SOME ESTIMATES FOR HARMONIC MEASURES. II

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(Communicated by Clifford J. Earle, Jr.)

Abstract. FitzGerald, Rodin, and Warschawski proved that, for a continuum of given diameter in the closed unit disc, the harmonic measure at the center is minimized when it is an arc on the circumference. A very simple proof of this result is given, using the method of the extremal metric.

In [1], FitzGerald, Rodin, and Warschawski gave the solution for two extremal problems for the harmonic measure of a continuum lying in the unit disc. These results may be stated as follows, using the following notations: $F$ is the closed unit disc $|z| < 1$, $E$ the open unit disc, $C$ a continuum in $F$ not containing the origin, $G$ the component of $E - C$ containing the origin, $\alpha$ the border entity of $G$ determined by $C$, and $\omega(0, \alpha, G)$ the harmonic measure of $\alpha$ at 0 with respect to $G$.

I. If $C$ has diameter $\delta$, $\omega(0, \alpha, G) \geq \frac{1}{2\pi} \theta$ where $\theta$ is the angular measure of an arc on $|z| = 1$ of diameter $\delta$, with equality precisely in that case.

II. If $C$ subtends an angle of measure $\theta$ equal at most to $\pi$ at 0, $\omega(0, \alpha, G) \geq \frac{1}{2\pi} \theta$ with equality if and only if $C$ is an arc of angular measure $\theta$ on $|z| = 1$.

In [2], the present author pointed out that by using triad modules a very simple proof of II can be given by the method of the extremal metric. Now we will observe that I is an immediate consequence of the extremal property of the Mori extremal domain.

The extremal property of the Mori extremal domain [3] can be stated as follows:

III. Let $C^*$ be a continuum not containing the origin or the point at infinity which has two points in $|z| \leq 1$ of distance $\geq \delta > 0$. Let $\Gamma^*$ be the homotopy...
class of rectifiable Jordan curves separating $C^*$ from 0 and $\infty$. Then the module $m(\Gamma^*)$ of $\Gamma^*$ is maximal precisely when $C^*$ is an arc of diameter $\delta$ on $|z| = 1$.

To derive I from III, we remark first, as in [2], that we may assume $C$ meets $|z| = 1$. Then I is equivalent to maximizing the triad module $m(0, \beta, G)$ where $\beta$ is the open boundary arc of $G$ on $|z| = 1$. Let $\tilde{C}$ be the reflection of $C$ in $|z| = 1$, $C^* = C \cup \tilde{C}$. Evidently $m(0, \beta, G) = 2m(\Gamma^*)$. Now I follows at once from III.

BIBLIOGRAPHY

