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WEAKLY CONVERGENT SEQUENCE COEFFICIENT OF PRODUCT SPACE

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ABSTRACT. W. L. Bynum introduced the weakly convergent sequence coefficient $\mathrm{WCS}(X)$ of the Banach space X as $\mathrm{WCS}(X) = \sup\{M:$ for each weakly convergent sequence $\{x_n\}$ in X, there is some $y \in \overline{\mathrm{co}}(\{x_n\})$ such that $M \cdot \limsup \|x_n - y\| \le A(\{x_n\})\}$. We consider the weakly convergent sequence coefficient of the l_p-product space $Z = (\prod_{i=1}^n X_i)_{l_p}$ of the finite non-Schur space $X_1, \ldots, X_n$ and show that $\mathrm{WCS}(Z) = \min\{\mathrm{WCS}(X_i): 1 \le i \le n\}$.

0. INTRODUCTION

Let X be a Banach space and $\{x_n\}$ a sequence of X. For $x \in X$, set $r(x, \{x_n\}) = \limsup_n \|x_n - x\|$. $A(\{x_n\}) = \lim_n \sup\{\|x_i - x_j\|: i, j \ge n\}$ and $r(\{x_n\}) = \inf\{r(x, \{x_n\}): x \in \overline{\mathrm{co}}(\{x_n\})\}$ are called *the asymptotic diameter of* $\{x_n\}$ and *the Chebyshev radius of* $\{x_n\}$ *relative to* $\overline{\mathrm{co}}(\{x_n\})$, respectively, where $\overline{\mathrm{co}}(\{x_n\})$ denotes the closed convex hull of $\{x_n\}$. If $A(\{x_n\}) = \lim_n \inf\{\|x_i - x_j\|: i, j \ge n,\ i \ne j\}$, then the sequence $\{x_n\}$ is called *the asymptotic equidistant sequence.*

In order to study the normal structure of Banach spaces, Bynum [1] introduced *the weakly convergent sequence coefficient* $\mathrm{WCS}(X)$ for a reflexive Banach space X as follows:

$$\mathrm{WCS}(X) = \sup\Big\{M: \text{ for each weakly convergent sequence } \{x_n\}, \text{ there is some } y \in \overline{\mathrm{co}}(\{x_n\}) \text{ such that } M \cdot \limsup_n \|x_n - y\| \le A(\{x_n\})\Big\}. \tag{1}$$

It is easy to prove that (see [2])

$$\mathrm{WCS}(X) = \inf\{A(\{x_n\})/r(\{x_n\}): \{x_n\} \text{ a weakly but not strongly convergent sequence in } X\}. \tag{2}$$

But, because of the indefiniteness of y in expression (1) and the existence of $r(\{x_n\})$ in the denominator of expression (2), it is inconvenient to apply and

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calculate $\mathrm{WCS}(X)$. We will improve expressions (1) and (2) on $\mathrm{WCS}(X)$ and use new expressions of $\mathrm{WCS}(X)$ to discuss the weakly convergent sequence coefficient of the l_p-product space of finite non-Schur Banach spaces.

In §1 we introduce the notion of the asymptotic equidistant sequence and show some results on it since we will use it to improve the expressions of $\mathrm{WCS}(X)$. In §2 we give four new expressions of $\mathrm{WCS}(X)$, which are better than (1) and (2). Finally, in §3, we give a result on the weakly convergent sequence coefficient of the l_p-product space of finite non-Schur Banach spaces.

We always denote the set of natural numbers by $\mathscr{N}$ and we say that $x_n \xrightarrow{w} x$ to denote $\{x_n\}$ weakly converges to x.

1. Asymptotic equidistant sequence

In this section we always consider X to be infinite-dimensional Banach space. For a sequence $\{x_n\}$ in X, we express $\lim_n \inf\{\|x_i - x_j\| : i, j \geq n, i \neq j\}$ by $A_1(\{x_n\})$.

Definition 1. A sequence $\{x_n\}$ in X is said to be an *asymptotic equidistant sequence* if $A_1(\{x_n\}) = A(\{x_n\})$.

It is clear that if $\{x_n\}$ is either strongly convergent or equidistant (i.e., for $i \neq j$, $\|x_i - x_j\| = \text{constant}$), then it is asymptotic equidistant.

In the following, we prove that we can choose an asymptotic equidistant subsequence from a bounded sequence.

Proposition 1. *Let $\{x_n\}$ be a bounded sequence in X and*

$$d = \sup\{d' : \text{there exists a subsequence } \{x_{n_i}\} \text{ of } \{x_n\} \text{ such that } \|x_{n_i} - x_{n_j}\| \geq d' \text{ when } i \neq j\}.$$

Then for every $\varepsilon > 0$ there exists a subsequence $\{y_k\}$ of $\{x_n\}$ such that $d - \varepsilon < \|y_i - y_j\| < d + \varepsilon$ when $i \neq j$.

Proof. For any $\varepsilon > 0$, by definition of d, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\|x_{n_i} - x_{n_j}\| > d - \varepsilon$ when $i \neq j$. For any subsequence $\{x_{n_k}(m)\}_{k=1}^{\infty}$ of $\{x_{n_k}\}$, if all distances from its first term $x_{n_1}(m)$ to the other terms, except those that are finite, are not smaller than $d + \varepsilon$, then the $x_{n_1}(m)$ is called a *deleted point.*

Consider the sequence $\{x_{n_k}\}$. If x_{n_1} is a deleted point, then delete x_{n_1} and all of the finite points whose distances to x_{n_1} are smaller than $d + \varepsilon$. Let the rest be $\{x_{n_k}(1)\}$ according to the original order. If $x_{n_1}(1)$ is a deleted point also, then delete $x_{n_1}(1)$ and the associated finite points and denote the rest by $\{x_{n_k}(2)\}$ according to the original order. If $x_{n_1}(2)$ is a deleted point, delete $x_{n_1}(2)$ and the associated finite points continuously, etc. But this process cannot proceed indefinitely, or else we get a sequence $x_{n_1}, x_{n_1}(1), x_{n_1}(2), \ldots$ of deleted points, any two points of which have distance $\geq d + \varepsilon$. This is a contradiction to the definition of d. So the sequence $\{x_{n_k}\}$ must have a point which is not deleted. Note it by $x_{n_1}(1)$ and let $\{x_{n_k}(1)\}$ be a subsequence of $\{x_{n_k}\}$ such that $\|x_{n_1}(1) - x_{n_i}(1)\| < d + \varepsilon$ for every $i > 1$.

Continue to consider the sequence $x_{n_2}(1), x_{n_3}(1), \ldots$. As mentioned above, we also can get a point $x_{n_2}(2)$ from this sequence, which is not deleted, and a subsequence $x_{n_2}(2), x_{n_3}(2), \ldots$ of this sequence such that $\|x_{n_2}(2) - x_{n_i}(2)\| < d+\varepsilon$ for every $i > 2$. Continue this process for the sequence $x_{n_3}(2), x_{n_4}(2), \ldots,$

etc. When this process is continued infinitely, we can finally get a subsequence $\{x_{n_k}(k)\}_{k=1}^{\infty}$ of $\{x_{n_k}\}$ such that $\|x_{n_i}(i) - x_{n_j}(j)\| < d + \varepsilon$ when $i \neq j$. The proof of the proposition is finished by letting $y_k = x_{n_k}(k)$. □

Proposition 2. *Let $\{x_n\}$ be a bounded sequence in X. Then there exists an asymptotic equidistant subsequence $\{y_k\}$ of $\{x_n\}$ such that $A_1(\{x_n\}) \leq A(\{y_k\}) \leq A(\{x_n\})$.*

Proof. Let $d_1 = \sup\{d'\colon$ there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $\|x_{n_i} - x_{n_j}\| \geq d'$ for $i \neq j\}$. Then $A_1(\{x_n\}) \leq d_1 \leq A(\{x_n\})$. By Proposition 1, there exists a subsequence $\{x_n(1)\}$ of $\{x_n\}$ such that $d_1 - 1 < \|x_i(1) - x_j(1)\| < d_1 + 1$ for $i \neq j$. Let $d_2 = \sup\{d'\colon$ there exists a subsequence $\{x_{n_i}(1)\}$ of $\{x_n(1)\}$ such that $\|x_{n_i}(1) - x_{n_j}(1)\| \geq d'$ for $i \neq j\}$. Then $A_1(\{x_n\}) \leq d_2 \leq d_1$ and, by Proposition 1 also, there exists a subsequence $\{x_n(2)\}$ of $\{x_n(1)\}$ such that $d_2 - \frac{1}{2} < \|x_i(2) - x_j(2)\| < d_2 + \frac{1}{2}$ for $i \neq j$. Generally, if $d_m = \sup\{d'\colon$ there exists a subsequence $\{x_{n_i}(m-1)\}$ of $\{x_n(m-1)\}$ such that $\|x_{n_i}(m-1) - x_{n_j}(m-1)\| \geq d'$ for $i \neq j\}$, then $A_1(\{x_n\}) \leq d_m \leq d_{m-1}$ and, by Proposition 1, there exists a subsequence $\{x_n(m)\}$ of $\{x_n(m-1)\}$ such that $d_m - 1/m < \|x_i(m) - x_j(m)\| < d_m + 1/m$ for $i \neq j$. Thus we can obtain a real sequence $\{d_m\}$ satisfying

$$A_1(\{x_n\}) \leq \cdots \leq d_m \leq d_{m-1} \leq \cdots \leq d_1 \leq A(\{x_n\}), \tag{3}$$

and a series of sequences $\{x_n(1)\}, \{x_n(2)\}, \ldots, \{x_n(k)\}, \ldots,$ where each sequence is a subsequence of the last one, satisfying

$$d_m - 1/m < \|x_i(m) - x_j(m)\| < d_m + 1/m \tag{4}$$

for each $m \in \mathscr{N}$ and $i \neq j$. Let $y_k = x_k(k)$. Then $\{y_k\}$ satisfies the requirement of the proposition. Indeed, let $d = \lim_m d_m$; it is sufficient to prove that $A_1(\{y_k\}) = A(\{y_k\}) = d$. For every $\varepsilon > 0$, by (4), there is $N \in \mathscr{N}$ such that $d - \varepsilon < d_k - 1/k < d_k + 1/k < d + \varepsilon$ when $k > N$. Hence, when $i, j > N$ and $i \neq j$, $d - \varepsilon < \|y_i - y_j\| < d + \varepsilon$, which shows that $A_1(\{y_k\}) > d - \varepsilon$ and $A(\{y_k\}) < d + \varepsilon$. By the arbitrariness of ε, $A_1(\{y_k\}) = A(\{y_k\}) = d$ holds. □

Recall the famous Riesz Lemma: for every $\varepsilon > 0$, there exists a sequence $\{x_n\}$ in $S(X)$, the unit sphere of X, such that $\|x_i - x_j\| > 1 - \varepsilon$ when $i \neq j$. Hence, by Proposition 2, there exist asymptotic equidistant sequences $\{x_n\}$ in $S(X)$ such that $A(\{x_n\}) > 1 - \varepsilon$ for every $\varepsilon > 0$. In addition, reviewing the parameter $\overline{J}(X)$[3,4] of X,

$$\overline{J}(X) = \sup\{r > 0\colon S(X) \text{ contains an infinite } r\text{-separated set}\},$$

we furthermore have

Proposition 3. *For every $\varepsilon > 0$, there exists an asymptotic equidistant sequence $\{x_n\}$ in $S(X)$ such that $\overline{J}(X) - \varepsilon \leq A(\{x_n\}) \leq \overline{J}(X)$.*

Proof. Obvious. □

Considering a weakly convergent sequence, we have the following result.

Proposition 4. *Let $x_n \xrightarrow{w} z$ and $\limsup_n \|x_n - z\| = a$. Then the sequence $\{x_n\}$ has an asymptotic equidistant subsequence $\{y_k\}$ such that $A(\{y_k\}) \geq a$.*

Proof. Through choice of subsequence, we can assume that $\lim\|x_n - z\| = a$. Then it follows that $A(\{y_k\}) \geq a$ for every asymptotic equidistant subsequence

$\{y_k\}$ of $\{x_n\}$. Otherwise, if $A(\{y_k\}) < a$ for some subsequence $\{y_k\}$ of $\{x_n\}$, then, for sufficiently large $n \in \mathcal{N}$, the diameter of $\overline{\text{co}}(\{y_k\}_n^\infty)$ is smaller than a. This contradicts the fact $z \in \overline{\text{co}}(\{y_k\}_n^\infty)$. □

Remark 1. For a non-Schur space X (i.e., weak and strong convergence of a sequence in X do not coincide), there exist sequences in $S(X)$ which weakly converge to zero. By the proof of Proposition 4, for a sequence $\{x_n\}$ in $S(X)$ with $x_n \xrightarrow{w} 0$, any of its asymptotic equidistant subsequences $\{y_k\}$ satisfies $A(\{y_k\}) \geq 1$.

2. Improvements of expressions of the WCS(X)

In order to study the weakly convergent sequence coefficient of product space we first give some improvements of expressions (1) and (2) on $\text{WCS}(X)$. It is assumed below that all Banach spaces are non-Schur spaces, and the area of spaces we discuss is extended from reflexive to this type of spaces. For a non-Schur space X, $\text{WCS}(X)$ is defined by (1) or (2) (both are equivalent) also.

Theorem 1.

$$\text{WCS}(X) = \sup\left\{M : x_n \xrightarrow{w} u \Rightarrow M \cdot \limsup_n \|x_n - u\| \leq A(\{x_n\})\right\}. \tag{5}$$

Proof. For convenience, let

$$W(X) = \sup\left\{M : x_n \xrightarrow{w} u \Rightarrow M \cdot \limsup_n \|x_n - u\| \leq A(\{x_n\})\right\}.$$

Since $x_n \xrightarrow{w} u$ implies $u \in \overline{\text{co}}(\{x_n\})$, it is easy to see that $W(X) \leq \text{WCS}(X)$. Now let $x_n \xrightarrow{w} u$. Due to separableness of $\overline{\text{co}}(\{x_n\})$, we can choose a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ by the diagonal method such that $\lim_k \|x_{n_k} - u\| = \limsup_n \|x_n - u\|$ and there exist $\lim_k \|x_{n_k} - y\|$ for all $y \in \overline{\text{co}}(\{x_n\})$. For each $k \geq 1$, consider the sequence $\{x_{n_j} : j \geq k\}$. For any $\varepsilon > 0$, by definition of $\text{WCS}(X)$, there exists $y_k \in \overline{\text{co}}(\{x_j : j \geq k\})$ such that

$$(\text{WCS}(X) - \varepsilon) \cdot \limsup_j \|x_{n_j} - y_k\| \leq A(\{x_{n_j} : j \geq k\}) \leq A(\{x_{n_j}\}).$$

Notice that $y_k \xrightarrow{w} u$ and by weakly lower semicontinuity of the functional $\limsup_j \|x_{n_j} - y\|$, we have

$$\begin{aligned}(\text{WCS}(X) - \varepsilon) \cdot \limsup_n \|x_n - u\| &= (\text{WCS}(X) - \varepsilon) \cdot \lim_j \|x_{n_j} - u\| \\ &\leq (\text{WCS}(X) - \varepsilon) \cdot \limsup_k \lim_j \|x_{n_j} - y_k\| \\ &\leq A(\{x_{n_j}\}) \leq A(\{x_n\}).\end{aligned}$$

This means that $\text{WCS}(X) - \varepsilon \leq W(X)$. By the arbitrariness of ε and the discussion above, we have shown that $\text{WCS}(X) = W(X)$. □

Theorem 2.

$$\text{WCS}(X) = \inf\left\{\frac{A(\{x_n\})}{r(u, \{x_n\})} : \{x_n\} \textit{ weakly but not strongly converges to } u\right\}. \tag{6}$$

Proof. Suppose

$$A(X) = \inf\left\{\frac{A(\{x_n\})}{r(u, \{x_n\})} : \{x_n\} \text{ weakly but not strongly converges to } u\right\}$$

and $x_n \xrightarrow{w} u$ but $\{x_n\}$ does not strongly converge. For every $\varepsilon > 0$, by Theorem 1, $(\mathrm{WCS}(X) - \varepsilon) \cdot \limsup_n \|x_n - u\| \leq A(\{x_n\})$, i.e.,

$$\mathrm{WCS}(X) - \varepsilon \leq \frac{A(\{x_n\})}{r(u, \{x_n\})}.$$

Hence $\mathrm{WCS}(X) - \varepsilon \leq A(X)$ and by the arbitrariness of ε it follows that $\mathrm{WCS}(X) \leq A(X)$. On the other hand, if $x_n \xrightarrow{w} u$, then $A(X) \cdot \limsup_n \|x_n - u\| \leq A(\{x_n\})$, which implies that $A(X) \leq \mathrm{WCS}(X)$. Thus $\mathrm{WCS}(X) = A(X)$. □

Theorem 3.

$$\mathrm{WCS}(X) = \inf\{A(\{x_n\}) : \{x_n\} \subset S(X) \textit{ and } x_n \xrightarrow{w} 0\}. \tag{7}$$

Proof. Let $W_0(X) = \inf\{A(\{x_n\}) : \{x_n\} \subset S(X) \text{ and } x_n \xrightarrow{w} 0\}$. By Theorem 2, it is clear that $\mathrm{WCS}(X) \leq W_0(X)$. Conversely, for every $\varepsilon > 0$, choose a sequence $\{x_n\}$ in X such that $x_n \xrightarrow{w} u$, $\{x_n\}$ does not strongly converge, and $A(\{x_n\})/r(u, \{x_n\}) < \mathrm{WCS}(X) + \varepsilon$. Let $y_n = x_n - u$. Then $y_n \xrightarrow{w} 0$. Choose a subsequence $\{y_{n_k}\}$ of $\{y_n\}$ such that $\lim_k \|y_{n_k}\| = \limsup_n \|y_n\| > 0$ and each $y_{n_k} \neq 0$; then $z_{n_k} = y_{n_k}/\|y_{n_k}\| \in S(X)$ and $z_{n_k} \xrightarrow{w} 0$. Hence

$$W_0(X) \leq A(\{z_{n_k}\}) = \frac{A(\{y_{n_k}\})}{\lim_k \|y_{n_k}\|} \leq \frac{A(\{x_n\})}{r(u, \{x_n\})} < \mathrm{WCS}(X) + \varepsilon.$$

By the arbitrariness of ε, it follows that $W_0(X) \leq \mathrm{WCS}(X)$ and so $\mathrm{WCS}(X) = W_0(X)$. □

From the results of Proposition 4 and Theorem 3 it follows immediately that

Theorem 4.

$$\mathrm{WCS}(X) = \inf\{A(\{x_n\}) : \{x_n\} \textit{ an asymptotic equidistant sequence in } S(X) \textit{ and } x_n \xrightarrow{w} 0\}. \tag{8}$$

It is obvious that it is better to use and calculate expressions (5)–(8) on $\mathrm{WCS}(X)$ rather than expressions (1) and (2).

3. On product spaces

The l_p-product space $Z = (\prod_{i=1}^n X_i)_{l_p}$ of Banach spaces $X_1, \ldots, X_n$ is, by definition, the set of all $z = (x_1, \ldots, x_n)$, where $x_i \in X_i$ for each $i \leq n$, either with the norm $\|z\| = (\sum_{i=1}^n \|x_i\|^p)^{1/p}$ when $1 \leq p < \infty$, or with the norm $\|z\| = \max\{\|x_i\| : 1 \leq i \leq n\}$ when $p = \infty$. Now we use expression (8) to discuss $\mathrm{WCS}(Z)$.

Theorem 5. *Let* $1 \leq p \leq \infty$, $Z = (\prod_{i=1}^n X_i)_{l_p}$, *and all* X_i $(1 \leq i \leq n)$ *be non-Schur spaces. Then* $\mathrm{WCS}(Z) = \min\{\mathrm{WCS}(X_i) : 1 \leq i \leq n\}$.

Proof. Without loss of generality, we assume $n = 2$, $X_1 = X$, and $X_2 = Y$. In view of Z having subspaces isometric and isomorphic to X and Y, it is clear that $\mathrm{WCS}(Z) \leq \min\{WCS(X), \mathrm{WCS}(Y)\}$. On the other hand, suppose $\{z_n\} \subset S(Z)$, $z_n \xrightarrow{w} 0$, and $\{z_n\}$ is an asymptotic equidistant sequence. Note

$z_n = (x_n, y_n)$, where $x_n \in X$ and $y_n \in Y$. Then $x_n \xrightarrow{w} 0$ and $y_n \xrightarrow{w} 0$. Let $\{n_k\}$ be a subsequence of the natural number sequence such that both $\{x_{n_k}\}$ and $\{y_{n_k}\}$ are asymptotic equidistant, and $\lim_k \|x_{n_k}\| = a$ and $\lim_k \|y_{n_k}\| = b$ both exist. For $1 \le p < \infty$, notice that $\|x_{n_k}\|^p + \|y_{n_k}\|^p = 1$ implies $a^p + b^p = 1$. It is easy to prove

$$\begin{aligned}\lim_{\substack{i,j\to\infty\\ i\neq j}} \|z_{n_i} - z_{n_j}\| &= \lim_{\substack{i,j\to\infty\\ i\neq j}} (\|x_{n_i} - x_{n_j}\|^p + \|y_{n_i} - y_{n_j}\|^p)^{1/p} \\ &\ge \{a^p[\mathrm{WCS}(X)]^p + b^p[\mathrm{WCS}(Y)]^p\}^{1/p} \\ &\ge \min\{\mathrm{WCS}(X), \mathrm{WCS}(Y)\}.\end{aligned}$$

For $p = \infty$, notice that $\max\{\|x_{n_k}\|, \|y_{n_k}\|\} = 1$ implies $\max\{a, b\} = 1$. It is easy to prove also

$$\begin{aligned}\lim_{\substack{i,j\to\infty\\ i\neq j}} \|z_{n_i} - z_{n_j}\| &= \lim_{\substack{i,j\to\infty\\ i\neq j}} \max\{\|x_{n_i} - x_{n_j}\|, \|y_{n_i} - y_{n_j}\|\} \\ &\ge \max\{a\mathrm{WCS}(X), b\mathrm{WCS}(Y)\} \\ &\ge \min\{\mathrm{WCS}(X), \mathrm{WCS}(Y)\}.\end{aligned}$$

Hence, $\mathrm{WCS}(Z) \ge \min\{\mathrm{WCS}(X), \mathrm{WCS}(Y)\}$ for $1 \le p \le \infty$, and the proof is complete. □

Recalling that Maluta's coefficient $D(X)$ of a Banach space X is defined by [5]

$$D(X) = \sup\left\{\frac{\limsup d(x_{n+1}, \mathrm{co}(\{x_j\}_1^n))}{\mathrm{diam}(\{x_n\})} : \{x_n\} \text{ a bounded nonconstant sequence in } X\right\}$$

and $\mathrm{WCS}(X) = 1/D(X)$ for reflexive space X [6], we obtain immediately

Corollary 1. *Let* $1 \le p \le \infty$, $Z = (\prod_{i=1}^n X_i)_{l_p}$, *and all* X_i $(1 \le i \le n)$ *be reflexive Banach spaces. Then*

$$D(Z) = \max\{D(X_i) : 1 \le i \le n\}.$$

Remark 2. When Landes discussed permanence properties of normal structures under a finite product of Banach spaces, he showed that [7] if both X and Y are reflexive spaces and $\mathrm{WCS}(X), \mathrm{WCS}(Y) > 1$, then $Z = (X \times Y)_{l_1}$ has the normal structure. Applying our Theorem 5, in fact, we can obtain $\mathrm{WCS}(Z) > 1$, which implies that Z has the normal structure.

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