PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 103, Number 1, May 1988

MORE NONDEVELOPABLE SPACES IN MOBI

J. CHABER

(Communicated by Dennis Burke)

ABSTRACT. We construct two spaces with a point-countable base, the first is not weakly θ-refinable, while the second is not a $\sigma^{\#}$-space. Both spaces can be represented as open and compact images of metacompact developable Hausdorff spaces. This answers questions of H. R. Bennett and R. F. Gittings.

The first example of a nonweakly θ-refinable space with a point-countable base was constructed, with the use of the continuum hypothesis, in [**GG**]. In the first section of this paper we apply an idea from [**GG**] in order to get a ZFC example of such a space. Our construction is simpler than the example of a metalindelöf space which is not weakly θ-refinable from [**Gr**].

In the second section we show that the space constructed in the first section is an open and compact image of a metacompact developable Hausdorff space. Thus this space is an element of MOBI_2, where MOBI_i is the minimal class of T_i-spaces containing all metric spaces and invariant under open and compact mappings.

In the third section we apply the techniques of the first two sections in order to construct a space in MOBI_2 which is not a $\sigma^{\#}$-space.

1. A nonweakly θ-refinable space with a point-countable base. It will be convenient to use the following definition [**BL**, Lemma 4]: a space Y is weakly θ-refinable if every open cover of Y has a refinement $\mathscr{A} = \bigcup\{\mathscr{A}_n : n \geq 1\}$ such that each $\mathscr{A}_n$ is discrete in $\bigcup \mathscr{A}_n$.

Our construction will be based on an example of a space with a point-countable base which does not have a σ-point finite base [**A**]. The points of the space are functions from countable ordinals into ω and the base of neighborhoods of a function s consists of sets of the form

$$B(s,m) = \{t \supset s : t = s \text{ or } t(\operatorname{dom} s) \geq m\}.$$

Let S be the space described above and put $B(s) = B(s,0)$ for $s \in S$. Clearly, every nonempty open subset of S contains a set $B(s)$ for an $s \in S$ and any intersection of a decreasing sequence of sets of the form $B(s)$ is again a set of this form. In particular, S is a Baire space.

Every open cover of the space S has a disjoint open refinement [**A**, **Mi**]. We shall destroy the covering properties of S with the help of the space ω_1 of the countable ordinals (see [**GG**]).

Put

$$S^* = \{\langle \alpha, s\rangle \in \omega_1 \times S : \alpha \leq \operatorname{dom} s\}$$

Received by the editors October 23, 1986 and, in revised form, March 23, 1987.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 54D18, 54C10.

and consider S^* with the topology inherited from $\omega_1 \times S$. We shall show that S^* is not weakly θ-refinable.

Let $U_\delta = \{\langle \alpha, s\rangle \in S^*: \alpha \le \delta\}$ and $\mathscr{U} = \{U_\delta: \delta \in \omega_1\}$. Suppose that S^* is weakly θ-refinable, then there exists a refinement $\mathscr{A} = \bigcup\{\mathscr{A}_n: n \ge 1\}$ of $\mathscr{U}$ such that each $\mathscr{A}_n$ is discrete in $\bigcup \mathscr{A}_n$.

By induction on $\alpha \in \omega_1$ construct sequences $\langle s_\alpha\rangle_{\alpha\in\omega_1}$ of elements of S, $\langle n_\alpha\rangle_{\alpha\in\omega_1}$ of natural numbers and $\langle \gamma_\alpha\rangle_{\alpha\in\omega_1}$ of countable ordinals such that for each $\alpha > 0$,

(i) $s_\alpha \supset \bigcup\{s_\beta: \beta < \alpha\}$ and $\langle \alpha, s_\alpha\rangle \in S^*$,

(ii) $(\{\alpha\} \times B(s)) \cap \bigcup \mathscr{A}_{n_\alpha} \ne \varnothing$ for any $s \supset s_\alpha$,

(iii) $\gamma_\alpha < \alpha$ and $(\gamma_\alpha, \alpha] \times B(s_\alpha)$ intersects at most one element of $\mathscr{A}_{n_\alpha}$.

Start by choosing $s_0 = \varnothing$, $n_0 = 0$ and $\gamma_0 = 0$. Suppose that s_β, n_β and γ_β have been defined for $\beta < \alpha$.

We shall define s_α by consecutively extending $\bigcup\{s_\beta: \beta < \alpha\}$. First choose $s' \supset \bigcup\{s_\beta: \beta < \alpha\}$ with $\operatorname{dom} s' \ge \alpha$. Clearly, any extension s_α of s' will satisfy (i). Since $\{\alpha\} \times B(s')$ is covered by $\{\bigcup \mathscr{A}_n: n \ge 1\}$, one can use the Baire property of S in order to find an $n_\alpha \ge 1$ and an $s'' \supset s'$ such that $\bigcup \mathscr{A}_{n_\alpha}$ is dense in $\{\alpha\} \times B(s'')$. Clearly, any extension s_α of s'' will satisfy (ii) with respect to this n_α. In particular, there exists an $s''' \supset s''$ such that $\langle \alpha, s'''\rangle \in \bigcup \mathscr{A}_{n_\alpha}$ and, since $\mathscr{A}_{n_\alpha}$ is discrete in $\bigcup \mathscr{A}_{n_\alpha}$, $\langle \alpha, s'''\rangle$ has a neighborhood of the form $(\gamma_\alpha, \alpha] \times B(s''', m)$ intersecting exactly one element of $\mathscr{A}_{n_\alpha}$. Clearly, $s_\alpha = s''' \cup \{\langle \operatorname{dom} s''', m\rangle\}$ satisfies (iii) with respect to these n_α and γ_α and the inductive step is complete.

Observe that (ii) and (iii) imply that for each $\beta > 0$, there exists an $A_\beta \in \mathscr{A}_{n_\beta}$ which intersects $\{\beta\} \times B(s)$ for all $s \supset s_\beta$.

In order to obtain a contradiction, use standard properties of ω_1 to find an $n \ge 1$, $\gamma \in \omega_1$ and uncountable set $E \subseteq \omega_1$ such that $n_\alpha = n$ and $\gamma_\alpha = \gamma$ for $\alpha \in E$. Pick a $\beta \in E$ and a $\delta \in \omega_1$ such that $A_\beta \subseteq U_\delta$. If $\alpha \in E$ and $\alpha > \beta$, then $A_\alpha \ne A_\beta$ while $(\gamma, \delta] \times B(s_\alpha) \supset \{\beta\} \times B(s_\alpha)$, so $(\gamma, \alpha] \times B(s_\alpha)$ intersects two different elements A_α and A_β of $\mathscr{A}_n$, which contradicts (iii).

It is easy to see that the space S^* has a point-countable base. In fact, it is a first-countable space with a co-countable neighbournet.

2. Spaces with a co-countable neighbournet. Recall that a neighbournet for a space Y is a relation $V \subset Y \times Y$ such that for each $y \in Y$, $y \in \operatorname{Int} V(y)$, where $R(y) = \{y' \in Y: \langle y, y'\rangle \in R\}$ for a relation $R \subset Y \times Y$ [**J1**]. We shall only consider neighbournets V such that $V(y)$ is open for $y \in Y$. The neighbournet V is called co-countable (co-finite) if $V^{-1}(y)$ is countable (finite) for $y \in Y$ [**J2**].

It is easy to check that first-countable T_1-spaces with a co-countable (co-finite) neighbournet have a point-countable base (are σ-discrete metacompact and developable [**J2**]).

Observe that the space S^* has a co-countable neighbournet. Indeed, the collection $\{B(\alpha, s): \langle \alpha, s\rangle \in S^*\}$, where $B(\alpha, s) = [0, \alpha] \times B(s)$, defines a co-countable neighbournet in S^*, for $\langle \beta, t\rangle \in B(\alpha, s)$ implies that $s \subset t$ and $\alpha \le \operatorname{dom} s \le \operatorname{dom} t$.

In [**J2**, Theorems 3, 4] (first-countable) T_1-spaces with co-countable neighbournets have been characterized as images of "nice" spaces under open countable-to-one mappings. We shall modify the construction of open mappings from [**J2**] in order to get open and compact mappings.

LEMMA. *If Y is a T_1-space with a co-countable neighbournet V, then there exists a Hausdorff space X with a co-finite neighbournet and an open and compact countable-to-one mapping f from X onto Y. Moreover, if Y is first-countable, then so is X.*

PROOF. As in [**J2**, Lemma 5], let $I(Y,V)$ be the set of all injections $\sigma\colon \{0,\dots,n\} \to Y$ such that $e(\sigma)$ defined as $\sigma(n)$ is in the intersection of all $V(\sigma(k))$, $k=0,\dots,n$. The latter restriction assures that $\{\sigma\colon e(\sigma)=y\} \subset \{\sigma\colon \sigma(\operatorname{dom}\sigma) \subset V^{-1}(y)\}$ for $y \in Y$, hence the natural projection e of $I(Y,V)$ onto Y is countable-to-one. The basic open sets containing σ are of the form

$$B(\sigma,U) = \{\tau \supset \sigma\colon \tau(\operatorname{dom}\tau \setminus \operatorname{dom}\sigma) \subset U\},$$

where U is an open subset of Y containing $e(\sigma)$.

Observe that, if $\tau \in B(\sigma,U) \cap B(\sigma',U')$, then $\sigma,\sigma' \subset \tau$ and, assuming that $\sigma \subset \sigma'$, $\sigma' \in B(\sigma,U)$.

Clearly, the projection $e\colon I(Y,V) \to Y$ is continuous, open [**J2**] and, from the observation above, it follows that the fibers of e are discrete. Thus $I(Y,V)$ is a T_1-space and, again, the observation above, implies that it is a collectionwise (in fact, monotonically) normal space. Moreover, the collection $\{B(\sigma,Y)\colon \sigma \in I(Y,V)\}$ defines a co-finite neighbournet on $I(Y,V)$.

We shall modify $I(Y,V)$ by adding a limit point to each infinite fiber of e.

Put $X = I(Y,V) \cup \{y \in Y\colon e^{-1}(y) \text{ is infinite}\}$ and consider X with a topology such that $I(Y,V)$ is open in X, while for $y \in X \setminus I(Y,V)$ basic neighbourhoods of y are of the form

$$B(y,U,F) = \{y\} \cup \bigcup \{B(\sigma,U)\colon \sigma \in e^{-1}(y) \setminus F\},$$

where U is an open subset of Y containing y and F is a finite subset of $e^{-1}(y)$.

It is easy to check that X is a Hausdorff space and the collection $\{B(\sigma,Y)\colon \sigma \in I(Y,V)\} \cup \{B(y,Y,\varnothing)\colon y \in X \setminus I(Y,V)\}$ defines a co-finite neighbournet on X. Clearly, if Y is a first-countable space, then X is first-countable too.

Define $f\colon X \to Y$ as the combination of e and the identity on $X \setminus I(Y,V)$. Clearly, f is a continuous open and compact countable-to-one mapping.

If Y in the above construction is the space S^*, then one gets

EXAMPLE. An open and compact mapping of a σ-discrete metacompact developable Hausdorff space onto the space S^* which is not weakly θ-refinable.

The example answers Question c from [**B2**] repeated in [**G**, Question 1]. Furthermore, if the space Y in the lemma is first-countable, then $I(Y,V)$ is developable and collectionwise normal, hence, it is then a metrizable space (see [**J2**]). Thus X is then a locally metrizable σ-discrete metacompact space and, therefore, X can be represented as an open finite-to-one image of a σ-discrete metrizable space. In particular, S^* is in the class MOBI_2, which gives an answer to Question 1 from [**B1**].

The above reasoning and the fact that open mappings with separable fibers preserve the property of having a co-countable neighbournet (see [**J2**, Proposition 2]) give

THEOREM. *For a Hausdorff space Y the following conditions are equivalent:*

(i) *Y is a first-countable space with a co-countable neighbournet,*

(ii) Y *is an open and compact countable-to-one image of a σ-discrete metacompact developable Hausdorff space,*

(iii) Y *is in the minimal class of Hausdorff spaces containing all σ-discrete metric spaces and invariant under open and compact mappings.*

In the next section we give a method of constructing spaces with co-countable neighbournets based on the construction of the space S and spaces $I(Y,V)$ and we use this method in order to answer the remaining part of Question 1 from [**G**] concerning the invariantness of $\sigma^{\#}$-spaces.

3. $\sigma^{\#}$-spaces. According to the standard terminology, a collection $\mathscr{A}$ of subsets of a space Y will be called $T_1(T_0)$-separating if for each two distinct points y and y' of Y there exists an $A \in \mathscr{A}$ such that $y \in A$ and $y' \notin A$ (or $y \notin A$ and $y' \in A$). The collection $\mathscr{A}$ is closure-preserving (σ-closure-preserving) if

$$\bigcup\{\overline{A}\colon A \in \mathscr{A}'\} = \overline{\bigcup\{A\colon A \in \mathscr{A}'\}}$$

for any subcollection $\mathscr{A}'$ of $\mathscr{A}$ (if $\mathscr{A}$ can be represented as a union of countably many closure-preserving subcollections). The space Y is called $\sigma^{\#}$-space if it has a σ-closure-preserving T_1-separating collection of closed sets.

We shall answer the part of Question 1 in [**G**] concerning $\sigma^{\#}$-spaces by constructing a Hausdorff first-countable space T with a co-countable neighbournet which is not a $\sigma^{\#}$-space. By virtue of the results of the previous section, the space T is then in MOBI_2.

One can check that the space S^* constructed in the first section is a $\sigma^{\#}$-space. Moreover, any space with a co-countable neighbournet has a closure-preserving cover by countable closed sets [**J2**, Proposition 1] and one can modify this cover so that it is, in addition, a T_0-separating collection (see 4.2). The example that we are going to construct shows that this does not imply the existence of a σ-closure-preserving T_1-separating closed collection.

EXAMPLE. A first-countable Hausdorff space T with a co-countable neighbournet which is not a $\sigma^{\#}$-space.

Let $Z = C_1 \cup C_2$ be the Alexandroff double circle. As in [**E**, 3.1.26], C_1 is the topological circle, C_2 is the set of isolated points of Z, p is the projection of C_1 onto C_2 from the joint center of C_1 and C_2 and, for $z \in C_1$, the basic open sets containing z are of the form $U_j(z) = V_j(z) \cup p(V_j(z)\setminus\{z\})$, where $j \geq 1$ and $V_j(z)$ is an open arc of length $1/j$ in C_1 centered at z.

We shall modify Z in order to obtain the space T. The points of T will be one-to-one functions from nonlimit countable ordinals into Z. For $s \in T$ with $\operatorname{dom} s = \alpha + 1$, by $e(s)$ we denote $s(\alpha)$. The basic open sets containing s are of the form

$$B(s,U) = \{t \supset s\colon e(t) \in U\},$$

where U is an open subset of Z containing $e(s)$.

It is easy to see that T is a first-countable Hausdorff (but not regular) space and the collection $\{B(s,Z)\colon s \in T\}$ defines a co-countable neighbournet in T.

Suppose that T is a $\sigma^{\#}$-space and let $\mathscr{E} = \bigcup\{\mathscr{E}_n\colon n \geq 1\}$ be a T_1-separating collection of closed subsets of T such that each $\mathscr{E}_n$ is closure-preserving. Choose a one-to-one sequence $\langle z_\beta\rangle_{\beta\in\omega_1}$ of elements of C_1 and denote $s_\alpha = \langle z_\beta\rangle_{\beta\leq\alpha} \in T$. For each $\alpha \in \omega_1$ find a decreasing sequence $\{W_n(z_\alpha)\}_{n\geq 1}$ of open subsets of Z containing $z_\alpha = e(s_\alpha)$ such that each $W_n(z_\alpha) = U_j(z_\alpha)$ for a $j \geq n$ and $B(s_\alpha, W_n(z_\alpha)) \cap \bigcup\{E \in \mathscr{E}_n : s_\alpha \notin E\} = \varnothing$.

From the fact that C_1 is hereditarily separable, it follows that there exists an $\alpha \in \omega_1$ such that for each $\beta > \alpha$ and $n \geq 1$

$$C_1 \cap \bigcup\{W_n(z_\gamma) : \gamma \geq \alpha\} = C_1 \cap \bigcup\{W_n(z_\gamma) : \gamma \geq \beta\}.$$

In particular, for each $n \geq 1$, there exists a $\beta_n > \alpha$ such that $z_\alpha \in W_n(z_{\beta_n})$ and since each $W_n(z_\beta)$ is of the form $U_j(z_\beta)$ for a $j \geq n$, we obtain that $p(z_\alpha) \in W_n(z_{\beta_n})$ and $\langle z_{\beta_n} \rangle_{n \geq 1}$ converges to z_α in C_1.

Let γ be a countable ordinal greater than $\sup\{\beta_n : n \geq 1\}$ and define an $s \in T$ with $\operatorname{dom} s = \gamma + 1$ by putting $s(\beta) = z_\beta$ for $\beta < \gamma$ and $s(\gamma) = p(z_\alpha) \in C_2$. If $n \geq 1$ and $E \in \mathscr{E}_n$ are such that $s \in E$ and $s_\alpha \notin E$, then $B(s_\alpha, W_n(z_\alpha)) \cap E = \varnothing$. Find an $m \geq n$ for which $z_{\beta_m} \in W_n(z_\alpha)$. Clearly, $s_{\beta_m} \in B(s_\alpha, W_n(z_\alpha))$ and, consequently, $s_{\beta_m} \notin E$, which implies $B(s_{\beta_m}, W_n(z_{\beta_m})) \cap E = \varnothing$. However, $e(s) = p(z_\alpha) \in W_m(z_{\beta_m}) \subset W_n(z_{\beta_m})$, hence $s \in B(s_{\beta_m}, W_n(z_{\beta_m}))$ and this contradicts $s \in E$.

4. Final remarks.

4.1. The space S from the first section is an easy example of a non-quasi-developable space in MOBI_2. In fact, the space S is not even a primitive $\sigma^{\#}$-space in the sense of [**Ch2**], which implies that S is neither a σ-space nor a space with a primitive base. An easy way to see that S is not a primitive $\sigma^{\#}$-space is to use a game-theoretic characterization of this property given, implicitly, in [**M**, Theorem 7.3].

Consider the following two-person game in a space Y: players I and II alternately choose nonempty subsets $C_1 \supset D_1 \supset C_2 \supset D_2 \supset \cdots$ of Y such that each D_n is relatively open in C_n. Player II (choosing D_n's) wins if $\bigcap\{C_n : n \geq 1\}$ contains at most one point. The space Y is a primitive $\sigma^{\#}$-space iff player II has a (stationary) winning strategy in this game. In S player I has a stationary winning strategy, namely, I wins by choosing at each stage of the game a set of the form $B(s)$ contained in the set recently chosen by II.

Observe that S is a $\sigma^{\#}$-space which is not a primitive $\sigma^{\#}$-space. This improves Example 1 in [**Ch2**] and shows that the term "primitive $\sigma^{\#}$-space" suggested in [**Ch2**] should be changed.

4.2. The following slight strengthening of Proposition 1 in [**J2**] (see the proof of (c) $\Rightarrow$ (a) in [**F**, Theorem 2.2]) clarifies the structure of T_1-spaces with a co-countable neighbournet:

PROPOSITION. *A T_1-space Y has a co-countable neighbournet iff there exists a partial order $\leq$ on Y such that $\{y' : y' \leq y\}$ is countable and $\{y' : y' \geq y\}$ is open for $y \in Y$.*

Observe that in the construction of X in §2, one can consider only sequences σ which are increasing with respect to the above partial order. The examples of spaces in MOBI_3 in [**Ch1**] were obtained in this way from spaces Y with the partial order having two levels.

4.3. Theorem 2.2 in [**F**] gives several conditions characterizing spaces with a co-countable neighbournet in the class of T_1-spaces with a point-countable base. We give another such condition, which shows that the reflection principles S_1 and S_2 from [**F**] are equivalent (see Conjecture 1 in [**F**, 6]).

PROPOSITION. *Let Y be a T_0-space with a point-countable base $\mathscr{U}$. For each $y \in Y$ fix an enumeration $e(y) = \langle U_i(y)\rangle_{i\geq 1} \in \mathscr{U}^{\aleph_0}$ (not necessarily one-to-one) of all the members of $\mathscr{U}$ containing y. Consider $\mathscr{U}^{\aleph_0}$ with the Baire metric and let $M = e(Y) \subset \mathscr{U}^{\aleph_0}$. Then Y has a co-countable neighbournet iff M is σ-discrete.*

PROOF. Suppose that Y has a co-countable neighbournet. Observe that e is one-to-one and $e^{-1}: M \to Y$ is continuous. Clearly, this gives a co-countable neighbournet on M and, since M is a metric space, it is σ-discrete (see [**F** or **J1**, 4.8]).

In order to prove the other implication, assume that $M' = e(Y') \subset M$ is $1/j$-discrete and define $V(y') = \bigcap\{U_i(y'): i \leq j\}$ for $y' \in Y'$. If $y \in Y$ and $y \in V(y')$ for a $y' \in Y'$, then $U_i(y')$ is a term of $e(y)$ for $i \leq j$ and, consequently, the number of such $y' \in Y'$ is countable. Since M is a countable union of such M', this shows that Y has a co-countable neighbournet.

4.4. The construction from [**B3**] showing that weak θ-refinability is not invariant under open and compact mappings can be generalized as follows.

PROPOSITION. *If a Hausdorff space Y can be covered by an open collection $\mathscr{G}$ such that each element of $\mathscr{G}$ has a co-finite (co-countable) neighbournet, then Y is an open and compact image of a Hausdorff space with a co-finite neighbournet.*

PROOF. Suppose first that each $G \in \mathscr{G}$ has a co-finite neighbournet V_G. Let e be the canonical mapping of the topological sum X' of the elements of $\mathscr{G}$ onto Y. Put $X = X' \cup \{y \in Y: e^{-1}(y) \text{ is infinite}\}$ and consider X with the topology such that X' is open in X and the basic open sets containing $y \in X\backslash X'$ are of the form

$$B(y, U, \mathscr{F}) = \{y\} \cup \bigcup\{G \cap V_G(y) \cap U: y \in G \in \mathscr{G}\backslash\mathscr{F}\},$$

where U is an open subset of Y containing y and $\mathscr{F}$ is a finite subcollection of $\mathscr{G}$.

It is easy to see that X is a Hausdorff space with a co-finite neighbournet and the natural extension f of e is an open and compact mapping of X onto Y.

If W_G is a co-countable neighbournet in $G \in \mathscr{G}$, then G in the above construction should be replaced with a space $X(G)$ constructed from $I(G, W_G)$ as in §2.

4.5. The nondevelopable Čech complete space with a point-countable base from [**D**] can be represented as an open and compact countable-to-one image of a meta-compact Moore space and, therefore, is in MOBI_3.

REFERENCES

[**A**] C. Aull, *Topological spaces with a σ-point finite base*, Proc. Amer. Math. Soc. **29** (1971), 411–416.

[**B1**] H. R. Bennett, *On Arhangel'skiĭ's class* MOBI, Proc. Amer. Math. Soc. **26** (1970), 178–180.

[**B2**] ——, *On quasi-developable spaces*, General Topology Appl. **1** (1971), 253–262.

[**B3**] ——, *On open-compact images of metacompact spaces*, Topology Proc. **4** (1979), 339–343.

[**BL**] H. R. Bennett and D. J. Lutzer, *A note on weak θ-refinability*, General Topology Appl. **2** (1972), 49–54.

[**Ch1**] J. Chaber, *Metacompactness and the class* MOBI, Fund. Math. **91** (1976), 211–217.

[**Ch2**] ——, *Primitive generalizations of σ-spaces*, Colloq. Math. Soc. János Bolyai 23, Topology, Budapest, 1978, pp. 259–268.

[**D**] S. W. Davis, *A nondevelopable Čech complete space with a point-countable base*, Proc. Amer. Math. Soc. **78** (1980), 139–142.

[**E**] R. Engelking, *General topology*, PWN, Warsaw, 1977.

[F] W. G. Fleissner, *Left separated spaces with point-countable bases*, Trans. Amer. Math. Soc. **294** (1986), 665–677.

[G] R. F. Gittings, *Open mapping theory*, Set-theoretic Topology, New York, 1977, pp. 141–191.

[Gr] G. Gruenhage, *On a Corson compact space of Todorčević*, preprint.

[GG] G. Gruenhage and R. J. Gardner, *Completeness and weak covering properties, and measure-compactness*, J. London Math. Soc. **18** (1978), 316–324.

[J1] H. J. K. Junnila, *Neighbornets*, Pacific J. Math. **76** (1978), 83–108.

[J2] ——, *Stratifiable preimages of topological spaces*, Colloq. Math. Soc. János Bolyai 23, Topology, Budapest, 1978, pp. 689–703.

[M] E. Michael, *A note on completely metrizable spaces*, Proc. Amer. Math. Soc. **96** (1986), 513–522.

[Mi] A. Miščenko, *Spaces with point-countable base*, Dokl. Akad. Nauk SSSR **144** (1962), 985–988.

INSTITUTE OF MATHEMATICS, UNIVERSITY OF WARSAW, 00 901 WARSZAWA, POLAND PKIN IXP

Current address: Department of Mathematics, Texas Tech University, Lubbock, Texas 79409