There is a gap in the proof of Theorem 2, pointed out to me by Dominik Noll. The assertion that $\pi(x) = \pi_*(x)$ lae is unjustified. Although it is true that for each $u$ and $v \in L^2(H)$, $\langle \pi(x)u, v \rangle = \langle \pi_*(x)u, v \rangle$, lae, the exceptional locally null set depends a priori on $u$ and $v$.

To remedy this, the conclusion of Theorem 2 should be replaced by the following. This is sufficient for the proof of Theorem 1.

Then there is a continuous homomorphism $\varphi_* : G \rightarrow H$ and for each open $\sigma$-compact subgroup $G_1 \subset G$ an open $\sigma$-compact subgroup $H_1 \subset H$ together with a filtered decreasing family $\{S_\alpha\}$ of closed normal subgroups of $H_1$ such that

1. $H_1 = \varprojlim H_1/S_\alpha$,

2. for each $\alpha$, $q_\alpha \varphi(x) = q_\alpha \varphi_*(x)$, for almost all $x \in G_1$, where $q_\alpha : H_1 \rightarrow H_1/S_\alpha$ is the canonical map. If $\varphi$ is a homomorphism, $\varphi = \varphi_*$.

To prove this we construct $\varphi_*$ as before. $\varphi_*(G_1)$ is contained in some $\sigma$-compact open subgroup $H_1 \subset H$. For each separable subset $E \subset L^2(H)$, $\lambda(H_1)E$ is also a separable set. Let $\{E_\alpha\}$ be the family of all closed separable subspaces of $L^2(H)$ invariant under $\lambda(H_1)$. Each $E_\alpha$ is invariant under $\pi_*(G_1)$ and $\{E_\alpha\}$ is a filtered increasing family whose union is $L^2(H)$. Put $S_\alpha = \{y \in H_1 \mid \lambda(y)|_{E_\alpha} = 1\}$. In view of the definition of the strong operator topology on the unitary group of $E_\alpha$ and the bicontinuity of $\lambda$ we see that $H_1 = \varprojlim H_1/S_\alpha$.

Because each $E_\alpha$ is separable and invariant under $\pi_*(G_1)$, $\pi_*(x)|_{E_\alpha} = \pi(x)|_{E_\alpha}$, for almost all $x \in G_1$. This leads to the equation $q_\alpha \varphi_*(x) = q_\alpha \varphi(x)$, for almost all $x \in G_1$. If $\varphi$ is a homomorphism, then $q_\alpha \varphi_*$ and $q_\alpha \varphi$ agree on a conull subgroup of $G_1$, which is all of $G_1$. Since this is true for each $\alpha$, $\varphi_*(x) = \varphi(x)$, for all $x \in G_1$. Because $G_1$ is arbitrary, $\varphi_* = \varphi$.

Zoltán Sasvári has also given an alternate proof of Theorem 1 [Proc. Amer. Math. Soc., to appear].