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A GENERIC TORELLI-TYPE THEOREM FOR SINGULAR ALGEBRAIC CURVES WITH AN INVOLUTION

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ABSTRACT. We prove a generic Torelli-type theorem for a special class of singular algebraic curves with an involution. In order to obtain this result we introduce an appropriate mixed Hodge structure on the anti-invariant part of the first homology group, and study its properties.

Let X be an irreducible projective algebraic curve with an involution σ. Suppose X has only ordinary singularities and let Σ be its singular locus. Let $\pi: N \to X$ be the normalization of X and let $\tau: N \to N$ be the involution induced by σ. We suppose that the following condition is satisfied:

(∗) The set of fixed points of σ coincides with Σ and the involution τ is without fixed points.

Following J. Carlson [2] one can introduce a polarized mixed Hodge structure (PMHS) on the anti-invariant part of $H_1(X, \mathbb{Z})$ with respect to σ, denoted by $H_1^-(X, \mathbb{Z})$:

$$0 \to H_1^-(N, \mathbb{Z}) \to H_1^-(X, \mathbb{Z}) \to A \to 0,$$

where $H_1^-(N, \mathbb{Z})$ is the anti-invariant part of $H_1(N, \mathbb{Z})$ with respect to τ and has a polarized Hodge structure (PHS) of weight -1; A has PHS of weight 0. It turns out that the latter is isomorphic to the lattice generated by a root-system of the type D_n when $\#\{\pi^{-1}(\Sigma)\} > 2$.

Using the generic Torelli theorem for the Prym map as proven by Friedman–Smith [4] and Kanev [5], the pair (N, τ) is uniquely determined by its Prym variety (equivalently by the PHS of $H_1^-(N, \mathbb{Z})$) if the following condition is satisfied.

$$N/\tau \text{ is a sufficiently general curve of genus } g \geq 7.$$

We prove the following result:

Theorem 7. *Let X be the curve which satisfies $(*)$. Suppose that N/τ is a general curve of genus $g \geq 15$. Then N, τ, and the set $\pi^{-1}(\Sigma)$ are uniquely determined by the PMHS of $H_1^-(X, \mathbb{Z})$.*

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As a consequence we obtain

Theorem 8. *If X satisfies the conditions of Theorem 7 and has only one singular point then X and σ are uniquely determined by the PMHS of $H_1^-(X, \mathbb{Z})$.*

1. Preliminaries

Let X be an irreducible projective curve with ordinary singularities, and $n: N \to X$ be its normalization for which

(i) there exist involutions $\tau: N \to N$ and $\sigma: X \to X$, such that τ has no fixed points and the fixed points of σ are the singularities of X;
(ii) $\pi \circ \tau = \sigma \circ \tau$.

Such curves can be constructed as follows:

(i) we fix (N, τ) to be a smooth irreducible projective algebraic curve with an involution without fixed points;
(ii) we choose a finite subset $\Omega \subset N$ such that $\Omega = \bigcup_{i=1}^k \Omega_i$, and for each i, $\tau(\Omega_i) = \Omega_i$ and $\Omega_i \cap \Omega_j = \varnothing$ if $i \neq j$;
(iii) we define $X = N/\rho$, provided with the factor-topology and the induced involution σ, where by definition for each $a, b \in N$
$$\{a \rho b\} \text{ iff } \{\text{either } a = b \text{ or } a, b \in \Omega_i \text{ for some } i\}.$$

The condition that X has only ordinary singularities uniquely determines X as an algebraic curve.

Obviously $M = N/\tau$ is a smooth projective algebraic curve and the induced morphism $\psi: M \to Y = X/\sigma$ is a normalization. Furthermore Y has at worst ordinary singularities.

Let $\phi: N \to M$ and $\lambda: X \to Y$ be the corresponding factor-morphisms.

2. Definition and properties of the main exact sequence

J. Carlson [2] constructed the following exact sequence for $\pi: N \to X$.

$$0 \to H_1(N, \mathbb{Z}) \xrightarrow{\pi_*} H_1(X, \mathbb{Z}) \xrightarrow{\partial} \zeta_x \to 0, \tag{1}$$

where ∂ is a boundary operator, $\zeta_x \subset \operatorname{Div}^0 N$ is a finitely-generated group, which by means of the natural polarization on $\operatorname{Div} N$ (for which the points of N form an orthonormal base) has a representation as an orthogonal sum: $\zeta_x = \bigoplus_{i=1}^k \zeta(y_i)$, where $y_i = \pi(\Omega_i)$. Furthermore $\zeta(y_i)$ is spanned by a root-system of type $A_{r(i)}$, where $r(i) = \#\{\Omega_i\} - 1$. The morphisms σ and τ act on (1). Since $\pi \circ \tau = \sigma \circ \pi$ for the anti-invariant part of the corresponding members of (1) we have:

$$H_1^-(N, \mathbb{Z}) \xrightarrow{\pi_*} H_1^-(X, \mathbb{Z}) \xrightarrow{\partial} \zeta_x.$$

Since $\tau(\Omega_i) = \Omega_i$ and τ has no fixed points we have $\zeta_x^- = \bigoplus_{i=1}^k \zeta^-(y_i) = \bigoplus_{i=1}^s \mathbb{Z}(x_i - \tau(x_i))$, which is an orthogonal sum. Here $2s = \#\{\Omega\}$ and $\Omega = \{x_i, \dots, x_s, \tau(x_i), \dots, \tau(x_s)\}$. Hence the nonzero elements of ζ_x^- with minimal length are

$$\{\pm(x_i - \tau(x_i)|x_i \in \Omega, \ i = 1, 2, \dots, s\}.$$

Let $A = \partial(H_1^-(X, \mathbb{Z}))$.

Lemma 1. (i) *If* $\#(\Omega) = 2$, *then* $A \cong \mathbb{Z}$, *a and* $(a, a) = 8$;

(ii) *If* $\#\{\Omega\} \geq 4$ *and* $R = \{a \in \zeta_x|(a, a) = 4\}$, *then* R *is a root-system of type* D_s $(2s = \#\{\Omega\})$ *and* R *generates* A.

Proof. (a) We claim that there is no element $b \in \zeta_x^-$ for which $(b, b) = 2$ and $b \in A$. Indeed if there exists such an element then $b = x - \tau(x) \in A$ with $x \in \Omega$, hence there exists $c \in C_1(N)$ for which $\partial(c) = x - \tau(x)$ and $\pi_*(c) \in H_1^-(X, \mathbb{Z})$. Furthermore $c + \tau(c)$ is a cycle in $H_1(N, \mathbb{Z})$, and $\pi_*(c) \in H_1^-(X, \mathbb{Z})$ implies that $\pi_*(c + \tau(c)) = 0$ in $H_1(X, \mathbb{Z})$. Since $H_1(N, \mathbb{Z}) \to H_1(X, \mathbb{Z})$ is injective, it follows that $c+\tau(c)$ is homologous to 0 in $H_1(N, \mathbb{Z})$.

Now consider $\phi: N \to M$. Since $M = N/\tau$, it follows that $\phi_*(c + \tau(c)) = 2\phi_*(c)$ is homologous to 0 in $H_1(M, \mathbb{Z})$. Since $H_1(M, \mathbb{Z})$ is torsionfree, $\phi_*(c)$ is also homologous to 0. Thus, as a loop in $\pi_1(M)$, $\phi_*(c)$ is in the commutator subgroup. Since $\pi_1(N) \subset \pi_1(M)$ is a normal subgroup of index 2, $\pi_1(N)$ contains the commutator subgroup. Thus $\phi_*(c)$ lies in the image of $\pi_1(N)$, which means that $\phi_*(c)$ lifts to a closed loop in N. This contradicts $\partial(c) = x - \tau(x)$.

(b) We claim that if $x, y \in \Omega$, then $a = x - \tau(x) \pm (y - \tau(y))$ is an element of A. It is sufficient to prove the statement for $a = x - \tau(x) + y - \tau(y)$ since $x - \tau(x) - (y - \tau(y)) = x - \tau(x) + \tau(y) - \tau(\tau(y))$. Let $c \in C_1(N)$ with $\partial(c) = x - \tau(y)$. Then $\partial(c - \tau(c)) = x - \tau(x) + y - \tau(y)$.

We complete the proof of Lemma 1 as follows:

Case (i). By (a) and (b) A is generated by $a = 2(x - y)$, where $\{x, y\} = \Omega$.

Case (ii). Put $e_i = x_i - \tau(x_i)$, where $\Omega = \{x_1, \dots, x_s, \tau(x_1), \dots, \tau(x_s)\}$. Then $R = \{e_i \pm e_j | i \neq j\}$, so R is a root-system of type D_s. Since $R = \{e_i \pm e_j\}$, it follows that R spans a sublattice R' of index 2 in $\bigoplus_{i=1}^s \mathbb{Z}e_i$. Then (b) implies $R' \subset A$, so that $R' = A$ or $A = \bigoplus_{i=1}^s \mathbb{Z}e_i$. By (a), the latter cannot happen, which proves that R generates A. Q.E.D.

Lemma 2. *The sequence*

$$0 \to H_1^-(N, \mathbb{Z}) \xrightarrow{\pi_*} H_1^-(X, \mathbb{Z}) \xrightarrow{\partial} A \to 0 \tag{2}$$

is exact.

Proof. The exactness at the first member of (2) is derived from the exactness of the first member of (1); the exactness in the third member of (2) is derived from the definition of A. It remains to prove that $\operatorname{Ker}\partial = \operatorname{Im}\pi_*$. Obviously $\operatorname{Ker}\partial \supset \operatorname{Im}\pi_*$. Let $a \in H_1^-(X, \mathbb{Z})$ and $\partial(a) = 0$, hence there exists $b \in H_1(N, \mathbb{Z})$ such that $\pi_*(b) = a$, since (1) is exact. We have

$$0 = a\sigma_*(a) = \pi_*(b) + \sigma_*\pi_*(b) = \pi_*(b + \tau(b)),$$

hence $b + \tau(b) = 0$, since π_* is an injection. Hence $b \in H_1^-(N, \mathbb{Z})$ and $\pi_*(b) = a$, i.e., $\operatorname{Ker}\partial \subset \operatorname{Im}\pi_*$. Q.E.D.

3. Construction of PMHS for $H_1^-(X, \mathbb{Z})$

From the Poincaré duality we get an exact sequence:

$$0 \to \widehat{A} \xrightarrow{\widehat{\partial}} H_-^1(X, \mathbb{Z}) \xrightarrow{\pi^*} H_-^1(N, \mathbb{Z}) \to 0,$$

where $\widehat{A} = \operatorname{Hom}_{\mathbb{Z}}(A, \mathbb{Z})$.

Following J. Carlson [2] we define a mixed Hodge structure on $H_-^1(X, \mathbb{Z})$:

(a) Weight filtration on $H_-^1(X, \mathbb{Z})$:

$$W_{-1} = 0, \qquad W_0 = \operatorname{Im}\widehat{\partial}, \qquad W_1 = H_-^1(X, \mathbb{Z});$$

define polarizations on W_0 and W_1/W_0 via the polarizations on A and on $H_1^-(N, \mathbb{Z})$ introduced above.

(b) Hodge filtration on $H_-^1(X, \mathbf{C}) = H_-^1(X, \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbf{C}$:

$F^0 = H_-^1(X, \mathbf{C})$;

$$F^1 = \left\{\omega \in H^0\left(X - \Sigma, \Omega^1(X - \Sigma)\right) \,\middle|\, \int_{X-\Sigma} \omega \wedge \overline{\omega} < \infty, \sigma^*\omega + \omega = 0\right\},$$

here Σ is the singular locus of X.

$F^2 = 0.$

Lemma 3. *The map π^* gives an isomorphism*

$$F^1 \cong H_-^0(N, \Omega_N^1).$$

It follows from Lemma 3 that $F^1 \cap W_0 = 0$, $W_1/W_0 \otimes_{\mathbb{Z}} \mathbf{C} \cong F^1 \oplus \overline{F^1}$ which means that we have MHS on $H_-^1(X, \mathbb{Z})$, W_0 has PHS of pure weight 0, and W_1/W_0 has PHS of pure weight 1 (cf. [2]).

Proof. Let $\alpha \in F^1$. Then $\pi^*(\alpha) \in H^0(N - \Omega, \Omega^1(N - \Omega))$. Since

$$\int_{N-\Omega} \pi^*\alpha \wedge \overline{\pi^*\alpha} = \int_{X-\Omega} \alpha \wedge \overline{\alpha} < \infty,$$

then $\pi^*\alpha \in H^0(N, \Omega_N^1)$. We have $\pi^*\alpha + \tau^* \circ \pi^*\alpha = \pi^*\left(\alpha + \sigma^*\alpha\right) = 0$ on $N - \Omega$. The 1-form $\pi^*\alpha$ is holomorphic, so $\pi^*\alpha + \tau^* \circ \pi^*\alpha = 0$ on N. Thus we have a linear map $\pi^*\colon F^1 \to H_-^0(N, \Omega_N^1)$. The inverse of this map is obviously defined since N and X are birationally isomorphic. Q.E.D.

Proposition 4. *The group $H_1^-(X, \mathbb{Z})$ has a polarized mixed Hodge structure for which*:

(i) $W_0H_1^-(X) = H_1^-(X, \mathbb{Z})$, $W_{-1}H_1^-(X) = \operatorname{Im}\pi_* = H_1^-(N, \mathbb{Z})$, $W_{-2}H_1^-(X) = 0$;

(ii) $F^1H_1^-(X)=0$, $F^0H_1^-(X)=\operatorname{ann}_{\mathbf{R}}(F^1H_-^1(X))\cong\overline{F^1H_-^1(X)^*}\oplus(A\otimes_{\mathbb{Z}}\mathbf{C})$, $F^{-1}H_1^-(X)=H_1^-(X,\mathbf{C})$, *here* $F^1H_-^1(X)^*=\operatorname{Hom}_{\mathbf{R}}(F^1H_-^1(X),\mathbf{R})$ *and the complex conjugation in* $H_-^1(X,\mathbf{C})$ *is induced by those in* $\mathbf{C}$ *through* $H_-^1(X,\mathbf{C})=H_-^1(X,\mathbb{Z})\otimes_{\mathbb{Z}}\mathbf{C}$;

(iii) $\operatorname{Gr}_{-1}^w=W_{-1}/W_{-2}$ *has a pure weight* -1, *the polarization of* $H_1^-(N,\mathbb{Z})$ *is transferred to* Gr_{-1}^w *through* π_*.

$\operatorname{Gr}_0^w=W_0/W_{-1}$ *has a pure weight* 0 *and the polarization of* A *is transferred to* Gr_0^w *through* ∂.

Proof. This is an immediate consequence of Lemma 3 and the definition of MHS on a dual group (cf. [2]). Q.E.D.

It follows that the Hodge structure on $\operatorname{Gr}_{-1}^w=W_{-1}H_1^-(X)$ is:

$$\begin{aligned}F^{-1}W_{-1}H_1^-(X)&=F^{-1}H_1^-(X)\cap[W_{-1}H_1^-(X)\otimes_{\mathbb{Z}}\mathbf{C}]\\&=F^1H_-^1(X)^*\oplus\overline{F^1H_-^1(X)^*}\\&=W_{-1}H_1^-(X);\end{aligned}$$

$$F^0W_{-1}H_1^-(X)=F^0H_1^-(X)\cap[W_{-1}H_1^-(X)\otimes_{\mathbb{Z}}\mathbf{C}]=\overline{F^1H_-^1(X)^*};$$
$$F^1W_{-1}H_1^-(X)=0.$$

4. Geometric description of the 1-motive map

By definition we have

$$L^0H_1^-(X)=[\operatorname{Gr}_0^w H_1^-(X)\otimes_{\mathbb{Z}}\mathbf{C}]\cap[\operatorname{Gr}_0^w H_1^-(X)]_{\mathbb{Z}},$$
$$J^0W_{-1}H_1^-(X)=[W_{-1}H_-^1(X)\otimes_{\mathbb{Z}}\mathbf{C}]/[W_{-1}H_1^-(X)+F^0W_{-1}H_1^-(X)].$$

It is clear from Proposition 4 and from the sequence (2) that $H_1^-(X,\mathbb{Z})$ is an extension of Gr_0^w by Gr_{-1}^w. For extensions of this type J. Carlson [2] has constructed a map 1-*motive*:

$$u\colon L^0H_1^-(X)\to J^0W_{-1}H_1^-(X),$$

which depends only on the mixed Hodge structure of $H_1^-(X,\mathbb{Z})$ and is given as follows:

(i) Let $\{\omega_i\}$ be a basis of Gr_{-1}^w and $\{\omega^i\}$ be the dual basis in $\left(\operatorname{Gr}_{-1}^w\right)^*$;

(ii) Let $\{\Omega^i\}\subset H_1^-(X,\mathbb{Z})$, for which $\pi^*(\Omega^i)=\omega^i$ for each i;

(iii) If $\gamma\in\operatorname{Gr}_0^w$ and $\Gamma\in H_1^-(X,\mathbb{Z})$ is such that $\partial(\Gamma)=\gamma$, then $\gamma\xrightarrow{u}[\Sigma_i\langle\Omega^i,\Gamma\rangle\omega_i]$, where $\langle\ ,\ \rangle$ is the canonical pairing between $H_1^-(X,\mathbb{Z})$ and $H_-^1(X,\mathbb{Z})$; and $[\alpha]$ is the class of α in $J^0W_{-1}H_1^-(X)$.

Let us recall that for (N, τ) a Prym variety $P(N, \tau) = H_-^{1,0}(N)^*/H_1^-(N, \mathbb{Z})$ is defined and a map $PA\colon \operatorname{Div}^0(N) \to P(N, \tau)$, where

$$PA(P - Q) = \left(\int_Q^P \phi_1, \dots, \int_Q^P \phi_{g-1}\right) \quad (\operatorname{mod} H_1^-(N, \mathbb{Z})).$$

Here $\{\phi_1, \dots, \phi_{g-1}\}$ is a basis of $H_-^{0,1}(N)$ and $H_1^-(N, \mathbb{Z})$ is injected into $H_-^{1,0}(N)^*$ by integration. PA is called the Abel–Prym's map.

Proposition 5. *The following diagram is commutative*

$$\begin{array}{ccc} L^0 H_1^-(X) & \xrightarrow{u} & J^0 W_{-1} H_1^-(X) \\ {\scriptstyle \partial}\downarrow\cong & & {\scriptstyle \nu}\downarrow\cong \\ A & \xrightarrow{\mu} & P(N, \tau) \end{array}$$

where ν is given by the identification of $W_{-1}H_1^-(X)$ with $H_1^-(N, \mathbb{Z})$ and by using the duality given by integration; μ is the Abel–Prym's map, restricted on $A \subset \operatorname{Div}^0 N$.

Proof. Obviously $L^0 H_1^-(X) = \operatorname{Gr}_0^w H_1^-(X)$ which is isomorphic to A through ∂ (cf. Proposition 4).

$$\begin{aligned} J^0 W_{-1} H_1^-(X) &= \frac{W_{-1}H_1^-(X) \otimes_{\mathbb{Z}} \mathbf{C}}{W_{-1}H_1^-(X) + F^0 W_{-1} H_1^-(X)} = \frac{F^1 H_-^1(X)^* \oplus \overline{F^1 H_-^1(X)^*}}{W_{-1}H_1^-(X) \oplus \overline{F^1 H_-^1(X)^*}} \\ &\cong \frac{F^1 H_-^1(X)^*}{W_{-1}H_1^-(X)} \cong \frac{H_-^{1,0}(X)^*}{H_1^-(N, \mathbb{Z})} \cong P(N, \tau). \end{aligned}$$

To calculate μ we introduce the well-known symplectic base of $H_1(N, \mathbb{Z})$:

$$\{a_1, b_2; a_{g+1}, b_{g+1}; \dots; a_{g-1}, b_{g-1}; a_{2g-1}, b_{2g-1}; a_g, b_g\}$$

for which

$$\begin{gathered} \tau_*(a_i) = a_{i+g}, \qquad \tau_*(b_i) = b_{i+g} \qquad \text{for } i = 1, 2 \dots, g-1; \\ \tau_*(a_g) = a_g, \qquad \tau_*(b_g) = b_g. \end{gathered}$$

Furthermore, choose a basis $\{\omega^1, \dots, \omega^{2g-1}\}$ of $H^{1,0}(N)$ with $\int_{a_i} \omega^j = \delta_i^j$ for $i, j = 1, 2, \dots, 2g-1$.

Then $H_-^{1,0}(N) = (\omega^1 - \omega^{g-1}) \cdot \mathbf{C} \oplus \cdots \oplus (\omega^{g-1} - \omega^{2g-1}) \cdot \mathbf{C}$, hence

$$H_-^{1,0}(N)^* = \left(\int_{a_1 - a_{g+1}}\right) \cdot \mathbf{C} \oplus \cdots \oplus \left(\int_{a_{g-1} - a_{2-1}}\right) \cdot \mathbf{C}.$$

In this case μ is given as follows:

$$\gamma \to \left[\sum_{i=1}^{g-1} \left(\int_\Gamma (\omega^i - \omega^{i+g}) , \left(\int_{a_i - a_{i+g}}\right)\right)\right],$$

which is exactly the map $\gamma \to PA(\gamma)$. Q.E.D.

In fact using the same basis of $H^{1,0}(N)$ we have a map $j\colon P(N, \tau) \to J(N)$, where $J(N)$ is the Jacobi variety of N and

$$j \circ \mu(\gamma) = \left[\left(\int_\Gamma(\omega^1 - \omega^{g+1}\right), \dots, \int_\Gamma(\omega^{g-1} - \omega^{2g-1}), 0, -\int_\Gamma(\omega^1 - \omega^{g+1}), \dots, -\int_\Gamma(\omega^{g-1} - \omega^{2g-1})\right].$$

It is clear that $j \circ \mu = (1-\tau) \circ Ab$, where $Ab\colon N \to J(N)$ is the Abel's map for N.

5. Proof of Theorem 8

For the Abel's map $Ab\colon N \to J(N)$ we have $(1-\tau) \circ Ab(D) = Ab((1-\tau)D)$ for each $D \in \mathrm{Div}^0 N$. Since each generator of A with minimal length has type $(1-\tau)(P-Q)$ then

$$\begin{aligned} j \circ \mu((1-\tau)(P-Q)) &= (1-\tau) \circ Ab \circ (1-\tau)(P-Q) = Ab \circ (1-\tau)^2(P-Q) \\ &= 2Ab \circ (1-\tau)(P-Q) = 2PA(P-Q). \end{aligned}$$

It follows that we must consider the map

$$\Phi\colon N \times N \to P(N, \tau) \subset J(N), \qquad (P, Q) \to 2 \cdot PA(P-Q).$$

Lemma 6. *Let N be a smooth projective curve with an involution τ without fixed points. If N is neither a 4-, nor 8-sheeted covering of $\mathbf{P}^1$, then $\Phi(P_1, Q_1) = \Phi(P_2, Q_2)$ if and only if: either $(P_1, Q_1) = (P_2, Q_2)$ or $(P_1, Q_1) = (\tau(Q_2), \tau(P_2))$ or $(P_1, P_2) = (Q_1, Q_2)$.*

Proof. $\Phi(P_1, Q_1) = \Phi(P_2, Q_2)$ iff $2PA(P_1 + Q_2 - Q_1 - P_2) = 0$ in $J(N)$. We consider $D = 2(1-\tau)(P_1 + Q_2 - P_2 - Q_1)$ as an element of $\mathrm{Div}^0 N$. If $D \neq 0$ then $D = D_+ - D_-$, $D_-^{\pm} > 0$, $\deg D_+ = \deg D_- = 4$ or 8 and $\operatorname{supp} D_+ \cap \operatorname{supp} D_- = \varnothing$. Since $Ab(D) = 0$ in $J(N)$, then by Abel's theorem for N we conclude that there exists a map $N \to \mathbf{P}^1$ of degree 4 or 8 which is impossible by hypothesis. Thus $D = 0$, which is possible only in the cases listed in the lemma. Q.E.D.

Theorem 7. *Let X be a curve of* 1, *for which N/τ is a generic curve of genus $g(N/\tau) \geq 15$. Then using the mixed Hodge structure of $H_1^-(X, \mathbb{Z})$, constructed in* 4, *we can get N, $\tau\colon N \to N$ and Ω.*

Proof. The mixed Hodge structure gives us $J^0W_{-1}H_1^-(X) \cong P(N, \tau)$. Using the generic Torelli theorem (V. Kanev [5], Friedman–Smith [4]) we obtain N and $\phi\colon N \to M$, which gives us (N, τ). Let R be the finite subset of $L^0H_1^-(X)$ defined as follows. If there exist elements b of $L^0H_1^-(X)$ with $(b, b) = 4$, then $R = \{a | (a, a) = 4\}$. Otherwise put $R = \{a | (a, a) = 8\}$ (see Lemma 1). Consider the 1-motive: $u\colon L^0H_1^-(X) \to J^0W_{-1}H_1^-(X)$. Since $g(N/\tau) \geq 15$ then N/τ is not a covering of $\mathbf{P}^1$ of degree 4 or 8 (see [1, p. 214]) implying N

is not a covering of $\mathbf{P}^1$ of degree 4 or 8. Then by Proposition 5 and Lemma 6 the set $u(R)$ uniquely determines the set Ω. Q.E.D.

Theorem 8. *Let X satisfy the conditions of Theorem 7. Suppose X has only one singular point. Then X and σ are uniquely determined by the PMHS defined in* 4.

Proof. By using Theorem 7 we reconstruct N, τ, and Ω; since X has an ordinary singular point, which is obtained by identification of the points of Ω, we recover X and σ. Q.E.D.

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