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KARP'S THEOREM IN ELECTROMAGNETIC SCATTERING THEORY

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ABSTRACT. Karp's Theorem for acoustic waves states that if the far field pattern of the scattered wave corresponding to a plane wave incident upon an obstacle is only a function of the scalar product of the directions of incidence and observation then the obstacle is a ball. In this paper we shall give the analogue of Karp's Theorem for the scattering of electromagnetic waves by a perfect conductor.

I. Introduction. Consider the scattering of a time harmonic acoustic plane wave moving in the direction $\boldsymbol{\alpha}$ by a bounded sound-soft obstacle D. Then the scattered acoustic field has the asymptotic behavior

$$u(\mathbf{x}) = \frac{e^{ikr}}{r} F(\hat{\mathbf{x}}; k, \boldsymbol{\alpha}) + O\left(\frac{1}{r^2}\right) \tag{1.1}$$

where $\mathbf{x} \in R^3$, $r = |\mathbf{x}|$, $\hat{\mathbf{x}} = \mathbf{x}/|\mathbf{x}|$, $k > 0$ is the wave number and F is the far field pattern. The acoustic inverse scattering problem is to determine the shape of D from a knowledge of F. In 1962 Karp exhibited an explicitly solvable problem in inverse scattering theory and the solution is now known as Karp's Theorem: If D is sound-soft and F is of the form

$$F(\hat{\mathbf{x}}; k, \boldsymbol{\alpha}) = F_0((\hat{\mathbf{x}}, \boldsymbol{\alpha}); k) \tag{1.2}$$

for some function F_0 where $(\cdot, \cdot)$ denotes the scalar product then D is a ball [**5**, **6**]. The proof of Karp's Theorem cannot be modified to include the cases of a sound-hard scattering obstacle, scattering by an inhomogeneous medium or electromagnetic scattering by a perfect conductor. In [**1**], one of us provided a new approach to proving Karp's Theorem for a sound-soft obstacle which also applied to the sound-hard obstacle and an inhomogeneous medium. In this paper we shall continue the story by extending the ideas of [**1**] to treat the case of the scattering of electromagnetic waves by a perfect conductor. A fundamental difference between [**1**] and the present work is that consideration must now be given to the polarization of the electromagnetic field and the fact that the far field pattern is a vector valued function.

II. Scattering by a perfect conductor. Let D denote a bounded domain in R^3 containing the origin with C^2 boundary ∂D and unit outward normal $\boldsymbol{\nu}$ to ∂D.

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The scattering of time harmonic electromagnetic waves by the perfect conductor D is described by the following exterior boundary value problem for Maxwell's equations: Find the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ satisfying

$$\begin{aligned} \operatorname{curl}\mathbf{E} - ik\mathbf{H} = 0 \\ \operatorname{curl}\mathbf{H} + ik\mathbf{E} = 0 \end{aligned} \quad \text{in } R^3\backslash\overline{D} \tag{2.1}$$

and the boundary condition

$$[\boldsymbol{\nu}, \mathbf{E}] = 0 \quad \text{on } \partial D \tag{2.2}$$

where $[\cdot,\cdot]$ denotes the vector product, $k > 0$ is the wave number and $\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s$, $\mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$, with $\{\mathbf{E}^i, \mathbf{H}^i\}$ being the incident field and $\{\mathbf{E}^s, \mathbf{H}^s\}$ the scattered field. The scattered field is required to satisfy the Silver-Müller radiation condition

$$[\mathbf{H}^s, \hat{\mathbf{x}}] - \mathbf{E}^s = o\left(\frac{1}{|\mathbf{x}|}\right), \qquad |\mathbf{x}| \to \infty, \tag{2.3}$$

uniformly for all directions $\hat{\mathbf{x}} = \mathbf{x}/|\mathbf{x}|$.

Much of our analysis will be based on the Stratton-Chu representation formula for solutions to Maxwell's equations (cf. [**2**]). If we assume that the incident field is an entire solution of Maxwell's equations and make use of the boundary condition (2.2), this representation formula can be written in the form

$$\operatorname{curl}\int_{\partial D} \frac{e^{ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|}[\boldsymbol{\nu}(\mathbf{y}), \mathbf{H}(\mathbf{y})]ds(\mathbf{y}) = \begin{cases} -\mathbf{H}^i(\mathbf{x}), & \mathbf{x} \in D, \\ \mathbf{H}^s(\mathbf{x}), & \mathbf{x} \in R^3\backslash\overline{D}. \end{cases} \tag{2.4}$$

In particular, from (2.4) it is seen that the scattered field has the asymptotic behavior

$$\begin{aligned} \mathbf{H}^s(\mathbf{x}) &= \frac{e^{ikr}}{r}\mathbf{F}(\hat{\mathbf{x}}) + O\left(\frac{1}{r^2}\right), \\ \mathbf{E}^s(\mathbf{x}) &= \frac{e^{ikr}}{r}[\mathbf{F}(\hat{\mathbf{x}}), \hat{\mathbf{x}}] + O\left(\frac{1}{r^2}\right) \end{aligned} \tag{2.5}$$

where $r = |\mathbf{x}|$ and the magnetic far field pattern $\mathbf{F}$ is a tangential vector field on the unit sphere $\partial\Omega$ in R^3 given by

$$\mathbf{F}(\hat{\mathbf{x}}) = \frac{ik}{4\pi}\left[\hat{\mathbf{x}}, \int_{\partial D} e^{-ik(\hat{\mathbf{x}},\mathbf{y})}[\boldsymbol{\nu}(\mathbf{y}), \mathbf{H}(\mathbf{y})]\, ds(\mathbf{y})\right] \tag{2.6}$$

for all $\hat{\mathbf{x}} \in \partial\boldsymbol{\Omega}$.

We now consider as incident fields the plane waves

$$\mathbf{E}^i(\mathbf{x}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = \boldsymbol{\beta}e^{ik(\mathbf{x},\boldsymbol{\alpha})}, \qquad \mathbf{H}^i(\mathbf{x}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = \boldsymbol{\gamma}e^{ik(\mathbf{x},\boldsymbol{\alpha})} \tag{2.7}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are orthogonal unit vectors and

$$\boldsymbol{\gamma} = [\boldsymbol{\alpha}, \boldsymbol{\beta}]. \tag{2.8}$$

The unit vector $\boldsymbol{\alpha}$ describes the direction of propagation of the plane wave and the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ describe the polarization of the electric and magnetic fields respectively. We indicate the corresponding dependence of the magnetic far field pattern on $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ by writing $\mathbf{F}(\mathbf{x}) = \mathbf{F}(\mathbf{x}; \boldsymbol{\alpha}, \boldsymbol{\beta})$. If D is a ball, it is obvious from symmetry considerations that the far field pattern $\mathbf{F}$ satisfies

$$\mathbf{F}(Q\hat{\mathbf{x}}; Q\boldsymbol{\alpha}, Q\boldsymbol{\beta}) = Q\mathbf{F}(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta}) \tag{2.9}$$

for all $\hat{\mathbf{x}}, \boldsymbol{\alpha}, \boldsymbol{\beta} \in \partial\boldsymbol{\Omega}$, $(\boldsymbol{\alpha}, \boldsymbol{\beta}) = 0$, and all rotations Q, i.e. for all real orthogonal matrices Q such that $\det Q = 1$. In the next section of this paper we shall show that the converse of this statement is also true.

III. Karp's Theorem. Before proceeding to the proof of the main theorem of this paper, we shall need to introduce some notation and prove a lemma. Let j_n denote a spherical Bessel function of order n and Y_n^m, $-n \leq m \leq n$, a spherical harmonic. From the Funk-Hecke theorem [4] we have the relationship

$$\int_{\partial\boldsymbol{\Omega}} e^{ik(\mathbf{x},\boldsymbol{\alpha})} Y_n^m(\boldsymbol{\alpha})\, ds(\boldsymbol{\alpha}) = 4\pi i^n j_n(k|\mathbf{x}|) Y_n^m(\hat{\mathbf{x}}). \tag{3.1}$$

We now introduce three sets X, Y, Z of solutions $\{\mathbf{E}, \mathbf{H}\}$ to the exterior boundary value problem for Maxwell's equations corresponding to three choices of incident fields. In particular, for the set X we take incident electric fields from the set

$$X^i = \{\mathbf{E}^i(\mathbf{x}) = \boldsymbol{\beta} e^{ik(\mathbf{x},\boldsymbol{\alpha})} \colon \boldsymbol{\alpha}, \boldsymbol{\beta} \in \partial\boldsymbol{\Omega}, (\boldsymbol{\alpha}, \boldsymbol{\beta}) = 0\},$$

for the set Y we take incident electric fields from the set

$$Y^i = \{\mathbf{E}^i(\mathbf{x}) = \operatorname{curl}\operatorname{curl} \mathbf{e}_j e^{ik(\mathbf{x},\boldsymbol{\alpha})} \colon \boldsymbol{\alpha} \in \partial\boldsymbol{\Omega}, j = 1, 2, 3\}$$

where $\mathbf{e}_j$, $j = 1, 2, 3$, denote the Cartesian unit coordinate vectors, and for the set Z we take incident electric fields from the set

$$Z^i = \{\mathbf{E}^i(\mathbf{x}) = \operatorname{curl}\operatorname{curl} \mathbf{e}_j j_n(k|\mathbf{x}|) Y_n^m(\hat{\mathbf{x}}) \colon j = 1, 2, 3, \\ n = 0, 1, 2, \dots, -n \leq m \leq n\}.$$

The corresponding incident magnetic fields are defined using Maxwell's equations.

We can now prove the following lemma:

LEMMA. *The set $\{[\boldsymbol{\nu}, \mathbf{H}] \colon \{\mathbf{E}, \mathbf{H}\} \in X\}$ is complete in the Hilbert space of square integrable tangential fields defined on ∂D.*

PROOF. Assume $\mathbf{g}$ is a square integrable tangential field such that

$$\int_{\partial D} (\mathbf{g}, [\boldsymbol{\nu}, \mathbf{H}])\, ds = 0 \tag{3.2}$$

for all $\{\mathbf{E}, \mathbf{H}\} \in X$. The lemma will be proved if we can show that $\mathbf{g}$ is identically zero. From

$$\operatorname{curl}\operatorname{curl} \mathbf{e}_j e^{ik(\mathbf{x},\boldsymbol{\alpha})} = [\boldsymbol{\alpha}, [\boldsymbol{\alpha}, \mathbf{e}_j]] e^{ik(\mathbf{x},\boldsymbol{\alpha})} \tag{3.3}$$

we see that (3.2) is also valid for all $\{\mathbf{E}, \mathbf{H}\} \in Y$. From (3.1) it follows that

$$\int_{\partial\Omega} \operatorname{curl}\operatorname{curl} \mathbf{e}_j e^{ik(\mathbf{x},\boldsymbol{\alpha})} Y_n^m(\boldsymbol{\alpha})\, ds(\boldsymbol{\alpha}) = 4\pi i^n \operatorname{curl}\operatorname{curl} \mathbf{e}_j j_n(k|\mathbf{x}|) Y_n^m(\hat{\mathbf{x}}). \tag{3.4}$$

Hence, denoting the solution of (2.1)–(2.3) with incident electric field $\mathbf{E}^i(\mathbf{x}) = \operatorname{curl}\operatorname{curl} \mathbf{e}_j e^{ik(\mathbf{x},\boldsymbol{\alpha})}$ by $\{\mathbf{E}_j(\mathbf{x}; \boldsymbol{\alpha}), \mathbf{H}_j(\mathbf{x}; \boldsymbol{\alpha})\}$, we see that

$$\mathbf{H}_{j,m}^n(\mathbf{x}) = \int_{\partial\Omega} \mathbf{H}_j(\mathbf{x}; \boldsymbol{\alpha}) Y_n^m(\boldsymbol{\alpha})\, ds(\boldsymbol{\alpha}) \tag{3.5}$$

is the magnetic field corresponding to the solution of (2.1)–(2.3) with incident electric field

$$\mathbf{E}_{j,m}^n(\mathbf{x}) = 4\pi i^n \operatorname{curl}\operatorname{curl} \mathbf{e}_j j_n(k|\mathbf{x}|) Y_n^m(\hat{\mathbf{x}}). \tag{3.6}$$

Thus, multiplying (3.2) by $Y_n^m(\boldsymbol{\alpha})$ and integrating over the unit sphere, we see that (3.2) is also valid for all $\{\mathbf{E}, \mathbf{H}\} \in Z$. But by Theorem 2.2 of [**3**], the set $\{[\boldsymbol{\nu}, \mathbf{H}]\colon \{\mathbf{E}, \mathbf{H}\} \in Z\}$ is complete in the Hilbert space of square integrable tangential fields defined on ∂D. Hence $\mathbf{g}$ is identically zero and the lemma is proved.

We can now prove Karp's Theorem for electromagnetic waves.

THEOREM. *Assume that the symmetry relation* (2.9) *is true for some fixed wave number k, all unit vectors $\hat{\mathbf{x}}, \boldsymbol{\alpha}, \boldsymbol{\beta}$, $(\boldsymbol{\alpha}, \boldsymbol{\beta}) = 0$, and all real orthogonal matrices Q with* $\det Q = 1$. *Then D is a ball.*

PROOF. For a fixed incident field, the vectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma} = [\boldsymbol{\alpha}, \boldsymbol{\beta}]$ form an orthonormal basis in R^3. Hence, we can write

$$\mathbf{F}(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = f_1(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta})\boldsymbol{\alpha} + f_2(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta})\boldsymbol{\beta} + f_3(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta})\boldsymbol{\gamma} \tag{3.7}$$

and condition (2.9) is equivalent to

$$f_j(Q\hat{\mathbf{x}}; Q\boldsymbol{\alpha}, Q\boldsymbol{\beta}) = f_j(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta}), \qquad j = 1, 2, 3. \tag{3.8}$$

From the identity

$$\begin{aligned} \int_{\partial\Omega} f_j(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta})\, ds(\hat{\mathbf{x}}) &= \int_{\partial\Omega} f_j(Q\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta})\, ds(\hat{\mathbf{x}}) \\ &= \int_{\partial\Omega} f_j(\hat{\mathbf{x}}; Q^{-1}\boldsymbol{\alpha}, Q^{-1}\boldsymbol{\beta})\, ds(\hat{\mathbf{x}}) \end{aligned} \tag{3.9}$$

we see that

$$\int_{\partial\Omega} \mathbf{F}(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta})\, ds(\hat{\mathbf{x}}) = c_1\boldsymbol{\alpha} + c_2\boldsymbol{\beta} + c_3\boldsymbol{\gamma} \tag{3.10}$$

where the constants c_1, c_2, c_3 are independent of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. By considering two incident waves with the same direction of propagation $\boldsymbol{\alpha}$, but with opposite polarization directions $\boldsymbol{\beta}$, since $\mathbf{E}^i(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = -\mathbf{E}^i(\hat{\mathbf{x}}; \boldsymbol{\alpha}, -\boldsymbol{\beta})$ and $\mathbf{H}^i(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = -\mathbf{H}^i(\hat{\mathbf{x}}; \boldsymbol{\alpha}, -\boldsymbol{\beta})$ we see that

$$\mathbf{F}(\hat{\mathbf{x}}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = -\mathbf{F}(\hat{\mathbf{x}}; \boldsymbol{\alpha}, -\boldsymbol{\beta}). \tag{3.11}$$

Therefore, from (3.10) we see that $c_1 = 0$.

We now set $n = 0$ in the conjugate of (3.1) and take the gradient to arrive at

$$-\frac{ik^2}{4\pi}\int_{\partial\Omega} \hat{\mathbf{x}} e^{-ik(\hat{\mathbf{x}}, \mathbf{y})}\, ds(\hat{\mathbf{x}}) = \operatorname{Im} \boldsymbol{\Phi}'(r)\hat{\mathbf{y}} \tag{3.12}$$

where $\hat{\mathbf{y}} = \mathbf{y}/|\mathbf{y}|$, $r = |\mathbf{y}|$,

$$\boldsymbol{\Phi}(r) = e^{ikr}/r, \tag{3.13}$$

and the prime denotes differentiation with respect to r. Substituting (2.6) into (3.10) now gives

$$k\int_{\partial D} \operatorname{Im}\{\boldsymbol{\Phi}'(r)\}[\hat{\mathbf{y}}, [\boldsymbol{\nu}(\mathbf{y}), \mathbf{H}(\mathbf{y})]]\, ds(\mathbf{y}) + c_2\boldsymbol{\beta} + c_3\boldsymbol{\gamma} = 0 \tag{3.14}$$

for all $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. On the other hand, setting $\mathbf{x} = 0$ in (2.4) gives

$$\frac{1}{4\pi}\int_{\partial D} \boldsymbol{\Phi}'(r)[\hat{\mathbf{y}}, [\boldsymbol{\nu}(\mathbf{y}), \mathbf{H}(\mathbf{y})]]\, ds(\mathbf{y}) = H^i(0) = \boldsymbol{\gamma} \tag{3.15}$$

and taking the curl of (2.4) and setting $\mathbf{x} = 0$ gives

$$\begin{aligned}(3.16)\quad \frac{1}{4\pi}\int_{\partial D}\Big\{\Big(k^2\Phi(r)+\frac{1}{r}\Phi'(r)\Big)\,[\boldsymbol{\nu}(\mathbf{y}),\mathbf{H}(\mathbf{y})]\\ +r\frac{d}{dr}\Big(\frac{\Phi'(r)}{r}\Big)\,\hat{\mathbf{y}}(\hat{\mathbf{y}},\boldsymbol{\nu}(\mathbf{y}),\mathbf{H}(\mathbf{y}))\Big\}\,ds(\mathbf{y}) = ikE^i(0) = ik\boldsymbol{\beta}\end{aligned}$$

where $(\cdot,\cdot,\cdot)$ denotes the triple product. Hence, substituting (3.15) and (3.16) into (3.14), we see that

$$\begin{aligned}(3.17)\quad \int_{\partial D}\{g_1(r)[\hat{\mathbf{y}},[\boldsymbol{\nu}(\mathbf{y}),\mathbf{H}(\mathbf{y})]]+g_2(r)[\boldsymbol{\nu}(\mathbf{y}),\mathbf{H}(\mathbf{y})]\\ +g_3(r)(\hat{\mathbf{y}},\boldsymbol{\nu}(\mathbf{y}),\mathbf{H})(\mathbf{y})\hat{\mathbf{y}}\}\,ds(\mathbf{y}) = 0\end{aligned}$$

for all $\boldsymbol{\alpha},\boldsymbol{\beta}$ where

$$(3.18)\qquad \begin{aligned} g_1(r) &= k\,\mathrm{Im}\,\Phi'(r)+\frac{c_3}{4\pi}\Phi'(r),\\ g_2(r) &= \frac{c_2}{4\pi ik}\Big(k^2\Phi(r)+\frac{1}{r}\Phi'(r)\Big),\\ g_3(r) &= \frac{c_2}{4\pi ik}r\frac{d}{dr}\Big(\frac{\Phi'(r)}{r}\Big).\end{aligned}$$

We now take the scalar product of (3.17) with a vector $\mathbf{a}\in R^3$ and write this as

$$(3.19)\qquad \int_{\partial D}(\mathbf{g}_a,[\boldsymbol{\nu},\mathbf{H}])\,ds = 0$$

with

$$(3.20)\qquad \mathbf{g}_a(\mathbf{y}) = g_1(r)[\mathbf{a},\hat{\mathbf{y}}]+g_2(r)\mathbf{a}+g_3(r)(\mathbf{a},\hat{\mathbf{y}})\hat{\mathbf{y}}.$$

By the lemma proved at the beginning of this section, (3.19) implies that

$$(3.21)\qquad [\mathbf{g}_a,\boldsymbol{\nu}] = 0\quad\text{on } \partial D.$$

Let $\mathbf{y}\in\partial D$ be fixed and choose $\mathbf{a}$ to be orthogonal to $\mathbf{y}$. Then from (3.20), (3.21) we have that

$$(3.22)\qquad g_1(r)[[\mathbf{a},\hat{\mathbf{y}}],\boldsymbol{\nu}(\mathbf{y})]+g_2(r)[\mathbf{a},\boldsymbol{\nu}(\mathbf{y})] = 0$$

for all $\mathbf{a}$ such that $(\mathbf{a},\mathbf{y}) = 0$. Taking the scalar product with $\mathbf{a}$, this implies that

$$(3.23)\qquad g_1(r)(\hat{\mathbf{y}},\boldsymbol{\nu}(\mathbf{y})) = 0.$$

Assume that $g_1(r)$ is not zero. Then $(\hat{\mathbf{y}},\boldsymbol{\nu}(\mathbf{y})) = 0$ and we can choose $\mathbf{a} = \boldsymbol{\nu}(\mathbf{y})$ in (3.22) to arrive at $g_1(r)\hat{\mathbf{y}} = 0$, i.e. $g_1(r) = 0$, a contradiction. Hence, since $\mathbf{y}\in\partial D$ can be chosen arbitrarily, we have that

$$(3.24)\qquad g_1(r) = 0$$

for all $\mathbf{y}\in\partial D$. This implies, by the analyticity of g_1, that $r = |\mathbf{y}|$ must be constant for $\mathbf{y}\in\partial D$, i.e. D is a ball.

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