THE HYPERSPACES OF SUBCONTINUA OF THE PSEUDO-ARC AND OF SOLENOIDS OF PSEUDO-ARCS ARE CANTOR MANIFOLDS

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Abstract. New proofs of the above facts are based on specific homogeneity properties of the pseudo-arc and of solenoids of pseudo-arcs.

The reader is referred to [5] for hyperspace theory. It is known that if $X$ is the pseudo-arc or a solenoid of pseudo-arcs (see [7] for the definition), then the hyperspace $C(X)$ of all nonvoid subcontinua of $X$ is 2-dimensional. It is proved in [6] that if $X$ is the pseudo-arc, then $C(X)$ is also a Cantor manifold, i.e., no 0-dimensional subset separates $C(X)$. In [2] a general theorem is presented that $C(X)$ has this property for an arbitrary metric, nondegenerate continuum $X$. Our proof of the theorem in the title is an application of the following result [3].

Lemma 1. If $X$ is an $n$-dimensional, locally compact, connected, homogeneous, metric space, then no $(n-2)$-dimensional subset separates $X (n \geq 1)$.

Lemma 2. If a dense, connected subset of a metric separable space $X$ is separated by no $n$-dimensional subset, then the space $X$ has the same property. □

(1) Let $X$ be the pseudo-arc. To show that $C(X)$ is a Cantor manifold it suffices to observe, by Lemmas 1 and 2, that the subspace $Y \subset C(X)$ of all nondegenerate, proper subcontinua of $X$ is connected, locally compact, homogeneous (see [1]), as well as 2-dimensional and dense.

(2) Let $X$ be a solenoid of pseudo-arcs with the continuous decomposition $D$ into pseudo-arcs such that $X/D$ is a solenoid $S$. The set $D$ as a subspace of $C(X)$ is homeomorphic to $S$. As in (1) the open subspace $Y$ of $C(X)$ is connected and dense. The set $Y\setminus D$ is dense in $Y$ and is the union of two disjoint, open, connected, 2-dimensional subsets $M = \{y \in Y: d \neq y \subset d \in D\}$.
and \( N = \{ y \in Y : d \neq y \supset d \in D \} \). It follows from [4] and from properties of solenoids of pseudo-arcs [7] that for every pair \( y_1, y_2 \in M(y_1, y_2 \in N) \) there exists a homeomorphism \( h : X \to X \) such that \( h(y_1) = h(y_2) \). The induced homeomorphism \( \hat{h} : C(X) \to C(X) \) satisfies \( \hat{h}(M) = M, \hat{h}(N) = N \) and \( \hat{h}(y_1) = y_2 \), so both \( M \) and \( N \) are homogeneous and, by Lemma 1, no 0-dimensional subset separates neither \( M \) nor \( N \). Suppose a 0-dimensional subset \( C \) separates \( Y \). Without loss of generality we may assume that \( C \) is a closed subset of \( Y \). It means \( Y \setminus C = A \cup B \), where \( A, B \) are nonvoid, disjoint and open subsets of \( C(X) \). In view of the above properties of \( M \) and \( N \) we may assume \( M \subset A \) and \( N \subset B \). Thus \( C \subset D \). If there is \( d \in D \setminus C \), then some order arc \( \alpha \subset C(X) \) passing through \( d \) joins \( M \) and \( N \), which is impossible, since \( \alpha \cap D = \{ d \} \) and \( C \) separates \( Y \) between \( M \) and \( N \). Therefore \( C = D \), hence \( C \) is not 0-dimensional, a contradiction.

**Remark.** A similar proof works for \( X \) being a solenoid. However in this case \( C(X) \) is the cone over \( X \), which is evidently a Cantor manifold.

### References