PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 112, Number 1, May 1991

σ-HEREDITARILY CLOSURE-PRESERVING k-NETWORKS AND g-METRIZABILITY

YOSHIO TANAKA

(Communicated by Dennis Burke)

Dedicated to Professor Ryosuke Nakagawa on his sixtieth birthday

ABSTRACT. We show that a regular space is g-metrizable if and only if it is a weakly first countable space with a σ-hereditarily closure-preserving k-network.

INTRODUCTION

Let X be a space. For each $x \in X$, let T_x be a collection of subsets of X such that any element of T_x contains x. Following Arhangel'skii [1], the collection $T_c = \bigcup\{T_x ; x \in X\}$ is a *weak base* for X if (a) and (b) below are satisfied (see [4] or [8]).

(a) For each $B_1, B_2 \in T_x$, there exists $B_3 \in T_x$ such that $B_3 \subset B_1 \cap B_2$.
(b) $U \subset X$ is open if and only if, for each $x \in U$, there exists $B \in T_x$ such that $B \subset U$.

We call the T_x a *weak neighborhood base* for x in X, and each element of T_x a *weak neighborhood* of x. A space X is *weakly first countable* ($=$ *g-first countable* [15], or X satisfies the *weak first axiom of countability* [1]) if X has a weak base T_c such that each T_x is countable. Every first countable space, or symmetric space (in the sense of [1, p. 125]) is weakly first countable. Every weakly first countable σ-space is a symmetric space (cf. [1]).

We recall that a space X is *sequential* (resp. a *k-space*) if $F \subset X$ is closed in X if and only if $F \cap C$ is closed in C for every convergent sequence C containing its limit point (resp. a compact subset C) in X. Every weakly first countable space is sequential (for example, [15]), and every sequential space is a k-space.

F. Siwiec [15] introduced the notion of g-metrizable spaces as a generalization of metric spaces. A space X is *g-metrizable* if it has a σ-locally finite weak base. A space X is metrizable if and only if it is a Fréchet, g-metrizable space

Received by the editors November 13, 1989 and, in revised form, April 3, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 54E99.
Key words and phrases. Hereditarily closure-preserving collection, k-network, weak base, g-metrizable space.

[15]. Here a space X is *Fréchet* if, whenever $x \in \overline{A}$, there exists a sequence in A converging to the point x. Other interesting properties of g-metrizable spaces are obtained by N. Jakovlev [8] (see also [18]).

Let $\mathscr{A} = \{A_\alpha ; \alpha \in A\}$ be a collection of subsets of a space X. We recall that $\mathscr{A}$ is *closure-preserving* if $\overline{\bigcup\{A_\alpha ; \alpha \in B\}} = \bigcup\{\overline{A}_\alpha ; \alpha \in B\}$ for any $B \subset A$. $\mathscr{A}$ is *hereditarily closure-preserving* if $\overline{\bigcup\{B_\alpha ; \alpha \in B\}} = \bigcup\{\overline{B}_\alpha ; \alpha \in B\}$ whenever $B \subset A$ and $B_\alpha \subset A_\alpha$ for each $\alpha \in B$. A *σ-hereditarily closure-preserving* collection is the union of countably many hereditarily closure-preserving collections. We shall use "σ-CP" (resp. "σ-HCP") instead of "σ-closure preserving" (resp. "σ-hereditarily closure-preserving") in this paper. Obviously, every σ-locally finite collection is σ-HCP.

Let $\mathscr{C}$ be a cover of a space X. We recall that $\mathscr{C}$ is *network* (= net) for X if for each $x \in X$ and neighborhood U of x, there exists $C \in \mathscr{C}$ such that $x \in C \subset U$. $\mathscr{C}$ is a *k-network* if whenever $K \subset U$ with K compact and U open in X, then $K \subset \bigcup \mathscr{C}' \subset U$ for some finite $\mathscr{C}' \subset \mathscr{C}$. A space is an *$\aleph$-space* [14] if it has a σ-locally finite k-network. Every σ-HCP weak base is a k-network [4].

L. Foged [4] proved that a space is g-metrizable if and only if it is a weakly first countable space with a σ-locally finite k-network. (Hence, a g-metrizable space is equal to a symmetric, $\aleph$-space.)

In view of Foged's result, Y. Kotake [11] asked whether every weakly first countable space with a σ-HCP k-network is g-metrizable. Related to this question, he also posed the following problem in [12]: Characterize a space having a σ-HCP weak base.

In this paper, we give an affirmative answer to the first question and, as for the second question, we show that every k-space with a σ-HCP weak base is precisely a g-metrizable space.

We assume that all spaces are regular, and all maps are continuous and onto.

Results

For a cover $\mathscr{C}$ of a space X, we consider the following condition $(*)$, which is labeled (C_2) in [17].

$(*)$ If A is a sequence converging to a point x and U is a neighborhood, then there exists $C \in \mathscr{C}$ such that $C \subset U$ and C contains a subsequence of A.

Obviously, every k-network satisfies $(*)$.

Lemma 1 ([17]). *Every σ-HCP cover satisfying $(*)$ is a k-network.*

Lemma 2. *Let X be a k-space, or a Lindelöf space. If X has a σ-HCP weak base, then it is weakly first countable.*

Proof. Let $\mathscr{B} = \bigcup\{\mathscr{B}_n ; n \in N\}$ be a σ-HCP weak base for X. Let X be a k-space. Since X has a σ-CP network of closed subsets, each point of X is a G_δ-set. Then any compact subset K of X is first countable, for each point of K is a G_δ-set in K. Then it is easy to show that a k-space X is sequential.

Let $x \in X$ be not isolated. Then there exists a sequence A converging to x with $A \not\ni x$. Note that any weak neighborhood of x contains A eventually, because $A \cup \{x\}$ is closed, but A is not closed in X, but each $\mathscr{B}_n$ is HCP and any infinite subset of A is not closed in X. Then for any $n \in N$, $T_x \cap \mathscr{B}_n$ is finite. Hence $T_x \cap \mathscr{B}$ is countable. Then X is weakly first countable. Next, let X be Lindelöf. Then for any nonisolated point x in X and any $n \in N$, $T_x \cap \mathscr{B}_n$ is countable. Indeed, suppose that $T_x \cap \mathscr{B}_n$ is not countable for some $n \in N$. Then there exists a subcollection $\{B_\alpha ; \alpha < \omega_1\}$ of $T_x \cap \mathscr{B}_n$. Since x is not isolated in X, for any B_α and any neighborhood G of x, $B_\alpha \cap G \cap (X - \{x\}) \neq \varnothing$. Then for each $\alpha < \omega_1$, there exists $x_\alpha \in B_\alpha \cap (X - \{x_\beta ; \beta < \alpha\}) \cap (X - \{x\})$. Then $\{x_\alpha ; \alpha < \omega_1\}$ is a discrete, closed subset of X with cardinality ω_1. But X is Lindelöf. This is a contradiction. Then for any $n \in N$, $T_x \cap \mathscr{B}_n$ is countable. Hence X is also weakly first countable.

Lemma 3 ([4]). *Every weakly first countable space with a σ-locally finite k-network is g-metrizable.*

Lemma 4. *Let X have a σ-CP k-network. Then, for each closed subset F of X, there exists a sequence $\{V_n ; n \in N\}$ of open subsets of X such that each V_n contains F, and for each sequence $\{x_n ; n \in N\}$ converging to a point $x \notin F$, some V_n contains only finitely many x_n, but does not contain x.*

Proof. Let $\mathscr{C} = \bigcup\{\mathscr{C}_n ; n \in N\}$ be a σ-CP k-network for X by closed subsets of X such that $\mathscr{C}_n \subset \mathscr{C}_{n+1}$ for each $n \in N$. For each $n \in N$, let $V_n = X - \bigcup\{C \in \mathscr{C}_n ; C \cap F = \varnothing\}$. Then each V_n contains F. Since $\mathscr{C}$ is a k-network with $\mathscr{C}_n \subset \mathscr{C}_{n+1}$, there exist $m, n \in N$ and a finite union A of elements of $\mathscr{C}_n$ such that $\{x_i ; i \geq m\} \cup \{x\} \subset A \subset X - F$. Then V_n contains only finitely many x_n, and does not contain x.

Lemma 5. *Let X have a weak base $\bigcup\{T_x ; x \in X\}$ such that each T_x is countable. Let D be a closed discrete subset of X. If X has a σ-HCP k-network, then there exists a σ-discrete collection $\mathscr{D}$ in X such that, for each $d \in D$, $\mathscr{D} \cap T_d$ is a weak base for d in X.*

Proof. First we note that, among regular spaces, if $\mathscr{A}$ is HCP, then so is $\{\overline{A} ; A \in \mathscr{A}\}$. Thus, X admits a σ-HCP k-network $\mathscr{C} = \bigcup\{\mathscr{C}_n ; n \in N\}$ of closed subsets such that $\mathscr{C}_n \subset \mathscr{C}_{n+1}$ for each $n \in N$. Let $D = \{x_\alpha ; \alpha \in A\}$. For each $n \in N$ and each $\alpha \in A$, let

$$\mathscr{C}_{n,\alpha} = \{C \in \mathscr{C}_n ; C \ni x_\alpha \text{ and } C \cap \{x_\beta ; \beta \neq \alpha\} = \varnothing\},$$

let

$$C(n, \alpha) = \bigcup \mathscr{C}_{n,\alpha},$$

and let

$$C^*(n, \alpha) = C(n, \alpha) - \bigcup\{C(n, \beta) ; \beta \neq \alpha\}.$$

For each $d \in D$, we can assume that $T_d = \{Q_n(d) ; n \in N\}$ is a decreasing weak neighborhood base for d in X consisting of closed subsets of X. For

each $x_\alpha \in D$, there exist $m, n \in N$ such that $Q_n(x_\alpha) \subset C^*(m, \alpha)$. Indeed, there exists $m \in N$ such that $Q_m(x_\alpha) \subset C(m, \alpha)$. To show this, suppose that $Q_n(x_\alpha) \not\subset C(n, \alpha)$ for any $n \in N$. Then there exists a sequence $A = \{a_n; n \in N\}$ such that $a_n \in Q_n(x_\alpha) - C(n, \alpha)$. Since the sequence A converges to x_α, and $\mathscr{C}$ is a k-network of closed subsets, there exists $C \in \mathscr{C}_n$ for some $n \in N$ such that C contains x_α and a subsequence of A, and $C \cap \{x_\beta; \beta \neq \alpha\} = \varnothing$. Then $C \subset C(m, \alpha)$ for $m \geq n$. Hence there exists $a_k \in C(k, \alpha)$ for some $k \geq n$. This is a contradiction. Then $Q_m(x_\alpha) \subset C(m, \alpha)$ for some $m \in N$. On the other hand, since $x_\alpha \notin \bigcup\{C(m, \beta); \beta \neq \alpha\}$, there exists $i \in N$ such that $Q_i(x_\alpha) \cap \bigcup\{C(m, \beta); \beta \neq \alpha\} = \varnothing$. Let $n \geq m, i$. Then $Q_n(x_\alpha) \subset C^*(m, \alpha)$. Now, for each $m, n \in N$, let $\mathscr{D}_{mn} = \{Q_n(x_\alpha); Q_n(x_\alpha) \subset C^*(m, \alpha), \alpha \in A\}$. For each $m \in N$, let $\mathscr{D}_m = \bigcup\{\mathscr{D}_{mn}; n \in N\}$, and let $D_m = \bigcup \mathscr{D}_m$. Then $D \subset \bigcup\{D_m; m \in N\}$ by the above arguments. We note that for each $m \in N$ and α, $\beta \in A$, if $\alpha \neq \beta$, then $C^*(m, \alpha) \cap C^*(m, \beta) = \varnothing$, and $\mathscr{C}_{m,\alpha} \cap \mathscr{C}_{m,\beta} = \varnothing$. But for each $Q_n(x_\alpha) \in \mathscr{D}_{mn}$, $Q_n(x_\alpha) \subset C^*(m, \alpha)$ and $Q_n(x_\alpha) \subset C(m, \alpha)$. Then, since each $\mathscr{C}_m$ is HCP, each $\mathscr{D}_{mn}$ is a disjoint, CP collection of closed subsets of X; hence, it is discrete in X. Then each $\mathscr{D}_m$ is a σ-discrete collection in X. Let $\mathscr{D} = \bigcup\{\mathscr{D}_m; m \in N\}$. Hence $\mathscr{D}$ is a σ-discrete collection in X such that, for each $d \in D$, $\mathscr{D} \cap T_d$ is a weak base for d in X.

Now, we prove the main theorem.

Theorem 6. *The following are equivalent*:

(a) *X is g-metrizable.*
(b) *X is a k-space with a σ-*HCP *weak base.*
(c) *X is a weakly first countable space with a σ-*HCP *k-network.*

Proof. Every g-metrizable space is sequential [15]; hence, it is a k-space. Then (a) $\Rightarrow$ (b) holds. We note that, for any sequence converging to a point x, any weak neighborhood of x contains the sequence eventually. Then (b) $\Rightarrow$ (c) follows from Lemmas 1 and 2. We prove (c) $\Rightarrow$ (a). By Lemmas 1 and 3, it suffices to show that X has a σ-locally finite cover satisfying $(*)$. Let $\mathscr{C} = \bigcup\{\mathscr{C}_n; n \in N\}$ be a σ-HCP k-network for X by closed subsets of X such that $\mathscr{C}_n \subset \mathscr{C}_{n+1}$ for each $n \in N$. For each $n \in N$, let $D_n = \{x \in X; \mathscr{C}_n$ is not point-finite at $x\}$. Since X is weakly first countable, it is sequential. Then, since each $\mathscr{C}_n$ is HCP, it follows that each D_n is closed, and discrete in X. By Lemma 4, for each $n \in N$, there exists a collection $\{V_{mn}; m \in N\}$ of open subsets of X satisfying the condition in Lemma 4 with respect to the closed subset D_n. For each $m, n \in N$, let $\mathscr{C}_{mn} = \{C - V_{mn}; C \in \mathscr{C}_n\}$. Let $\mathscr{C}' = \bigcup\{\mathscr{C}_{mn}; m, n \in N\}$. Since each $\mathscr{C}_{mn}$ is a point-finite and CP collection of closed subsets of X, it is locally finite in X. Hence $\mathscr{C}'$ is a σ-locally finite collection of closed subsets of X. Let $\bigcup\{T_x; x \in X\}$ be a weak base for X such that each T_x is countable. Since each D_n is a closed discrete subset of X, by Lemma 5 there exists a σ-discrete collection $\mathscr{D}_n$ in X such that, for each $d \in D_n$, $\mathscr{D}_n \cap T_d$ is a weak base for d in X. Let $\mathscr{D} = \bigcup\{\mathscr{D}_n; n \in N\}$.

Let $\mathscr{P} = \mathscr{C}' \cup \mathscr{D}$. Then $\mathscr{P}$ is a σ-locally finite cover of X. We show that $\mathscr{P}$ satisfies $(*)$. Let U be open in X and A be a sequence converging to a point x in U. Let $S = X - \bigcup\{D_n ; n \in N\}$. If $x \in S$, then there exists an element C of $\mathscr{C}'$ such that $C \subset U$ and C contains a subsequence of A. If $x \notin S$, then there exists an element D of $\mathscr{D}$ such that $D \subset U$ and D contains A eventually. Then $\mathscr{P}$ satisfies $(*)$. Hence X has a σ-locally finite cover $\mathscr{P}$ satisfying $(*)$.

Lemma 7 ([15]). *Every Fréchet, g-metrizable space is metrizable.*

Remark 8. (1) As for networks (= nets), the following (a) holds by [16]: (a) X has a σ-HCP network $\Leftrightarrow$ X has a σ-CP network $\Leftrightarrow$ X has a σ-discrete network ($\Leftrightarrow$ X is a σ-space). In (a), we cannot replace "network" by "k-network." Indeed, not every space with a σ-HCP k-network has a σ-locally finite k-network; see Theorem 14. However, among weakly first countable spaces, we have the following (b) in view of [8, Theorem 5] (or the proof of [4, Theorem 2.4]) and Theorem 6: (b) Let X be weakly first countable. Then X has a σ-HCP k-network $\Leftrightarrow$ X has a σ-discrete k-network $\Leftrightarrow$ X has a σ-discrete weak base ($\Leftrightarrow$ X is a g-metrizable space).

(2) In Theorem 6, we cannot weaken "σ-HCP k-network" (resp. "σ-HCP weak base") to "σ-CP k-network" (resp. "σ-CP weak base"), by (a) and (b) below: (a) There exists a separable, first countable space X with a σ-CP open base, but X is not g-metrizable; (b) There exists a weakly first countable, CW-complex X, but X is not g-metrizable. As for (a), the nonmetrizable, separable, first countable M_1-space X in [3, Example 9.2] is the desired space by Lemma 7.

As for (b), let X be the CW-complex in [17, p. 336] such that X is the quotient, finite-to-one image of a metric space, but X has no σ-HCP k-networks. Note that every CW-complex has a σ-CP open base by [3, Corollary 8.6], and that every quotient, finite-to-one image of a metric space is weakly first countable in view of [1, p. 125]. Then X is the desired space.

In the following corollary, (a) $\Leftrightarrow$ (b) is due to [2], and (a) $\Leftrightarrow$ (c) is Guthrie's unpublished result.

Corollary 9. *The following are equivalent*:

(a) *X is metrizable.*
(b) *X has a σ-HCP open base.*
(c) *X is a first countable space with a σ-HCP k-network.*

Proof. As for (b), note that every space with a σ-HCP open base is first countable by [2, Lemma 4]. Then the corollary follows from Theorem 6 and Lemma 7.

A space is *g-second countable* [15] if it has a countable weak base. The following Corollary is obtained from Theorem 6 and Lemma 2.

Corollary 10. *The following are equivalent*:

(a) X *is a* g*-second countable space.*
(b) X *is a Lindelöf space with a* σ-HCP *weak base.*
(c) X *is a Lindelöf, weakly first countable space with a* σ-HCP k*-network.*

The author has the following question in view of Theorem 6 and Corollary 10.

Question. Is every space with a σ-HCP weak base g-metrizable?

Let us give some partial answers to this question. We recall that a space X is a *c-space* (or X has countable tightness) if, whenever $x \in \overline{A}$, then $x \in \overline{C}$ for some countable $C \subset A$. Every g-metrizable (more generally, every sequential) space is a c-space; see [13, p. 123], for example. A space is *meta-Lindelöf* if every open cover has a point-countable open refinement.

Proposition 11. *Let* X *have a* σ-HCP *weak base. Then* (a) *or* (b) *below implies that* X *is* g*-metrizable.*

(a) (CH). X *is a separable space, or a* c*-space.*
(b) X *is a meta-Lindelöf* c*-space.*

Proof. For case (a), let X be a separable space. Then, as is well known, X has an open base of cardinality $\leq 2^{\omega}$, hence of cardinality $\leq \omega_1$. Let $\mathscr{B} = \bigcup\{\mathscr{B}_n ; n \in N\}$ be a σ-HCP weak base for X. Let $x \in X$ be not isolated in X. Then for any $n \in N$ and for any $A \in T_x \cap \mathscr{B}_n$, x is not isolated in A. Hence in view of [2, Corollary 2], $T_x \cap \mathscr{B}_n$ is countable for each $n \in N$. Then $T_x \cap \mathscr{B}$ is countable. Thus X is weakly first countable, hence a k-space. Next, let X be a c-space. Then it is easy to show that $F \subset X$ is closed in X if $F \cap S$ is closed for every separable closed subset S of X. Let S be any separable closed subset of X. Since S is closed in X, it has a σ-HCP weak base. Since S is separable, it is a k-space, as seen above. Thus any separable closed subset of X is a k-space. To show that X is a k-space, for $F \subset X$, let $F \cap C$ be closed for every compact subset C of X. Let S be any separable closed subset of X. Then $F \cap K$ is closed for every compact subset K of S. Since S is a k-space, $F \cap S$ is closed. Then F is closed in X. Thus X is a k-space. Then for case (a), X is a k-space. Hence it is g-metrizable, by Theorem 6.

For case (b), note that every separable closed subset of a meta-Lindelöf space X is Lindelöf. But every Lindelöf closed subset of X is a k-space by Lemma 2. Since X is a c-space, X is g-metrizable in view of the proof for case (a).

Lemma 12. *Let* $f: X \to Y$ *be a closed map. If* X *has a* σ-HCP k*-network, then so does* Y.

Proof. Let $\mathscr{C}$ be a σ-HCP k-network. Then $f(\mathscr{C})$ is a σ-HCP cover of Y. Then, by Lemma 1, it suffices to show that $f(\mathscr{C})$ satisfies $(*)$. To show this, let A be a sequence converging to y in Y, and let U be a neighborhood of y. Since f is a closed map such that each point of X is a G_δ-set, in view of

[17, Lemma 1.6] there exists a convergent sequence B in X such that $f(B)$ is a subsequence of A. Thus there exists $C \in \mathscr{C}$ such that $f(C) \subset U$ and $f(C)$ contains a subsequence of A. Hence $f(\mathscr{C})$ satisfies $(*)$.

As is well known, every perfect image of a metric space is metrizable. However, not every perfect image of a countable, g-metrizable space is g-metrizable [18]. We recall that every closed image of a metric space is metrizable if and only if it is first countable. As for g-metrizability, we have the following analogue by Lemma 12 and Theorem 6:

Theorem 13. *Let $f: X \to Y$ be a closed map such that X is g-metrizable. Then Y is g-metrizable if and only if it is weakly first countable.*

Finally, among Fréchet spaces, let us give a survey of characterizations for nice properties by means of k-networks or weak bases. We recall that a map $f: X \to Y$ is an *s-map* if $f^{-1}(y)$ is separable for each $y \in Y$. For M_3-spaces, see [3].

Theorem 14. *Let X be a Fréchet space. Then we have the following:*

(a) *X is a metric space $\Leftrightarrow$ X has a σ-locally finite weak base $\Leftrightarrow$ X has a σ-HCP weak base.*
(b) *X is the closed s-image of a metric space $\Leftrightarrow$ X has a σ-locally finite k-network.*
(c) *X is the closed image of a metric space $\Leftrightarrow$ X has a σ-HCP k-network.*
(d) *X is an M_3-space $\Leftrightarrow$ X has a σ-CP k-network $\Leftrightarrow$ X has a σ-CP weak base.*

Proof. (a) follows from Theorem 6 and Lemma 7. (This is also shown in [11]. Indeed, every Fréchet space with a σ-HCP weak base has a σ-HCP open base. Then X is metrizable by Corollary 9.) (b) is due to [6], (c) is due to [5], and the equivalence in the first part of (d) is due to [10]. Let X be an M_3-space. By [7] or [9], X has a σ-CP quasi-base $\mathscr{B}$; that is, for $x \in X$ and a neighborhood U of x, there exists $B \in \mathscr{B}$ such that $x \in \operatorname{int} B \subset B \subset U$. For $x \in X$, let $T_x = \{B \in \mathscr{B}; x \in \operatorname{int} B\}$. Then $\bigcup\{T_x; x \in X\}$ is a σ-CP weak base for X. Thus X has a σ-CP weak base. Conversely, let $\mathscr{B}$ have a σ-CP weak base. Since X is Fréchet, for any $x \in X$ and for any $B \in T_x \cap \mathscr{B}$, $x \in \operatorname{int} B$. Hence $\mathscr{B}$ is a σ-CP quasi-base for X. Hence X is an M_3-space by [3, Theorem 2.2].

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DEPARTMENT OF MATHEMATICS, TOKYO GAKUGEI UNIVERSITY, KOGANEI-SHI, TOKYO (184), JAPAN