PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 106, Number 4, August 1989

ON THE BLOWUP OF u_t AT QUENCHING

KENG DENG AND HOWARD A. LEVINE

(Communicated by Barbara L. Keyfitz)

ABSTRACT. Let Ω be a bounded convex domain in $\mathbf{R}^n$ with smooth boundary. We consider the problems (C): $u_t = \Delta u + \varphi(u)$ in $\Omega \times (0,T)$, while $u = 0$ on $\partial\Omega \times (0,T)$ and $u(x,0) = u_0(x)$. Here $\varphi(u)$: $(-\infty, A) \to (0,\infty)$ $(A > 0)$ satisfies $\varphi'(u) \geq 0$, $\varphi''(u) \geq 0$, and $\lim_{u \to A^-} \varphi(u) = +\infty$, while u_0 satisfies $\Delta u_0(x) + \varphi(u_0(x)) \geq 0$. We show that if u quenches (reaches A in finite time), then the quenching points are in a compact subset of Ω and u_t blows up. We also extend the result to the third boundary value problem.

1. INTRODUCTION

In his paper [1], Kawarada studied the following initial boundary value problem:

$$
\text{(A)} \qquad \begin{aligned}
&u_t = u_{xx} + \frac{1}{1-u}, && 0 < x < L, && t > 0,\\
&u(0,t) = u(L,t) = 0, && && t > 0,\\
&u(x,0) = 0, && 0 \leq x \leq L. && .
\end{aligned}
$$

He showed the following:

(i) If $L > 2\sqrt{2}$, then $u(L/2,t)$ reaches one in finite time.
(ii) If $u(L/2,t)$ reaches one in finite time, then $u_t(L/2,t)$ becomes unbounded in finite time.

When (ii) occurs, Kawarada says that u quenches in finite time. Unfortunately, his methods do not appear to extend readily to more general cases. Therefore a weaker definition was posed in [2,4] where u is said to quench if (i) occurs. For more general problems of parabolic type, some results were obtained over the past few years by several authors (see [2,3,4,5,6,7]).

In [12] the authors remark that Kawarada's proof of (ii) is incomplete. They give a complete proof of (ii) using elementary arguments for a more general class of nonlinearities which includes those of the form $(1-u)^{-\beta}$ for $\beta \geq 1$.

Received by the editors July 20, 1988 and, in revised form, November 22, 1988.
1980 *Mathematics Subject Classification* (1985 *Revision*). 35K55, 35K60.
This research was supported by the Air Force office of Scientific Research under Grant AFOSR 88-0031. The United States Government is authorized to reproduce and distribute reprints for governmental purposes not withstanding any copyright notation therein..

In [13], the author has shown that if $\beta \geq 3$, then the behavior of u at a quenching point is asymptotically precisely the same as that for the solution of the initial value problem $y' = (1-y)^{-\beta}$, $y(0) = 0$ on $(0,T)$. In [14], he extended these results to radial solutions in sufficiently large balls.

Recently, A. Acker and B. Kawohl in [8] investigated an analogous problem in several dimensions.

$$\text{(B)} \qquad \begin{aligned} &u_t = \Delta u + f(u), && x \in B_a, && t > 0, \\ &u = 0, && x \in \partial B_a, && t > 0, \\ &u(x,0) = u_0(x), && x \in B_a. \end{aligned}$$

where B_a is a ball in $\mathbf{R}^n$ with center at the origin. Under the assumptions that $u(x,t) = u(r,t)$ where $r = |x|$ and the initial values satisfy $u(r,0) \geq 0$, $u_r(r,0) \leq 0$, $u_t(r,0) \geq 0$ and $u_{rt}(r,0) \leq 0$ and with some mild restrictions on the nonlinearity $f(u)$, they proved that if u quenches, then the only quenching point is the origin and $u_t(0,t)$ is unbounded.

In this paper, we consider the more general problem:

$$\text{(C)} \qquad \begin{aligned} &u_t = \Delta u + \varphi(u), && x \in \Omega, && 0 < t < T, \\ &u = 0, && x \in \partial\Omega, && 0 < t < T, \\ &u(x,0) = u_0(x), && x \in \Omega. \end{aligned}$$

Here Ω is a bounded convex domain in $\mathbf{R}^n$ with smooth boundary. $\varphi(u)$: $(-\infty, A) \to (0,\infty)$ $(A > 0)$ satisfies $\varphi'(u) \geq 0$, $\varphi''(u) \geq 0$, and $\lim_{u \to A^-} \varphi(u) = +\infty$; while the initial datum satisfies $0 \leq u_0 < A$ and $\Delta u_0 + \varphi(u_0) \geq 0$.

In §2, we prove that if u quenches in finite or infinite time, then the quenching points are in a compact set. In §3, we show that u_t blows up at finite quenching time. In §4, we extend the result in §3 to the third boundary value problem. Our arguments are based on modifications of those in [9] for blowup problems.

Our result also allows us to obtain blowup results. By means of transformation $v = -\ln(1-u)$, the differential equation in (C) becomes

$$v_t = \Delta v + e^v \varphi(1 - e^{-v}) - |\nabla v|^2.$$

Thus, as remarked in [8], blowup of solutions of this equation is equivalent to the quenching of solutions (C). Thus, following [8], we may say that the set of blowup points for the Dirichlet initial-boundary value problem for the above equation be in a compact subset of Ω and whenever v blows up in finite time so does $e^{-v} v_t$.

In [11], the authors considered the Dirichlet initial boundary value problem for

$$v_t = \Delta v - |\nabla v|^q + |v|^{p-1} v$$

and showed that if $1 < q \leq 2p/(p+1)$, $p > 1$, these solutions blow up in finite time. Clearly the case $q = 2$ is excluded from their result. On the other hand, if we consider the equation

$$v_t = \Delta v + \varepsilon e^{(\beta+1)v} - |\nabla v|^2,$$

then, with $\varphi(u) = \varepsilon(1-u)^{-\beta}$, our results tell us that for $\beta > 0$, some solutions v (with v_t) blow up if ε is sufficiently large. (Blowup results are well known for these last two equations when $|\nabla v|^2$ is *not* present on the right-hand side.)

2. The location of the quenching points

We begin with an introduction of some remarks and definitions. We show that the quenching points lie in a compact subset of Ω.

Let ν be a unit vector in $\mathbf{R}^n$ and let T_λ be the hyperplane $\nu \cdot x = \lambda$. Let the plane move continuously toward Ω with the same normal, i.e., decrease λ, until it begins to intersect $\overline{\Omega}$. From that moment on, at every stage the plane T_λ will cut off from Ω an open cap $\Sigma(\lambda)$ associated with ν. Note that ν is the outer normal of $\partial\Omega$ at the point p of the boundary which T_λ first touches. For simplicity, we use λ to denote the distance from p to T_λ, and denote $\Omega \times \{t = \eta\}$ by Ω_η $(0 < \eta < T)$.

Lemma 2.1. *If $\varphi(u)\colon (-\infty, A) \to (0, \infty)$ is continuously differentiable and $u_0(x) \geq 0$, then for every η $(0 < \eta < T)$ at every point p_0 on $\partial\Omega_\eta \times (\eta, T)$, there is a cap $\Sigma(\lambda_0)$, such that $\partial u/\partial n_{p_0} < 0$ for any $(x, t) \in \Sigma(\lambda_0) \times (\eta, T)$.*

Proof. Since $u > 0$ in $\Omega_\eta \times [\eta, T)$ and $u = 0$ on $\partial\Omega_\eta \times (\eta, T)$, this follows from the similar argument in [9]. □

Now we call $D(\lambda_0) = \Sigma(\lambda_0) \times (\eta, T)$ the cylinder.

For fixed $\eta \in (0, T)$, $\partial\Omega_\eta$ is a compact set. Hence $\lambda_{\max} = \max\{\lambda_0\}$ and $\lambda_{\min} = \min\{\lambda_0\}$ exist, and $\lambda_{\max} \geq \lambda_{\min} > 0$. Let $\Omega_{\lambda_0} = \{p | \overrightarrow{p_{\lambda_0} p} \cdot \overrightarrow{p_{\lambda_0} p_0} < 0\}$, with p_0 on $\partial\Omega_\eta$ and p_{λ_0} on T_{λ_0} with $\overrightarrow{p_{\lambda_0} p_0}$ perpendicular to T_{λ_0}.

Let

$$\Omega'_\eta = \bigcap_{p_{\lambda_0} \in T_{\lambda_0}} \Omega_{\lambda_0}.$$

Ω'_η is the complement of the union of all $\Sigma(\lambda_0)$'s with respect to Ω_η. Clearly $\Omega'_\eta \subset\subset \Omega_\eta$.

Now we state our main result as follows:

Theorem 2.2. *Assume that the conditions in Lemma 2.1 hold, and $\varphi'(u) \geq 0$ for $0 \leq u < A$, $0 \leq u_0(x) < A$. Then the quenching points are in a compact set of $\Omega \times (\eta, T)$ in the sense that $\{x \mid \lim_{t \to T^-} u(x, t) = A\}$ is a subset of $\overline{\Omega}'_\eta$.*

Proof. For any point $p_*(x^*, t^*) \in (\Omega_\eta \backslash \Omega'_\eta) \times (\eta, T)$, there is a point $p_0(x^0, t^*) \in \partial\Omega \times \{t = t^*\}$ such that the line L_0 through p_0 and p_* has the same direction as n_{p_0} at p_0. By Lemma 2.1, p_* is contained in a cylinder $D(\lambda_0)$. We construct a new cylinder $D(\lambda_*)$ by using T_{λ_*} instead of T_{λ_0}, where $0 < \lambda_* < \lambda_0$ is such that p_* remains in $D(\lambda_*)$. From the conclusion of Lemma 2.1, we know that $\partial u/\partial n_{p_0} < 0$ in $D(\lambda_*)$; in particular for all (x, t) on $T_{\lambda_*} \cap (\Omega \times (\eta, T))$.

Without loss of generality, we may assume $n_{p_0} = (1, 0, \dots, 0)$, $x = (x_1, x')$, $(x' = (x_2, \dots, x_n))$, and (x, t^*) is on the line L_0. We let $p_0 = (x_1^0, x', t^*)$, $p_* = (x_1^*, x', t^*)$, and the point $\bar{p} = (\bar{x}_1, x', t^*)$ where the line L_0 intersects T_{λ_*}, i.e., $T_{\lambda_*} = \{x_1 = \bar{x}_1\}$. Obviously, $\partial u / \partial n_{p_0} = \partial u / \partial x_1$.

Now we define a function in $D(\lambda_*)$ by:

$$F(x, t) = \frac{\partial u}{\partial x_1} + c(x_1 - \bar{x}_1).$$

Here c is a positive constant to be determined. We have

$$F_t = u_{x_1 t}$$
$$\Delta F = \Delta u_{x_1}$$

so $F_t - \Delta F = \varphi'(u) u_{x_1} \le 0$ in $D(\lambda_*)$. On the boundary of $D(\lambda_*)$, we have at $T_{\lambda_*} = \{x_1 = \bar{x}_1\}$, $F = u_{x_1} < 0$, on $D(\lambda_*) \cap \{t = \eta\}$,

$$F = u_{x_1}(x, \eta) + c(x_1 - \bar{x}_1) \le \max_{D(\lambda_*) \cap \{t = \eta\}} u_{x_1}(x, \eta) + c\lambda_{\max} < 0$$

provided $c < -\max u_{x_1}(x, \eta) / \lambda_{\max}$.

To show that $F < 0$ on $\partial D(\lambda_*) \cap (\partial \Omega \times (\eta, T))$, we consider the problem:

$$\begin{aligned} &v_t = \Delta v + \varphi(0), && x \in \Omega, && 0 < t < T, \\ &v = 0, && x \in \partial\Omega, && 0 < t < T, \\ &v(x, 0) = 0, && x \in \Omega. \end{aligned}$$

Since $\varphi'(u) \ge 0$, we have $u \ge v$ by the maximum principle, and $u \not\equiv v$. It follows from the maximum principle that

$$\frac{\partial u}{\partial n} < \frac{\partial v}{\partial n} \le -c_1 < 0 \quad \text{on } \partial\Omega \times (0, T).$$

In particular,

$$\frac{\partial u}{\partial x_1} = \frac{\partial u}{\partial n} \cos\langle n, x_1 \rangle \le -c_1 \cos\langle n, x_1 \rangle \quad \text{on } \partial D(\lambda_*) \cap (\partial\Omega \times (\eta, T)).$$

We may assume

$$\cos\langle n, x_1 \rangle \ge c_2 > 0$$

for some $c_2 > 0$ and every point $(x, t) \in \partial D(\lambda_*) \cap (\partial\Omega \times (\eta, T))$. Therefore on $\partial D(\lambda_*) \cap (\partial\Omega \times (\eta, T))$,

$$\frac{\partial u}{\partial x_1} + c(x_1 - \bar{x}_1) \le -c_1 c_2 + c\lambda_{\max} < 0 \quad \text{if } c < \frac{c_1 c_2}{\lambda_{\max}}.$$

Letting $c = \min\{-\max u_{x_1}(x, \eta)/\lambda_{\max}, c_1 c_2 / \lambda_{\max}\}$, we find that

$$F(x, t) \le 0 \quad \text{in } D(\lambda_*),$$

or

$$-u_{x_1} \ge c(x_1 - \bar{x}_1).$$

Thus for any point $p_* = (x_1^*, x', t^*)$, we see that

$$\int_{\bar{x}_1}^{x_1^*} (-u_{x_1})\, dx_1 \geq c \int_{\bar{x}_1}^{x_1^*} (x_1 - \bar{x}_1)\, dx_1$$
$$u(\bar{x}_1, x', t^*) - u(x_1^*, x', t^*) \geq c(x_1^* - \bar{x}_1)^2/2$$

or

$$\begin{aligned} u(x_1^*, x', t^*) &\leq u(\bar{x}_1, x', t^*) - c(x_1^* - \bar{x}_1)^2/2 \\ &\leq A - c(x_1^* - \bar{x}_1)^2/2. \end{aligned}$$

Since $p_*(x^*, t^*)$ is an arbitrary point in $(\Omega_\eta \backslash \Omega'_\eta) \times (\eta, T)$, the set of quenching points lies in a compact set.

Remark 2.1. In Theorem 3.3 of [9], the authors define (with J instead of F)

$$F(x, t) = u_{x_1} + c(x_1 - \bar{x}_1)\mathscr{F}(u)$$

where $\mathscr{F}$ is required to satisfy (2.23) of [9], i.e.

$$\int^{\infty} [\mathscr{F}(s)]^{-1}\, ds < \infty$$

among other conditions. Our $\mathscr{F}(\equiv 1)$ does not satisfy this condition.

Remark 2.2. We do not require that T be finite in the proof of the theorem and the result thus holds for $0 < T \leq \infty$.

Remark 2.3. If $u_0(x)$ satisfies the condition (2.2) in [10] on a part of Ω, then we can locate the quenching points more precisely. Especially, if Ω is a ball in $\mathbf{R}^n$ and u is a radial solution with $\partial u_0/\partial r \leq 0$, then the center of the ball is the only quenching point.

Remark 2.4. If in (C) we replace φ by $\varepsilon\varphi$ where $\varepsilon > 0$, then it is known that for all sufficiently large ε, the set of quenching points is not empty [2,3,4,7].

3. The blowup of u_t at quenching

As an application of Theorem 2.2, we now show that when u quenches then u_t blows up. Here we use a modification of an argument of [9].

Theorem 3.1. *Assume, in addition to the hypotheses in Theorem* 2.2, *that* $\varphi''(u) \geq 0$ *for* $0 \leq u < A$ *and* $\Delta u_0 + \varphi(u_0) \geq 0$ *in* Ω. *Then if* u *quenches in finite time,* u_t *blows up.*

Proof. Let Ω''_η be the set $\{x \mid \operatorname{dist}(x, \Omega'_\eta) \leq \frac{1}{2}\lambda_{\min}\}$, it is clear that $\Omega'_\eta \subset\subset \Omega''_\eta \subset\subset \Omega_\eta$.

Consider the function $G(x,t) = u_t - \delta\varphi(u)$ in $\Omega''_\eta \times (\eta, T)$, where δ is an undetermined positive constant.

$$\begin{aligned} G_t &= u_{tt} - \delta\varphi'(u)u_t, \\ \Delta G &= \Delta u_t - \delta\varphi''(u)|\nabla u|^2 - \delta\varphi'(u)\Delta u, \\ G_t - \Delta G &= \varphi'(u)u_t + \delta\varphi''(u)|\nabla u|^2 - \delta\varphi'(u)\varphi(u), \\ &= \varphi'(u)(G + \delta\varphi(u)) + \delta\varphi''(u)|\nabla u|^2 - \delta\varphi'(u)\varphi(u) \\ &= \varphi'(u)G + \delta\varphi''(u)|\nabla u|^2, \end{aligned}$$

so

$$G_t - \Delta G - \varphi'(u)G \geq 0.$$

From the maximum principle, it follows that G cannot take negative minimum in $\Omega''_\eta \times (\eta, T)$.

On the parabolic boundary, $\varphi(u) \leq c$ in Ω''_η, while on $\partial\Omega''_\eta \times (\eta, T)$ (by Theorem 2.2), $\varphi(u) \leq c$ also. On the other hand, using the condition $\Delta u_0 + \varphi(u_0) \geq 0$ in Ω, we see that $u_t > 0$ in $\Omega \times (0, T)$. Thus $u_t \geq c_1 > 0$ for (x,t) on $\partial\Omega''_\eta \times (\eta, T)$ and (x, η) in Ω'_η. Hence if $\delta < c_1/c$, then $G > 0$ in Ω''_η. It follows that $G \geq 0$ in $\Omega''_\eta \times (\eta, T)$, i.e. $u_t \geq \delta\varphi(u)$. Thus $\lim_{u \to A^-} u_t = +\infty$.

Remark. In the proof of the theorem, c_1 depends on T. This means that the method does not apply to the case $T = \infty$. This is in agreement with the observation in [4] where it was shown that when Ω is an interval and when $\lim_{t\to\infty} u(x_0, t) = A$, then $\lim_{t\to\infty} u_t(x_0, t) = 0$.

4. The blowup of u_t for the Robin condition

In this section, we extend some of the previous results to the following problem:

$$\text{(D)} \qquad \begin{aligned} &u_t = \Delta u + \varphi(u), && x \in \Omega, && 0 < t < T, \\ &\partial u/\partial n + \beta u = 0, && x \in \partial\Omega, && 0 < t < T, \\ &u(x, 0) = u_0(x), && x \in \Omega \end{aligned}$$

where $\beta = \beta(x,t) \geq 0$, $\beta_t \leq 0$.

Because the main theorem in §2 is based on Lemma 2.1, which strongly depends on the Dirichlet problem (C), it cannot be applied to problem (D). Instead, we use other arguments to achieve the same result as in Theorem 3.1.

Lemma 4.1. *If $\varphi(u) > 0$, $\varphi'(u)$ exists for $0 \leq u < A$, and $\lim_{u\to A^-} \varphi(u) = +\infty$, then there exists a number A_0 $(0 < A_0 < A)$, such that $u\varphi'(u) \geq \varphi(u)$ if $u \geq A_0$.*

Proof. Let $\Phi(u) = u/\varphi(u)$. We find that $\Phi(0) = 0$, $\Phi(u) > 0$ for $0 < u < A$, and $\lim_{u\to A^-} \Phi(u) = 0$. Thus setting $\Phi(A) = 0$, we have that $\Phi(u)$ is continuous in $[0, A]$, and differentiable in $(0, A)$. Let A_0 be the point nearest A where $\Phi(u)$ obtains its maximum, then $\Phi'(A_0) = 0$ and $\Phi'(u) \leq 0$, or $u\varphi'(u) \geq \varphi(u)$ for $u \geq A_0$.

Lemma 4.2. *If the conditions in Lemma* 4.1 *are satisfied,* $\varphi'(u) \geq 0$ *for* $0 \leq u < A$ *and* $\Delta u_0 + \varphi(u_0) \geq 0$ *in* Ω, *then if* u *is the solution of the problem* (D), $u_t \geq c > 0$ *for* $(x,t) \in \Omega \times (\eta, T)$.

Proof. Set $v(x,t) = u_t(x,t)$. Then $v(x,t)$ satisfies

$$\begin{aligned} &v_t = \Delta v + \varphi'(u)v, && x \in \Omega, \quad 0 < t < T, \\ &\partial v/\partial n + \beta v \geq 0, && x \in \partial\Omega, \quad 0 < t < T, \\ &v(x,0) = \Delta u_0 + \varphi(u_0) \geq 0, && x \in \Omega. \end{aligned}$$

By the maximum principle, $v > 0$ in $\Omega \times (0,T)$. Since $\varphi'(u) \geq 0$,

$$v_t \geq \Delta v \quad \text{in } \Omega \times (0,T).$$

Now consider the related problem:

$$\begin{aligned} &w_t = \Delta w, && x \in \Omega, \quad \eta < t < T, \\ &\partial w/\partial n + \beta w = 0, && x \in \partial\Omega, \quad \eta < t < T, \\ &w(x,\eta) = v(x,\eta), && x \in \Omega. \end{aligned}$$

Since $v(x,t) \geq w(x,t)$ and $w(x,t) \geq c$ in $\Omega \times (\eta, T)$, the conclusion follows.

Now we can use the technique adopted in [9] to show the following:

Theorem 4.2. *Under the same hypotheses as in Theorem* 3.1, *we have that* u_t *blows up at quenching.*

Proof. Set $G(x,t) = u_t - \delta\varphi(u)$.

As before,

$$G_t - \Delta G - \varphi'(u)G \geq 0 \quad \text{in } \Omega \times (\eta, T).$$

By applying Lemma 4.2,

$$G(x,\eta) > 0 \quad \text{for } x \in \Omega \quad \text{if } \delta < c/\max_{\Omega_\eta} \varphi(u(x,\eta)).$$

Then we claim that $G(x,t) \geq 0$ for all $(x,t) \in \partial\Omega \times (\eta, T)$ provided $\delta \leq c/\varphi(A_0)$.

Assume the contrary: G takes a negative value at some point $(x_0, t_0) \in \partial\Omega \times (\eta, T)$.

Let $G(x^*, t^*)$ be a negative minimum on $\partial\Omega \times [\eta, t_0]$. From the choice of δ and the fact that G cannot have negative minimum in $\Omega \times [\eta, t_0]$, it is clear that $u(x^*, t^*) > A_0$, and that

$$\frac{\partial G}{\partial n} + \beta G < 0 \quad \text{at } (x^*, t^*)$$

or

$$\left(\frac{\partial u_t}{\partial n} + \beta u_t\right) - \delta\left(\frac{\partial \varphi}{\partial n} + \beta\varphi\right) < 0.$$

It follows that

$$\frac{\partial \varphi}{\partial n} + \beta\varphi > 0 \quad \text{at } (x^*, t^*).$$

Note that

$$\frac{\partial \varphi}{\partial n} = \varphi'(u)\frac{\partial u}{\partial n} = -\beta u \varphi'(u).$$

We find that $\varphi(u) - u\varphi'(u) > 0$ for $u = u(x^*, t^*) > A_0$.

This leads to a contradiction.

Therefore we have shown that $G(x, t) \geq 0$ in $\Omega \times (\eta, T)$, which yields the desired result.

References

1. H. Kawarada, *On the solutions of initial-boundary value problems for* $u_t = u_{xx} + \frac{1}{1-u}$, Publ. Res. Inst. Math. Sci. **10** (1975), 729–736.
2. A. Acker and W. Walter, *The quenching problem for nonlinear parabolic equations*, Lecture Notes in Math., vol. 564, Springer-Verlag, New York, 1976.
3. ——, *On the global existence of solutions of parabolic differential equations with a singular nonlinear term*, Nonlinear Anal. **2** (1978), 449–505.
4. H. A. Levine and J. T. Montgomery, *The quenching of solutions of some nonlinear parabolic equations*, SIAM J. Math. Anal. **11** (1980), 842–847.
5. H. A. Levine, *The quenching of solutions of linear parabolic and hyperbolic equations with nonlinear boundary conditions*, SIAM J. Math. Anal. **14** (1983), 1139–1153.
6. ——, *The phenomenon of quenching: A survey*, Trends in the Theory and Practice of Nonlinear Analysis (V. Lakshmikantham, ed.) Elsevier Science Publ., North Holland, 1985, pp. 275–286.
7. ——, *Quenching, nonquenching, and beyond quenching for solutions of some parabolic equations*, Ann. Mat. Pura Appl. (in press).
8. A. Acker and B. Kawohl, *Remarks on quenching*, Nonlinear Anal. TMA 13 (1989), 53–61.
9. A. Friedman and B. McLeod, *Blow-up of positive solutions of semilinear heat equations*, Indiana Univ. Math. J. **34** (1985), 425–447.
10. B. Gidas, W.-M. Ni, and L. Nirenberg, *Symmetry and related properties via the maximum principle*, Comm. Math. Phys. **68** (1979), 209–243.
11. M. Chipot and F. B. Weissler, *Some blow up results for a nonlinear parabolic equation with a gradient term*, IMA Preprint Series no. 298.
12. C. Y. Chan and M. K. Kwong, *Quenching phenomena for singular nonlinear parabolic equations*, Nonlinear Analysis, TMA 12 (1988), 1377–1383.
13. J. Guo, *On the quenching behavior of the solution of a semilinear parabolic equation*, J. Math. Anal. Appl. (in print).
14. ——, *On the semilinear elliptic equation*, $\Delta w - \frac{1}{2} y \cdot \nabla w + \lambda w - w^{-\beta} = 0$ in R^N, (in print).

Department of Mathematics, Iowa State University, Ames, Iowa 50011