EXISTENCE OF SYMMETRIC SKEW BALANCED STARTERS FOR ODD PRIME POWERS

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ABSTRACT. Strong starters and skew starters have been widely used in various combinatorial designs. In particular skew balanced starters and symmetric skew balanced starters are crucially used in the construction of completely balanced Howell rotations. Let $n = 2^m k + 1$ be an odd prime power where $m \geq 2$ and k is an odd number. The existence of symmetric skew balanced starters for $GF(n)$ has been proved for $m \geq 2$ and $k \neq 1, 3, 9$. In this paper, we present a new approach which gives a uniform proof of the existence of symmetric skew balanced starters for all $m \geq 2$ and $k \geq 3$.

1. Introduction. Let n be an odd prime power. A set of $p = (n-1)/2$ pairs $(x_1, y_1), \ldots, (x_p, y_p)$ is called a skew balanced starter (SBS) if

(i) $\{x_1, y_1, \ldots, x_p, y_p\} = GF(n) - \{0\}$,

(ii) $\{\pm(x_i - y_i): i = 1, \ldots, p\} = GF(n) - \{0\}$,

(iii) $\{\pm(x_i + y_i): i = 1, \ldots, p\} = GF(n) - \{0\}$,

(iv) the multiset of differences $\{x_i - x_j : i, j = 1, \ldots, p, \ i \neq j\}$ and the multiset of differences $\{y_i - y_j : j = 1, \ldots, p, \ i \neq j\}$ together contain every nonzero element of $GF(n)$ the same number of times.

An SBS is called *symmetric* and denoted by SSBS if in addition:

(v) $\{x_1, \ldots, x_p\} = \{-x_1, \ldots, -x_p\}$ (which implies $\{y_1, \ldots, y_p\} = \{-y_1, \ldots, -y_p\}$).

Write $n = ek + 1$ where $e = 2^m$ and k is odd. Note that condition (v) implies that p must be even, hence $m \geq 2$. It is well known [3] that an SBS of order $n = 2k + 1$ yields a complete balanced Howell rotation (also known as balanced Room square) of order $n + 1$ and an SSBS of order $n = 2^m k + 1$ for $m \geq 2$ yields a complete balanced Howell rotation of order $2(n + 1)$. Berlekamp and Hwang [1] gave a construction for SBS of order $n = 2k + 1$ for all odd $k > 1$. The construction of SSBS has been extensively studied in the literature (see [5] for a review). In particular, Du and Hwang [3] gave a construction which works for (i) $m = 2$ and $k > 1$, (ii) $m = 3$ and $k > 1$, (iii) $m \geq 4$ and $k > 9e^3$. Later, Yu and Hwang [7] showed that the construction of Du and Hwang works for all $m \geq 2$ and $k \geq 4e$. They also conjectured that the construction works for all $m \geq 2$ and $k > 1$. Recently, Yu and Hwang [8] proved the conjecture except for the two cases of $k = 3$ and $k = 9$. In this paper, we present an alternative approach to show the existence of SSBS including the two cases $k = 3$ and $k = 9$.

2. Some preliminary results. From now on we will only be concerned with the case $m \geq 2$. Let $GF^*(n)$ denote the multiplicative group of $GF(n)$. Let x be a generator of $GF^*(n)$ and for any element $y \in GF(n)$ we write $T_x(y) = z$ if $y = x^z$. An element $y \in GF^*(n)$ is called a *desirable element* if it satisfies the following three conditions:

(i) $T_x(y) \equiv -1 \pmod{e}$.

(ii) $T_x(y - 1) \equiv T_x(x - 1) \pmod{2}$.

(iii) $T_x(y + 1) \equiv T_x(x + 1) \pmod{2}$.

Du and Hwang [3] proved that if y is a desirable element, then the following pairing (referred to as the DH pairing in [8]) is an SSBS for $GF(n)$:

$$(x^{i2^m+2j+1}, x^{i2^m+2j+2}), \qquad i = 0, 1, \ldots, k-1; \; j = 0, 1, \ldots, 2^{m-2}-1,$$
$$(x^{i2^m+2^{m-1}+2j+2}y, x^{i2^m+2^{m-1}+2j+2}),$$
$$i = 0, 1, \ldots, k-1; \; j = 0, 1, \ldots, 2^{m-2}-1.$$

Partition the element of $GF^*(n)$ into e residue classes $C_t = \{x^{t+ie} : i = 1, \ldots, k-1\}$, $t = 0, 1, \ldots, e-1$. Yu and Hwang [8] noticed that if for some odd t the class C_t contains two elements u and v such that

(a) $T(u - 1) \equiv T(v - 1) + 1 \pmod{2}$,

(b) $T(u + 1) \equiv T(v + 1) + 1 \pmod{2}$, and

(c) one of the two elements, say u, is a generator,

then v^{-1} is a desirable element. They proved that except for $k = 9$ and $k = 3$, such a class C_t exists for $m \geq 2$. Furthermore, for $k = 9$ there exists a C_t which contains two elements u and v satisfying (a) and (b).

Here, we first show that for the existence of SSBS, condition (c) is superfluous.

THEOREM 1. *Suppose that for some odd t the class C_t contains two elements u and v such that* (a) *and* (b) *hold. Then the following pairing is an SSBS for $GF(n)$:*

$$(x^{i2^m+2j+2}u, x^{i2^m+2j+2}), \qquad i = 0, 1, \ldots, k-1; \; j = 0, 1, \ldots, 2^{m-2}-1,$$
$$(x^{i2^m+2^{m-1}+2j+2}v, x^{i2^m+2^{m-1}+2j+2}), \qquad i = 0, 1, \ldots, k-1; \; j = 0, 1, \ldots, 2^{m-2}.$$

PROOF. We first prove that for any $i, i' \in \{0, 1, \ldots, k-1\}$ and $j, j' \in \{0, 1, \ldots, 2^{m-2}-1\}$, $x^{i2^m+2j+2}u \neq x^{i'2^{m'}+2^{m-1}+2j'+2}v$. Suppose to the contrary that

$$x^{i2^m+2j+2}u = x^{i'2^{m'}+2^{m-1}+2j'+2}v.$$

Since $T(x^{i2^m+2j+2}u) \equiv 2j + 2 + t \pmod{e}$ and

$$T(x^{i'2^{m'}+2^{m-1}+2j'+2}v) \equiv 2^{m-1} + 2j' + 2 + t \qquad \pmod{e},$$

we have $2j \equiv 2^{m-1} + 2j' \pmod{e}$, i.e. $j - j' \equiv 2^{m-2} \pmod{e}$. However, $|j - j'| \leq 2^{m-2} - 1$ and $e = 2^m$, hence it is impossible that $j - j' \equiv 2^{m-2} \pmod{e}$.

Now, we verify conditions (i)–(v) in the definition of SSBS. Note that t is odd. Therefore $x^{i2^m+2j+2}u$ and $x^{i2^m+2^{m-1}+2j+2}v$, $i = 0, 1, \ldots, i-1$, $j = 0, 1, \ldots, 2^{m-1}-1$, run through x^{2s+1}, $s = 0, 1, \ldots, 2^{m-1}k$. It follows that (i) holds. Conditions (ii) and (iii) can be easily derived from (a) and (b). Condition (iv) holds since $(x^1, x^3, \ldots, x^{2p-1})$, $(1, x^2, \ldots, x^{2p-2})$ form a difference set by the Bose theorem [2]. Finally, condition (v) is satisfied since $T(-1) \equiv 2^{m-1}k \equiv 0 \pmod{2}$. $\square$

The pairing in Theorem 1 is a generalization of the DH pairing. In fact, if y is a desirable element, then $x^{-1}, y \in C_{-1}$. Let $u = x^{-1}$, $v = y$. Then u and v satisfy (a) and (b). Substituting them into Theorem 1, we can obtain the original DH pairing. As a corollary of Theorem 1, we can conclude that SSBS exists for $k = 9$ (not necessarily for the DH pairing). Thus we only have the case $k = 3$ left. In the following, however, we will present an approach which works for every case rather than a singular case.

A class C_t is called a *desirable class* if t is odd and C_t contains two elements u, v satisfying (a) and (b). Following [8], denote $O(E)$ to be the set of all odd (even) nonnegative integers less than e. An element $y \in GF^*(n)$ will be called an (E, E) type if $T(y-1)$ is even and $T(y+1)$ is even. Similarly, we define the (E, O) type, the (O, E) type and the (O, O) type. The numbers of elements in C_t of types (O, O), (E, E), (E, O), (O, E) are denoted by $N_t(O, O)(N_t(E, E), N_t(E, O), N_t(O, E))$. Clearly, for an odd t C_t is desirable if and only if $N_t(O, O)N_t(E, E) + N_t(E, O)N_T(O, E) > 0$.

THEOREM 2. *Let t be odd. Then C_t is desirable if and only if the following two conditions hold:*

(a') *C_t contains two elements u and v such that $T(u-1) \equiv T(v-1)+1 \pmod 2$.*

(b') *C_t contains two elements u' and v' such that $T(u' + 1) \equiv T(v' + 1) + 1 \pmod 2$.*

PROOF. If C_t is desirable then (a') and (b') hold clearly. We now show that conditions (a') and (b') imply conditions (a) and (b). Suppose that u and v, and also u' and v', do not satisfy conditions (a) and (b). Then necessarily, $T(u + 1) \equiv T(v + 1) \equiv x \pmod 2$ and $T(u' - 1) \equiv T(v' - 1) \equiv v \pmod 2$ for some x and y. Since $T(u - 1)$ and $T(v - 1)$ have different parities, one of them, say, $T(u - 1)$ is congruent to $y + 1 \pmod 2$. Similarly, since $T(u' + 1)$ and $T(v' + 1)$ have different parities, one of them, say, $T(u' + 1)$ is congruent to $x + 1 \pmod 2$. Then u and u' satisfy conditions (a) and (b). $\square$

3. Main results. The cyclotomic number (s, t) for given $n = ek+1$ is defined as the number of elements $y \in GF^*(n)$ such that $T_x(y-1) \equiv s \pmod e$ and $T_x(y) \equiv t \pmod e$. Define $\beta = \exp(2\pi i/e)$ and let $J(u, v)$ be the Jacobi sum

$$\sum_{\substack{y \in GF^*(n) \\ y \neq 1}} x^u(y)x^v(1 - y)$$

for a character x on $GF(n)$ of order e. The following properties of the Jacobi sum can be found in [6]:

LEMMA 1.

$$J(0,0) = n - 2,$$
$$J(0,v) = J(u,0) = -1,$$
$$J(u,v) = J(v,u) \ \text{has absolute value either } \sqrt{n} \text{ or } 1,$$
$$J(u,v) = (-1)^{uk} J(u,-u-v),$$
$$J(u,v)J(-u,-v) = n \quad \text{for } u, v \text{ and } u+v \not\equiv 0 \pmod{e},$$
$$J(u,v) = \sum_{s=0}^{e-1} \beta^{us} \sum_{t=0}^{e-1} \beta^{-(u+v)t}(s,t).$$

It is easily verified and well known [7] that

LEMMA 2.

$$\sum_{t \in O} (-\beta)^{tu} = \sum_{t \in O} \beta^{tu} = 0, \quad \text{if } u \not\equiv 0, \frac{e}{2} \pmod{e}$$
$$= -\frac{e}{2}, \quad \text{if } u = \frac{e}{2} \pmod{e},$$
$$\sum_{t \in E} (-\beta)^{tu} = \sum_{t \in E} \beta^{tu} = 0, \quad \text{if } u \not\equiv 0, \frac{e}{2} \pmod{e}$$
$$= \frac{e}{2}, \quad \text{if } u = \frac{e}{2} \pmod{e},$$
$$\sum_{t=1}^{e-1} \beta^{tu} = \sum_{t=1}^{e-1} (-\beta)^{tu} = -1, \quad \text{if } u \not\equiv 0 \pmod{e}.$$

LEMMA 3.

$$(s,t) = \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} (-1)^u \beta^{-su-tv} J(u,v).$$

PROOF.

$$(s,t) = \frac{1}{e^2} \sum_{i=0}^{e-1} \sum_{j=0}^{e-1} (i,j) \sum_{u=0}^{e-1} \beta^{u(i-j-s+t)} \sum_{w=0}^{e-1} \beta^{w(t-j)}$$

$$= \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{w=0}^{e-1} \beta^{tw-u(s-t)} \sum_{k=0}^{e-1} \beta^{ui} \sum_{k=0}^{e-1} \beta^{-(w+u)j}(i,j)$$

$$= \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{w=0}^{e-1} \beta^{t(w+u)-us} J(u,w)$$

$$= \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} \beta^{-tv-us} J(u,-u-v) \quad \text{setting } v = -w - u$$

$$= \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} \beta^{-tv-us} (-1)^{uk} J(u,v) \quad \text{by Lemma 1}$$

$$= \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} (-1)^u \beta^{-tv-us} J(u,v) \quad \text{since } k \text{ is odd.} \quad \square$$

LEMMA 4.

$$\sum_{s \in O}(s,t) = \begin{cases} (n-1-A_t)/2e, & \text{if } t \neq 0, \\ (n-e+1-A_t)/2e, & \text{if } t = 0, \end{cases}$$

$$\sum_{s \in E}(s,t) = \begin{cases} (n-1+A_t)/2e, & \text{if } t \neq 0, \\ (n-e+1+A_t)/2e, & \text{if } t = 0, \end{cases}$$

where $A_t = \sum_{v=0}^{e-1} \beta^{-tv} J(e/2, v)$.

PROOF.

$$\sum_{s \in O}(s,t) = \frac{1}{e^2} \sum_{s \in O} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} (-1)^u \beta^{-su-tv} J(u,v)$$

$$= \frac{1}{e^2} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} (-1)^u \beta^{-tv} J(u,v) \sum_{s \in O} \beta^{-su}$$

$$= \frac{1}{e^2} \left\{ \frac{e}{2} \sum_{v=1}^{e-1} \beta^{-tv} J(0,v) - \frac{e}{2} \sum_{v=0}^{e-1} (-1)^{(n-1)/2} \beta^{-tv} J\left(\frac{e}{2}, v\right) \right\}$$

$$= \frac{1}{2e} \left\{ n - 2 - \sum_{v=1}^{e-1} \beta^{-tv} - \sum_{v=0}^{e-1} \beta^{-tv} J\left(\frac{e}{2}, v\right) \right\}$$

$$= \frac{1}{2e} \left\{ n - 2 - \sum_{v=1}^{e-1} \beta^{-tv} - A_t \right\}$$

$$= \begin{cases} (n-1-A_t)/2e, & \text{if } t \neq 0, \\ (n-e+1-A_t)/2e, & \text{if } t = 0. \end{cases}$$

Similarly, we can verify the other equation. □

LEMMA 5. $\sum_{t=0}^{e-1} A_t^2 = e^2(ke - 2k + 1)$.

PROOF.

$$\sum_{t \in O} A_t^2 = \sum_{t \in O} \sum_{u=0}^{e-1} \sum_{v=0}^{e-1} J\left(\frac{e}{2}, u\right) J\left(\frac{e}{2}, v\right) \beta^{-t(u+v)}$$

$$= \sum_{u=0}^{e-1} J\left(\frac{e}{2}, u\right) \sum_{v=0}^{e-1} J\left(\frac{e}{2}, v\right) \sum_{t \in O} \beta^{-t(u+v)}$$

$$= \sum_{u=0}^{e-1} J\left(\frac{e}{2}, u\right) \left\{ J\left(\frac{e}{2}, -u\right) \cdot \frac{e}{2} - J\left(\frac{e}{2}, \frac{e}{2} - u\right) \frac{e}{2} \right\}$$

$$= \frac{e}{2} \left\{ (e-2)n + 2 - \sum_{u=0}^{e-1} J\left(\frac{e}{2}, u\right)^2 \right\},$$

since

$$J\left(\frac{e}{2}, \frac{e}{2} - u\right) = (-1)^{(e/2)k} J\left(\frac{e}{2}, \frac{e}{2} - \left(\frac{e}{2} - u\right)\right) = J\left(\frac{e}{2}, u\right)$$

and

$$J\left(\frac{e}{2}, u\right) J\left(\frac{e}{2}, -u\right) = \begin{cases} n, & \text{if } u \neq 0, e/2, \\ 1, & \text{if } u = 0, e/2. \end{cases}$$

Similarly,

$$\sum_{t \in E} A_t^2 = \frac{e}{2} \left\{ (e-2)n + 2 + \sum_{u=0}^{e-1} J\left(\frac{e}{2}, u\right)^2 \right\}.$$

Therefore,

$$\sum_{t=0}^{e-1} A_t^2 = e((e-2)n + 2) = e((e-2)(ke+1) + 2)$$

$$= e^2(ke - 2k + 1). \quad \square$$

LEMMA 6.

$$\sum_{t=1}^{e-1} \left(\sum_{s \in O}(s,t) \right) \left(\sum_{s \in E}(s,t) \right) \geq \frac{1}{4}(k(k-1)e - (k-1)^2).$$

PROOF. By Lemma 3 and Lemma 4, we have

$$\sum_{t=1}^{e-1} \left(\sum_{s \in O}(s,t) \right) \left(\sum_{s \in E}(s,t) \right) = \sum_{t=1}^{e-1} \frac{1}{4e^2} \left((n-1)^2 - A_t^2 \right)$$

$$= \frac{1}{4e^2} \left((e-1)(n-1)^2 - \sum_{t=1}^{e-1} A_t^2 \right)$$

$$\geq \frac{1}{4e^2} \left((e-1)(n-1)^2 - e^2(ke - 2k + 1) \right)$$

(by Lemma 3, A_0 is a real number, so $A_0^2 \geq 0$)

$$= \frac{1}{4e^2} \left((e-1)(ke)^2 - e^2(ke - 2k + 1) \right)$$

$$= \frac{1}{4}(k(k-1)e - (k-1)^2). \quad \square$$

Let P denote the number of odd t satisfying (a').

LEMMA 7. $P \geq e/4 + 1$ for $k \geq 3$.

PROOF. Note that $\sum_{s \in O}(s,t)$ and $\sum_{s \in E}(s,t)$ are two nonnegative integers such that $\sum_{s \in O}(s,t) + \sum_{s \in E}(s,t) = k$. Since k is odd, we have

$$\left(\sum_{s \in O}(s,t) \right) \left(\sum_{s \in E}(s,t) \right) \leq \frac{k+1}{2} \cdot \frac{k-1}{2} = \frac{1}{4}(k^2 - 1).$$

For any $t \in \{1, 2, \ldots, e-1\}$ condition (a') does not hold if and only if

$$\left(\sum_{s \in O}(s,t) \right) \left(\sum_{s \in E}(s,t) \right) = 0.$$

Thus, we obtain

$$\sum_{t=1}^{e-1} \left(\sum_{s \in O}(s,t) \right) \left(\sum_{s \in E}(s,t) \right) \leq \left(P + \frac{e}{2} - 1 \right) \cdot \frac{1}{4}(k^2 - 1).$$

By Lemma 5, we have

$$(P + e/2 - 1)(k^2 - 1) \geq k(k - 1)e - (k - 1)^2.$$

Therefore,

$$P \geq \frac{k}{k+1}e - \frac{k-1}{k+1} - \frac{e}{2} + 1$$
$$\geq \frac{3}{4}e - \frac{e}{2} + \frac{2}{k+1} = \frac{e}{4} + \frac{2}{k+1}.$$

Since P is an integer, $P \geq e/4 + 1$. □

LEMMA 8. *There exists a desirable class for $k \geq 3$ and $m \geq 2$.*

PROOF. Let Q denote the number of odd t satisfying (b'). We claim that $P = Q$. To see this, it suffices to prove that t satisfies (a') if and only if $e/2 + t$ satisfies (b'). We verify it as follows.

$$T(u - 1) \equiv T(v - 1) + 1 \quad (\mathrm{mod}\ 2)$$
$$\Leftrightarrow T(-u + 1) \equiv T(-v + 1) + 1 \quad (\mathrm{mod}\ 2)$$
$$\Leftrightarrow T(x^{ek/2}u + 1) \equiv T(x^{ek/2}v + 1) \quad (\mathrm{mod}\ 2).$$

Clearly, $u, v \in C_t$ if and only if $x^{ek/2}u, x^{ek/2}v \in C_{ek/2+t} = C_{e/2+t}$. This concludes our claim. Now, we have $P + Q \geq 2(e/4 + 1) = e/2 + 2$. Hence, there exists an odd t satisfying both (a') and (b'). □

THEOREM 3. *SSBS exists for all $k \geq 3$ and $m \geq 2$.*

PROOF. By Lemma 8 and Theorem 1. □

4. Some concluding remarks. Our approach also proves the conjecture of Yu and Hwang [7]. It suffices to prove that for $k = 3$ and 9 there exist two elements u and v satisfying conditions (a), (b), (c). Note that for $k = 3$ every C_t for an odd t contains 2 generators. Therefore, at least one of the two elements in a desirable class is a generator. For $k = 9$ every C_t for odd t contains 6 generators. If C_t does not contain two elements u, v satisfying (a), (b) and (c), then one of the following cases occurs:

(1) $N_t(O, O)N_t(E, E) + N_t(O, E)N_t(E, O) = 0$.

(2) $N_t(O, O)N_t(E, E) \neq 0$; $N_t(O, E)N_t(E, O) = 0$, and there is no generator of (O, O) type or (E, E) type in C_t.

(3) $N_t(O, O)N_t(E, E) = 0$, $N_t(O, E)N_t(E, O) \neq 0$, and there is no generator of (O, E) type or (E, O) type in C_t.

Lemma 7 rejects the possibility of (1). If (2) occurs, then $N_t(O, O) + N_t(E, E) = 3$, so

$$N_t(O, O)N_t(E, E) + N_t(O, E)N_t(E, O) \leq 2.$$

Similarly, if (3) occurs then the above inequality again holds. Thus we have

$$\sum_{t \in O}\{N_t(O, O)N_t(E, E) + N_t(O, E)N_t(E, O)\} \leq e.$$

However, Yu and Hwang [8] have proved

$$\sum_{t \in O} \{N_t(O,O)N_t(E,E) + N_t(O,E)N_t(E,O)\} \geq \frac{1}{16}k(k-4)e > e,$$

for $k = 9$, a contradiction.

Finally we show that no desirable element can exist for $k = 1$. Note that for $k = 1$ each of the e cyclotomic classes contains only one element. Therefore the only element which satisfies condition (i): $T_x(y) = -1 \pmod{e}$ is $y = x^{-1}$. But

$$x - 1 = -x(x^{-1} - 1) = x^{e/2+1}(x^{-1} - 1).$$

Therefore

$$T_x(x^{-1} - 1) \not\equiv T_x(x - 1),$$

i.e., condition (ii) of a desirable element is violated.

Of course the above proof only shows that the DH pairing does not work for $k = 1$. The nonexistence of SSBS for $k = 1$ remains an open conjecture [5]. However, when conditions (iii) and (v) of SSBS are waived (an SSBS then becomes a *balanced starter*), then Hanner [4] proved that balanced starters exist for all odd prime powers.

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