1}\,dr
\end{aligned}$$

as required. This completes the proof of Theorem 1.

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