PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 112, Number 3, July 1991

A CORRECTION TO "ON EXTENSIONS OF MODELS OF STRONG FRAGMENTS OF ARITHMETIC"

ROMAN KOSSAK

(Communicated by Andreas R. Blass)

Theorem 3.1 in [Kos] says that if M is a model of $I\Delta_0 + \exp$ and N is an end extension of M such that $N \models I\Delta_0$ and for some $a \in M$ there is a function f which is coded in N whose domain is included in $< a$ and whose range is unbounded in M, then M has continuum many automorphisms.

Richard Kaye has pointed out to me that, while the result is correct, the proof given in [Kos] is insufficient. The problem is discussed in [Kaye] where a full proof of a stronger theorem is also given. Here our purpose is to explain briefly where the gap is and how to fill it.

In the proof of [Kos, Theorem 3.1], I claimed that if $b \in M$ is such that $2^{a^n} < b$, for all standard n, then there is an automorphism of M which is the identity function on $< a$ and which moves elements below b. But it is only proved that, under the above assumptions, there are e_1, $e_2 < b$, $e_1 \neq e_2$, which satisfy the same Δ_0 formulas with parameters less than a. This is not enough, however, to prove the existence of the automorphism. (On the contrary, as shown in [Kaye], if b is not large enough such automorphisms do not exist.) Let me outline what has to be done.

Let $2_1(x) = 2^x$, and let $2_{n+1}(x) = 2_1(2_n(x))$. The correct version of the claim is: if M, N, and a are as in Theorem 3.1 and for every standard n we have $2_n(a) < b$, then there is an automorphism of M which is the identity function on $< a$ and moves elements below b.

In the proof, Kaye uses a counting argument to show that there are e_1, $e_2 < b$, $e_1 \neq e_2$, which satisfy the same Δ_0 formulas with parameters less than $2_n(a)$, for all standard n. But now this is enough to construct the automorphism (using an argument from [Kot, Lemma 4.4]).

To finish the proof of the theorem we need to consider two cases. Case 1: there is a $b \in M$ such that $2_n(a) < b$ for all standard n. Case 2: $M = \sup_{n\in\omega} 2_n(a)$.

Received by the editors April 25, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 03C50, 03C52, 03H15.

©1991 American Mathematical Society
0002-9939/91 $1.00 + $.25 per page

In the first case, we apply the claim. In the second, there is a $c \in N$ which codes the sequence $2_n(a)$, $n \in \omega$. Then, instead of constructing an automorphism fixing $< a$ pointwise, it is enough to find an automorphism fixing c and moving elements inside M, and this is a much simpler task.

BIBLIOGRAPHY

[Kaye] Richard Kaye, *Model theoretic properties characterizing Peano arithmetic* (to appear).

[Kos] Roman Kossak, *On extensions of models of strong fragments of arithmetic*, Proc. Amer. Math. Soc. **108** (1990), 223–232.

[Kot] Henryk Kotlarski, *On elementary cuts in recursively saturated models of Peano arithmetic*, Fund. Math. **120** (1984), 205–222.

MATHEMATICAL INSTITITE, POLISH ACADEMY OF SCIENCES, ŚNIADECKICH 8, P. O. BOX 137, 00-950 WARSAW, POLAND