PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 114, Number 4, APRIL 1992

LAYERS OF COMPONENTS OF $\beta([0, 1] \times N)$ ARE INDECOMPOSABLE

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(Communicated by James E. West)

ABSTRACT. We examine the structure of certain subcontinua of the Stone-Čech compactification of the reals. Let N denote the integers, let $X = [0, 1] \times N$, and let C be a component of $X^* = \beta X - X$. It is known that C admits an upper semicontinuous decomposition G into maximal nowhere dense subcontinua of C so that C/G is a Hausdorff arc. The elements of G are called layers. It has been shown that the layers of C that contain limit points of a countable increasing or decreasing sequence of cut points of C are nondegenerate indecomposable continua (various forms of this fact have been proven by Bellamy and Rubin, Mioduszewski, and Smith). We show that all the layers of C are indecomposable. *

Let $A = [0, \infty)$. We wish to examine certain subcontinua of $A^* = \beta A - A$ the remainder of the Stone-Čech compactification of A. Let N denote the integers. Then $X = [0, 1] \times N$ is a subspace of A and X^* is a subspace of A^*. It has been shown that certain layers of components of X^* are indecomposable. (See Mioduszewski [M], Bellamy and Rubin [BR], and Smith [S4].) It is the purpose of this paper to show that every "layer" of components of X^* is indecomposable. This answers a question posed by Mioduszewski in [M]. A layer of a component C of X^* is a maximal nowhere dense (in C) subcontinuum of C.

First we introduce some notation. Let $Z(X)$ denote the zero sets of X (the closed subsets of X). Elements of βX are identified with ultrafilters of zero sets of X (closed sets of X). Let I_n denote $[0, 1] \times \{n\}$ for each $n \in N$. If $x \in X^*$, let $u_x = \{\{n | I_n \cap H \neq \varnothing\} | H \in x\}$. It can be verified that if $x \in X^*$ then $u_x \in N^*$ and if $u \in N^*$ then $C(u) = \{x | u_x = u\}$ is a component of X^*.

We wish to state some properties of the layers of components of X^* and a useful characterization.

There is a natural order induced by the topology on the layers of $C(u)$. Let $p, q \in C(u)$; then we say that $p <_u q$ if and only if there exist $H \in p$, $K \in q$, and $D \in u$ so that every element of $H \cap I_n$ precedes every element of $K \cap I_n$ with respect to the usual order on I_n for all $n \in D$ (i.e., $H \cap I_n <_n K \cap I_n \ \forall n \in D$).

Received by the editors August 15, 1988 and, in revised form, October 19, 1990.
1991 *Mathematics Subject Classification*. Primary 54D40, 54F20.
Key words and phrases. Stone-Čech compactifications, nonmetric continua, indecomposable continua.
Presented at the Spring Topology Conference, Sacramento, CA, April 11–13, 1991.

* The theorem was recently discovered independently by J. Zhu.

The subscript " u " of the symbol " $<_u$ " may be omitted if the meaning is clear from the context. Let Ω denote the set of all sequences $S = \{S(n)\}_{n=1}^{\infty}$ so that $S(n) \in I_n$. If $u \in N^*$ and $S \in \Omega$ then define $A(u, S) = \{H \in Z(X) | \exists D \in u$ so that $\{S(n) | n \in D\} \subset H\}$. $A(u, S)$ is an ultrafilter and hence an element of X^*. In fact $A(u, S) \in C(u)$. If $\{i | S(i) = 1\} \in u$ then $A(u, S)$ is an end point of $C(u)$. Similarly if $\{i | S(i) = 0\} \in u$ then $A(u, S)$ is an end point of $C(u)$. Except for these two cases, if $S \in \Omega$ then $A(u, S)$ is a cut point of $C(u)$. (The converse cannot be assumed. See Baldwin and Smith [BS].) Note that $\{A(u, S) | S \in \Omega\}$ is dense in $C(u)$.

The following observations are not difficult to verify (see Baldwin and Smith [BS], Mioduszewski [M], and Smith [S4]).

Observation 1. *If p and q are two points of $C(u)$ and $p >_u q$ then there is a cut point $A(u, S)$ with $S \in \Omega$ so that $p <_u A(u, S) <_u q$.*

Observation 2. *If $p = A(u, S)$ for some $S \in \Omega$ and $q \in C(u)$, with $q \neq p$, then either $q <_u p$ or $p <_u q$.*

Observation 3. *If S_1 and S_2 are two elements of Ω, $u \in N^*$, and $A(u, S_1) <_u A(u, S_2)$ then $\{p \in C(u) | A(u, S_1) <_u p <_u A(u, S_2)\}$ is open in $C(u)$ and $\{p \in C(u) | A(u, S_1) \leq_u p \leq_u A(u, S_2)\}$ is a subcontinuum of $C(u)$, and hence is not nowhere dense.*

Lemma 1. *Every cut point of $C(u)$ is a layer of $C(u)$.*

Proof. If $x \in C(u)$ then define L_x and R_x as follows:

$$L_x = \{p \in C(u) | p <_u x\} \qquad R_x = \{p \in C(u) | x <_u p\}.$$

If p and q are points of $C(u)$ and $p <_u q$ then there is a sequence $S \in \Omega$ so that $p <_u A(u, S) <_u q$. Therefore we have

Claim 1.1. If $x \in C(u)$ then L_x and R_x are connected.

Also since $\{A(u, S) | S \in \Omega\}$ is dense in $C(u)$ we have

Claim 1.2. $\mathrm{Cl}_{C(u)}(R_x \cup L_x) = C(u)$.

We also need to prove

Claim 1.3. If $x \in M$ and M is a nowhere dense subcontinuum of $C(u)$ containing x then $M \cap L_x = \varnothing$ and $M \cap R_x = \varnothing$.

Proof. Suppose $x \in C(u)$ and M is a subcontinua of $C(u)$ that contains x and $M \cap R_x \neq \varnothing$. Then there exists $y \in M \cap R_x$ so that $x <_u y$. By Observation 1 there exist S_1 and S_2 in Ω so that $x <_u A(u, S_1) <_u A(u, S_2) <_u y$. Since $A(u, S_1)$ is a cut point of $C(u)$ and separates x from y, it follows that $A(u, S_1) \in M$. Similarly $A(u, S_2) \in M$; and in fact, if $A(u, S_1) < A(u, S) < A(u, S_2)$ for $S \in \Omega$ then $A(u, S) \in M$. Therefore since M is compact, $\{z \in C(u) | A(u, S_1) \leq z \leq A(u, S_2)\} \subset M$. So by Observation 3, M is not nowhere dense. This is a contradiction. Therefore $M \cap R_x = \varnothing$ and similarly $M \cap L_x = \varnothing$, which verifies Claim 1.3.

Suppose now that x is a cut point of $C(u)$ and $\{x\}$ is not a layer of $C(u)$. Then there is a nowhere dense nondegenerate subcontinuum M of $C(u)$ containing x. Let $y \in M - \{x\}$ Since x is a cut point of $C(u)$, $C(u) - \{x\}$ is the union of two mutually separated point sets H and K. Since

$L_x \cup R_x \subset C(u) - \{x\}$, by Claims 1.1 and 1.2 either $L_x \subset H$ and $R_x \subset K$ or $R_x \subset H$ and $L_x \subset K$. Without loss of generality assume $R_x \subset H$ and $L_x \subset K$. Furthermore by Claim 3 $M \cap (L_x \cup R_x) = \varnothing$.

Claim 1.4. y is a limit point of R_x.

Proof. Let O be an open set containing y. Let $A \in y$, $B \in x$, and U be a basic open set in βX so that

(i) $A \cap B = \varnothing$,

(ii) $A \subset U$,

(iii) $\mathrm{Cl}_{\beta X} U \subset 0$,

(iv) $B \cap \mathrm{Cl}_{\beta X} U = \varnothing$,

(v) the number of components of $U \cap I_n$ is finite for all $n \in N$ (this can be done since $A \cap I_n$ is compact). Furthermore choose A so that $\{(0, i)\} \notin A$ for all i.

Let $N' = \{i | U \cap I_i \neq \varnothing\}$. Then $N' \in u$.

Let v_i be the rightmost component of $U \cap I_i$ (with respect to the usual order on $I_i = [0, 1] \times \{i\}$). Then if $i \in N'$ let $p_i \in v_i$ be a point such that $p_i > A \cap I_i$. If $P \in \Omega$ is such that $P(i) = p_i$ then $A(u, P) \in C(u)$, $A(u, P) > y$, and $A(u, P) \in U$.

Note that $p_i | i \in N'\} \cap B = \varnothing$ and $\{B \cap I_j | j \in N'\} \in x$.

Consider $B' = \{B \cap ([1, p_j] \times \{j\}) | j \in N'\}$. If $B' \in x$ then $x >_u A(u, P)$ so $x >_u y$. So $y \in L_x$, which contradicts Claim 1.3. Therefore $B'' = \{B \cap ([p_j, 0] \times \{j\}) | j \in N'\}$ is in x. Therefore $A(u, P) > x$. Therefore O contains a point of R_x. So y is a limit point of R_x. Thus Claim 1.4 is established.

Similarly it can be shown that y is a limit point of L_x. So y is a limit point of both H and K. But H and K are mutually separated. This is a contradiction so the lemma must be true.

Lemma 2. *If $p \in C(u)$ and $L = \{q \in C(u) |$ neither $p <_u q$ nor $q <_u p\}$ then L is a layer of $C(u)$.*

Proof. Let $p \in L$ and let $L = \{q \in C(u) |$ neither $p <_u q$ nor $q <_u p\}$. Then $L = \bigcap\{\{x \in C(u) | A(u, s) \leq_u x \leq_u A(u, r)\} | A(u, s) < p < A(u, r)\}$. So L is a continuum. In order to verify that L is nowhere dense we only need to observe that if L is not nowhere dense in $C(u)$ then L would contain at least two cut points q_1 and q_2 of $C(u)$. But then one of them, q_i, is not p so either $q_i < p$ or $p < q_i$, which is a contradiction. In order to verify that L is a maximal nowhere dense subcontinuum we only need to observe that if L' is a subcontinuum of $C(u)$ with $L \subset L'$ then L' contains a point q so that either $p < q$ or $q < p$. In either case L' contains at least two cut points of $C(u)$ and hence is not nowhere dense in $C(u)$. Therefore L is a layer of $C(u)$.

Lemma 3. *If L is a maximal nowhere dense subcontinuum of $C(u)$ and $p \in L$ then $L = \{q |$ neither $p <_u q$ nor $q <_u p\}$.*

Proof. If p is a cut point or an end point of $C(u)$ then the lemma follows from the observations above and Lemma 1.

Suppose that L is a nondegenerate maximal nowhere dense subcontinuum of $C(u)$. Let $p \in L$. Then since L is nondegenerate, it follows from the above observations that L contains no cut point of $C(u)$. Therefore if $q \in L$ then

$p \not< q$ and $q \not< p$. Therefore $\widehat{L} = \{x | p \not< x \text{ and } x \not< p\}$ contains L. But $\widehat{L}$ is nowhere dense, so since L is maximal, it follows that $L = \widehat{L}$.

Notice that if G is the collection of layers of $C(u)$ then G is upper semicontinuous and $C(u)/G$ is a Hausdorff arc.

Lemmas 1, 2, and 3 (with a slight modification for the end points of $C(u)$) establish that L is a layer in $C(u)$ if and only if for each point $p \in L$,

$$L = \bigcap \{\{x \in C(u) | A(u, s) \leq x \leq A(u, r)\} | A(u, s) < p < A(u, r);\ r, s \in \Omega\}.$$

We are now prepared to prove the main result.

Theorem. *Let $u \in N^*$ and let L be a layer of $C(u)$. Then L is indecomposable.*

Proof. Suppose that M is an arbitrary property subcontinuum of L. We will prove that M is nowhere dense in L, which implies that L is indecomposable. Let $\widehat{U}$ and $\widehat{V}$ be disjoint open sets in βX so that $\mathrm{Cl}_{\beta X} \widehat{U} \cap \mathrm{Cl}_{\beta X} \widehat{V} = \varnothing$, $M \cap \widehat{V} \neq \varnothing$, $M \cap \mathrm{Cl}_{\beta X} \widehat{U} = \varnothing$, and $\widehat{U} \cap L \neq \varnothing$. Let U and V be open sets in βX so that

$$U \subset \mathrm{Cl}_{\beta X} U \subset \widehat{U}, \qquad V \subset \mathrm{Cl}_{\beta X} V \subset \widehat{V},$$
$$M \cap V \neq \varnothing, \qquad L \cap U \neq \varnothing,$$

and for each n the number of components of $I_n \cap U$ and of $I_n \cap V$ is finite.

Let $p \in M \cap V$.

Claim 1. If k is an integer, $H \in p$, $H \subset V \cap X$, and J is the set of all integers n such that there is a sequence $U_1^n, U_2^n, \ldots, U_{i_n}^n$ of components of $U \cap I_n$ and a sequence $V_1^n, V_2^n, \ldots, V_{i_n}^n$ of components of $V \cap I_n$ that intersect H such that $U_1^n < V_1^n < U_2^n < V_2^n < \cdots < U_{i_n}^n < V_{i_n}^n$ and $k \leq i_n$, then $J \in u$.

Proof. Let k be a positive integer and suppose the hypothesis of the claim and that $J \notin u$. So there is a sequence $\{s^i\}_{i=0}^{2k+3}$ of elements of Ω such that for each $n \in N - J$, $0 = s^0(n) \leq s^1(n) < s^2(n) < \cdots < s^{2k+1}(n) < s^{2k+2}(n) \leq s^{2k+3}(n) = 1$ and

$$U \cap I_n \subset \{x \in I_n | s^i(n) \leq x \leq s^{i+1}(n) \text{ and } i \text{ is even}\},$$
$$V \cap I_n \subset \{x \in I_n | s^i(n) \leq x \leq s^{i+1}(n) \text{ and } i \text{ is odd}\}.$$

Note that if the first component of $V \cap I_n$ precedes the first component of $U \cap I_n$ it may be ignored. Let $H^i = \bigcup_{n \in N-J} \{x \in I_n | s^i(n) \leq x \leq s^{i+1}(n)\}$. Since $J \notin u$ we have $N - J \in u$ and then $\widehat{H} = \{x \in H \cap I_n | n \in N - J\} \in p$. Then $\widehat{H} \subset \bigcup_{i=1,\ i \text{ odd}}^{2k+1} H^i$, so $H^1 \cup H^3 \cup \cdots \cup H^{2k+1} \in p$, and hence $H^i \in p$ for some odd integer i. But then $A(u, s^i) \leq_u p \leq_u A(u, s^{i+1})$ and $U \cap \{z \in C(u) | A(u, s^i) \leq_u z \leq_u A(u, s^{i+1})\} = \varnothing$. This contradicts the fact that $U \cap L \neq \varnothing$, because $L \subset \{z \in C(u) | A(u, s^i) \leq_u z \leq_u A(u, s^{i+1})\}$. This establishes the claim.

For each n let $B_1^n, B_2^n, \ldots, B_{k_n}^n$ be the components of $I_n - U$ that intersect V listed in order. If $H \in p$ let $B(H) = \{B_i^n | B_i^n \cap H \neq \varnothing\}$ and let $\mathscr{B} = \{B(H) | H \in p\}$. Define $\mathrm{Ls}(\mathscr{B}) = \bigcap_{B \in \mathscr{B}} \mathrm{Cl}_{\beta X}(\bigcup B)$.

Claim 2. $\mathrm{Ls}(\mathscr{B})$ is a continuum, $p \in \mathrm{Ls}(\mathscr{B})$, and $\mathrm{Ls}(\mathscr{B})$ is a component of $X^* - U$.

Various forms of this claim have appeared elsewhere (see Blass [Bl], Smith [S1], and van Douwen [vD]). We will outline a proof for completeness.

Proof. Assume the hypothesis of the claim. Then the collection $\{\mathrm{Cl}_{\beta X}(\bigcup B(H)) | H \in p\}$ is a collection of compact sets in βX so the common part $\mathrm{Ls}(\mathscr{B})$ is compact. Since $p \in \mathrm{Cl}_{\beta X}(\bigcup B(H))$ for each $H \in p$ we have $p \in \mathrm{Ls}(\mathscr{B})$. Suppose that $\mathrm{Ls}(\mathscr{B})$ is not a continuum but is the union of two mutually exclusive compact sets C_1 and C_2. Let W_1 and W_2 be open sets in βX containing C_1 and C_2 respectively with disjoint closures. Then there exists an element $B(H)$ in $\mathscr{B}$ so that $\mathrm{Cl}_{\beta X}(\bigcup B(H)) \subset W_1 \cup W_2$. But $X \cap W_1 \cap (\bigcup B(H))$ and $X \cap W_2 \cap (\bigcup B(H))$ are both compact and $(W_1 \cap (\bigcup B(H))) \cup (W_2 \cap (\bigcup B(H))) \in p$ so $W_i \cap (\bigcup B(H)) \in p$ for $i = 1$ or $i = 2$. (Note that each element of $B(H)$ is connected and lies in $W_1 \cup W_2$.) Assume $i = 1$. Let $\widehat{H} = W_1 \cap (\bigcup B(H))$. Then $\widehat{H} \in p$ and $\mathrm{Cl}_{\beta X}(\bigcup B(\widehat{H})) \subset \mathrm{Cl}_{\beta X}(W_1)$, and hence $\mathrm{Cl}_{\beta X}(\bigcup(B(\widehat{H}))) \cap W_2 = \varnothing$ and so $\mathrm{Ls}(\mathscr{B}) \cap W_2 = \varnothing$, which is a contradiction. It is easy to verify that $\mathrm{Ls}(\mathscr{B})$ is a component of $(\bigcup_{n=1}^{\infty} \bigcup_{i=1}^{k_n} B_i^n)^* = X^* - U$.

Let $\widehat{M}$ denote $\mathrm{Ls}(\mathscr{B})$. Since $\widehat{M}$ is a component of $X^* - U$, $p \in \widehat{M}$, $M \subset X^* - U$, and $p \in M$, it follows that $M \subset \widehat{M}$.

For each B_i^n let $p_i^n \in B_i^n \cap V$, and let $D = \{p_i^n\}_{i=1}^{k_n}{}_{,}{}^{\infty}{}_{n=1}$. If $d \in D$ then let $B(d)$ denote B_i^n where $d = p_i^n$. Let $D^* = \mathrm{Cl}_{\beta X}(D) - D$.

Claim 3. Every component of $X^* \cap \mathrm{Cl}_{\beta X}(\bigcup\{B_i^n | n \in N,\ 1 \le i \le k_n\})$ contains exactly one element of D^*.

Proof. Let $R = \{(n, i) | n \in N,\ 1 \le i \le k_n\}$. If $r \in R$ with $r = (n, i)$ let $Br = B_i^n$ and $dr = p_i^n$. If $\alpha \in R^*$ then let

$$d\alpha = \bigcap\{\mathrm{Cl}_{\beta X}\{dr | r \in J\} | J \in \alpha\}.$$

Then $d\alpha$ is a single point of D^* and $d\alpha = dy$ iff $\alpha = y$. Furthermore C is a component of $X^* \cap \mathrm{Cl}_{\beta X}(\bigcup\{B_i^n | n \in N,\ 1 \le i \le k_n\})$ if and only if there is an $\alpha \in R^*$ so that $C = B\alpha$ where

$$B\alpha = \bigcap\{\mathrm{Cl}_{\beta X}\{\mathrm{Br} | r \in J\} | J \in \alpha\}.$$

(See the argument to Claim 2 above and also [S1].) Furthermore $B\alpha = By$ if and only if $\alpha = y$. Therefore $d\alpha \in By$ if and only if $\alpha = y$; and this verifies Claim 3.

We have $D^* \subset V$ by construction. Consider the sets $\mathrm{Even} = \bigcup\{B_i^n | i$ is even$\}$, $\mathrm{Odd} = \bigcup\{B_i^n | i$ is odd$\}$. Then either $\mathrm{Even} \in p$ or $\mathrm{Odd} \in p$. Without loss of generality assume $\mathrm{Even} \in p$. For i even, $1 \le i \le k_n$, and n a positive integer let $E_i^n = \{x \in I_n | x \in B_{i-1}^n \cup B_i^n$ or $B_{i-1}^n <_n x <_n B_i^n\}$. Then $B_i^n \subset E_i^n$. Let $E(J) = \{E_i^n | E_i^n \cap J \neq \varnothing\}$ and let $\mathscr{E} = \{E(J) | J \in p\}$. Let $\mathrm{Ls}(\mathscr{E}) = \bigcap_{E \in \xi} \mathrm{Cl}_{\beta X}(\bigcup E)$, which is a continuum containing p by the same argument used to prove Claim 2. Since $\mathrm{Even} \in p$ and $\mathrm{Even} \subset E(\mathrm{Even})$ we have $\widehat{M} = \mathrm{Ls}(\mathscr{B}) \subset \mathrm{Ls}(\mathscr{E})$.

Claim 4. $\mathrm{Ls}(\mathscr{E}) \subset L$.

Proof. Suppose that the claim is not true. Since $p \in \mathrm{Ls}(\mathscr{E})$ and $\mathrm{Ls}(\mathscr{E}) \not\subset L$, there exists an element $s \in \Omega$ so that $A(u, s) \in \mathrm{Ls}(\mathscr{E})$ and $A(u, s) \in V$.

But by Claim 1 $E(\{s_n | n \in N\}) \notin p$. So $p \notin \mathrm{Cl}_{\beta X}(\bigcup E(\{s(n) | n \in N\}))$, and hence $p \notin \mathrm{Cl}_{\beta X}(\bigcup\{E_i^n | s(n) \in E_i^n,\ n \in N,\ i \text{ even},\ i \leq i \leq k_n\})$. So $E = \{E_i^n | s(n) \in E_i^n,\ n \in N,\ i \text{ even},\ i \leq i \leq k_n\}$ is an element of $\mathscr{E}$ and $A(u, s) \notin \mathrm{Cl}_{\beta X}(\bigcup E)$, which is a contradiction.

Let $z = d\alpha$ be the element of D^* that lies in $\widehat{M}$. Let $\hat{\alpha}$ be the element of R^* defined as follows: $\hat{\alpha} = \{G \in R |$ there is a $J \in \alpha$ so that $\{(n, i-1) | (n, i) \in J\} \subset G\}$. It is not difficult to verify that $\hat{\alpha} \in R^*$. But since $\{(n, i) | i$ is even$\} \in \alpha$ and $\{(n, i-1) | i$ is even$\} \in \hat{\alpha}$, it follows that $\hat{\alpha} \neq \alpha$, so $d\hat{\alpha} \neq d\alpha$. Yet by construction since $B_i^n \cup B_{i-1}^n \subset E_i^n$ for i even, $1 \leq i \leq k_n$, it is easy to see that $d\hat{\alpha} \in \mathrm{Ls}(\mathscr{E})$. Therefore $d\hat{\alpha}$ is a point of D^* distinct from $d\alpha$, $d\hat{\alpha} \in \mathrm{Ls}(\mathscr{E}) \subset L$ so $d\hat{\alpha} \in L$, and since $\widehat{M} \cap D^* = \{d\alpha\}$, it follows that $d\hat{\alpha} \notin \widehat{M}$, and hence $d\hat{\alpha} \notin M$. So $d\hat{\alpha}$ is a point of $L \cap V$ that is not in M. So it follows that M is nowhere dense in L, and hence L is indecomposable.

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