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EMBEDDINGS AND IMMERSIONS OF A 2-SPHERE IN 4-MANIFOLDS

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ABSTRACT. Let M be $CP^2\#(-CP^2)\#P_1\#\cdots\#P_{m+k}$, where $P_1, \ldots, P_{m+k}$ are copies of $-CP^2$. Let $h, g, g_1, \ldots, g_{m+k}$ be the images of the standard generators of $H_2(CP^2; Z)$, $H_2(-CP^2; Z)$, $H_2(P_1; Z), \ldots, H_2(P_{m+k}; Z)$ in $H_2(M; Z)$ respectively. Let $\xi = ph + qg + \sum_{i=1}^m r_i g_i$ be an element of $H_2(M; Z)$. Suppose $\xi^2 = l > 0$, $p^2 - q^2 \geq 8$, $|p| - |q| \geq 2$, and $r_i \neq 0$, $i = 1, \ldots, m$. If $2(m + l - 2) \geq p^2 - q^2$, then ξ cannot be represented by a smoothly embedded 2-sphere. If $2(m+r+[(l-r-1)/4]-1) \geq p^2-q^2$ for some r with $0 \leq r \leq l-1$, then for a normal immersion f of a 2-sphere representing ξ the number of points of positive self-intersection $d_f \geq [(l - r - 1)/4] + 1$.

1. INTRODUCTION

The problem that prevents automatic extension of higher-dimensional surgery techniques to dimension 4 is the failure of the Whitney trick for codimension 2 submanifolds of 4-manifolds. In the early 1950s, Rohlin [R_1] pointed out that not every 2-dimensional homotopy/homology class of a 4-manifold can be represented by a smoothly embedded 2-sphere. Although not relevant to the problem of doing surgery in dimension 4, the question of representing such a class is still of great interest.

In 1961 Kervaire and Milnor started the investigation [KM], followed by Wall [W], Boardman [B], Tristram [T], Hsiang and Szczarba [HS], and Rohlin [R_2]; but until Donaldson's paper [D] was published, the results about $S^2 \times S^2$ had not been complete. In 1984 Kuga [K] obtained the necessary and sufficient condition of representing a 2-dimensional homology class of $S^2 \times S^2$ by applying Donaldson's theorem and "blowing down" of Kervaire and Milnor. Since then, Gompf [G], Lawson [La], Luo [Lu], Suciu [S], and D.-Y. Gan and J.-H. Guo [GG] used the same method of Kuga to obtain further information about representing a 2-dimensional homology class by a smoothly embedded or immersed 2-sphere for some other 4-manifolds.

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Let M be $CP^2\#(-CP^2)\#P_1\#\cdots\#P_{m+k}$, where $P_1, \dots, P_{m+k}$ are $m+k$ copies of $-CP^2$. Let $h, g, g_1, \dots, g_{m+k}$ be the images of the standard generators of $H_2(CP^2; Z)$, $H_2(-CP^2; Z)$, $H_2(P_1; Z), \dots, H_2(P_{m+k}; Z)$ in $H_2(M; Z)$ respectively. Let $\xi = ph + qg + \sum_{i=1}^m r_i g_i$ be an element of $H_2(M; Z)$. We have

Theorem 1. *Suppose* $\xi^2 = l > 0$, $p^2 - q^2 \geq 8$, $|p| - |q| \geq 2$, $r_i \neq 0$, $i = 1, \dots, m$, *and* $2(m + l - 2) \geq p^2 - q^2$. *Then* ξ *cannot be represented by a smoothly embedded* 2*-sphere.*

Remark 1. If $l = 1$ and $k = 0$, we get a stronger version of Theorem 1 of [GG].

Let $f: S^2 \to M$ be a normal immersion representing ξ and d_f the number of points of positive self-intersection of $f(S^2)$. We have

Theorem 2. *Suppose* $\xi^2 = l > 0$, $p^2 - q^2 \geq 8$, $|p| - |q| \geq 2$, $r_i \neq 0$, $i = 1, \dots, m$, *and* $2(m + r + [(l - r - 1)/4] - 1) \geq p^2 - q^2$ *for some* r *with* $0 \leq r \leq l - 1$. *Then* $d_f \geq [(l - r - 1)/4] + 1$.

Remark 2. The estimate in Theorem 2 is the best one. For example, let $\xi = 5h + 3g$; then $l = p^2 - q^2 = 16$ and $m = 0$. Letting $r = 7$ Theorem 2 implies $d_f \geq 3$ for normal immersion $f: S^2 \to M$ representing $5h + 3g$. Choose five copies of CP^1 in CP^2 such that one of the intersections is tripled and the others are doubled. We can cap off a $-CP^2$ with three copies of CP^1 in it with just a triple intersection to eliminate the original triple intersection. Then tubing the four intersections along another copy of CP^1, we obtain a normal immersion of a 2-sphere with three positive intersections.

Remark 3. In Theorems 1 and 2, q and each r_i are equal in position, so one can choose any one of them as q if the hypotheses are satisfied.

2. Proof of Theorem 1

For convenience we assume $q \geq 0$, $p, r_i > 0$, $i = 1, \dots, m$. The other cases are similar.

Suppose that Theorem 1 is false, i.e., ξ can be represented by a smoothly embedded 2-sphere. Adding $l - 1$ copies of $-CP^2$, $P_{m+k+1}, \dots, P_{m+k+l-1}$, we obtain $M' = M\#P_{m+k+1}\#\cdots\#P_{m+k+l-1}$. Set

$$\eta = ph + qg + \sum_{i=1}^{m} r_i g_i + \sum_{i=m+k+1}^{m+k+l-1} g_i \in H_2(M'; Z).$$

We have $\eta^2 = 1$, and η can also be represented by a smoothly embedded 2-sphere S. Then surger the tubular neighbourhood of S from M' to obtain a simply connected smooth 4-manifold N. We get

$$M' = N\#CP^2.$$

Let Q_X denote the intersection form of manifold X. Thus we have

$$Q_{M'} \sim Q_N \oplus Q_{CP^2},$$

but $Q_{M'} = \langle 1\rangle \oplus (m + k + l)\langle -1\rangle$ and $Q_{CP^2} = \langle 1\rangle$; so Q_N is negative definite. By Donaldson's theorem [D], we obtain

$$Q_N \sim (m + k + l)\langle -1\rangle.$$

Therefore there exist $2(m+k+l)$ homology classes in $H_2(N;Z)$ with self-intersection number -1. The images of them in $H_2(M';Z)$ have self-intersection number -1 and intersection number 0 with η.

Let $a = xh + yg + \sum_{i=1}^{m+k+l-1} z_i g_i \in H_2(M';Z)$ such that $a \cdot \eta = 0$ and $a \cdot a = -1$. Then $x, y, z_1, \dots, z_{m+k+l-1}$ satisfy the following Diophantine equations:

$$(1) \qquad \begin{cases} px - qy - \displaystyle\sum_{i=1}^{m} r_i z_i - \sum_{i=m+k+1}^{m+k+l-1} z_i = 0, \\ x^2 - y^2 - \displaystyle\sum_{i=1}^{m} z_i^2 - \sum_{m+1}^{m+k} z_i^2 - \sum_{i=m+k+1}^{m+k+l-1} z_i^2 + 1 = 0. \end{cases}$$

It is sufficient to prove (1) has at most $2(m+k+l)-1$ integral solutions.

Discarding the z_i's, $1 \le i \le m$ and $m+k+1 \le i \le m+k+l-1$, which are zero, and renumbering the nonzero ones and corresponding r_i's (note that $r_i = 1$ for $m+k+1 \le i \le m+k+l-1$) as $z_1, \dots, z_s$ and $r_1, \dots, r_s$, $0 \le s \le m+l-1$, and renumbering the original $z_{m+1}, \dots, z_{m+k}$ as $z_{m+l}, \dots, z_{m+k+l-1}$, one obtains

$$(1') \qquad \begin{cases} px - qy - \displaystyle\sum_{i=1}^{s} r_i z_i = 0, \\ x^2 - y^2 - \displaystyle\sum_{i=1}^{s} z_i - \sum_{i=m+l}^{m+k+l-1} z_i^2 + 1 = 0. \end{cases}$$

One may eliminate x to obtain

$$(2) \qquad \begin{aligned} &(p^2 - q^2)y^2 - 2q\left(\sum_1^s r_i z_i\right) y - \left(\sum_1^s r_i z_i\right)^2 \\ &\quad + p^2\left(\sum_1^s z_i^2 + \sum_{m+l}^{m+k+l-1} z_i^2 - 1\right) = 0. \end{aligned}$$

As a quadratic equation of y, its discriminant times $(p^2-q^2)^2/4$ is

$$\Delta = p^2\left(\sum_1^s r_i z_i\right)^2 - p^2(p^2-q^2)\left(\sum_1^s z_i^2 - 1\right) - p^2(p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right).$$

Set

$$\delta_1 = \Delta/p^2 = \left(\sum_1^s r_i z_i\right)^2 - (p^2-q^2)\left(\sum_1^s z_i^2 - 1\right) - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right).$$

We have

$$(3) \qquad \delta_1 = \delta - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right)$$

where $\delta = (\sum_1^s r_i z_i)^2 - (p^2-q^2)(\sum_1^s z_i^2 - 1)$.

Case 1. $s = 0$. If $\sum_{m+l}^{m+k+l-1} z_i^2 > 1$, (2) has no solution; if $\sum_{m+l}^{m+k+l-1} z_i^2 = 1$, (2) has $2k$ solutions; if $\sum_{m+l}^{m+k+l-1} z_i^2 = 0$, (2) has two solutions. All together, we have at most $2(k+1)$ solutions.

Case 2. $s = 1$. For $z_1 = \pm 1$, (2) becomes

$$(p^2 - q^2)y^2 \mp 2qr_1 y - r_1^2 + p^2 \left(\sum_{m+l}^{m+k+l-1} z_i^2 \right) = 0.$$

If $\sum_{m+l}^{m+k+l-1} z_i^2 = 0$, the solutions of (2) are $\pm(-r_1/(p+q), r_1/(p-q))$. Since $p+q \geq p$ and $r_i^2 < p^2 - q^2$, $r_1/(p+q)$ is not an integer and $r_1/(p-q)$ is an integer if and only if $(p-q)|r_1$. Since $p - q \geq 2$ and there are at least two r_i's which equal 1 (otherwise, $\sum_{i=1}^{m+l-1} r_i^2 \geq \sum_{i=1}^{m+l-2} r_i^2 + 1 \geq 4(m+l-2)+1 > p^2 - q^2$, contradicting $\sum_{i=1}^{m+l-1} r_i^2 = p^2 - q^2$), (2) has at most $2(m+l-3)$ integral solutions. If $\sum_{m+l}^{m+k+l-1} z_i^2 > 0$, then

$$y = \frac{(\pm qr_1 \pm p\sqrt{r_1^2 - (p^2 - q^2)(\sum_{m+l}^{m+k+l-1} z_i^2)})}{(p^2 - q^2)}.$$

Since $r_1^2 < p^2 - q^2$, (2) has no solution.

For $z_1^2 \geq 4$, since $2(m+l-2) > p^2 - q^2$, we have $r_1 \leq r_1^2 < \frac{1}{2}(p^2 - q^2) < (z_1^2 - 1)(p^2 - q^2)/z_1^2$ and $\delta = r_1^2 z_1^2 - (p^2 - q^2)(z_1^2 - 1) < 0$, hence $\delta_1 < 0$. Thus (2) has no solution.

Case 3. $2 \leq s \leq m+l-2$. We have $p^2 - q^2 - \sum_1^s r_i^2 = p^2 - q^2 - \sum_1^{m+l-1} r_i^2 + \sum_{s+1}^{m+l-1} r_i^2 = 1 + \sum_{s+1}^{m+l-1} r_i^2 \geq 1 + m + l - s - 1 = m + l - s$ and $\sum_1^s z_i^2 \geq s$. Thus

$$\left(p^2 - q^2 - \sum_1^s r_i^2\right)\left(\sum_1^s z_i^2\right) \geq (m+l-s)s. \tag{4}$$

By hypothesis we have

$$2(m+l-2) \geq p^2 - q^2. \tag{5}$$

If we have

$$\left(p^2 - q^2 - \sum_1^s r_i^2\right)\left(\sum_1^s z_i^2\right) > p^2 - q^2 \tag{6}$$

we get

$$\begin{aligned} \frac{\sum_1^s r_i^2}{p^2 - q^2} &= \frac{p^2 - q^2 - (p^2 - q^2 - \sum_1^s r_i^2)}{p^2 - q^2} \\ &< \frac{(p^2 - q^2 - \sum_1^s r_i^2)(\sum_1^s z_i^2 - 1)}{(p^2 - q^2 - \sum_1^s r_i^2)(\sum_1^s z_i^2)} = \frac{\sum_1^s z_i^2 - 1}{\sum_1^s z_i^2} \end{aligned} \tag{7}$$

and hence

$$\begin{aligned}\delta_1 &= \left(\sum_1^s r_i z_i\right)^2 - (p^2-q^2)\left(\sum_1^s z_i^2 - 1\right) - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right)\\ &\le \left(\sum_1^s r_i^2\right)\left(\sum_1^s z_i^2\right) - (p^2-q^2)\left(\sum_1^s z_i^2 - 1\right) - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right)\\ &< -(p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right) \le 0.\end{aligned}$$

Thus (2) has no solution.

Note that $(m+l-s)s > 2(m+l-2)$ unless $s=2$ or $s=m+l-2$; so we obtain (6) for $2<s<m+l-2$. For $s=2$ or $s=m+l-1$, if the Cauchy-Schwarz inequality $(\sum_1^s r_i z_i)^2 \le (\sum_1^s r_i^2)(\sum_1^s z_i^2)$ is a strict inequality, we have $\delta_1 < 0$ also. When the Cauchy-Schwarz inequality becomes equality, we obtain $z_i = r_i$, $i=1,\dots,s$, or $z_i = -r_i$, $i=1,\dots,s$.

If $s=2$ and (4) becomes equality, we have $z_1^2+z_2^2=2$ and $p^2-q^2-r_1^2-r_2^2 = m+l-2$. It follows that $z_1^2=z_2^2=r_1^2=r_2^2=1$ and $2(m+l-2)=2(p^2-q^2-2) > p^2-q^2$. So we get the strict inequality in (5).

If $s=m+l-2$ and (4) becomes equality, we have $\sum_1^s z_i^2 = m+l-2$ and $p^2-q^2-\sum_1^s r_i^2 = 2$. It implies $z_1^2=\cdots=z_s^2=r_1^2=\cdots=r_s^2=1$ and $m+l-2=p^2-q^2-2$; thus $2(m+l-2)=2(p^2-q^2-2)>p^2-q^2$, and we also get the strict inequality in (5).

Thus (2) has no solution for $2 \le s \le m+l-2$.

Case 4. $s=m+l-1$. Suppose $r_i=1$ for $i=n+1,\dots,m+l-1$. Since $\sum_{i=1}^{m+l-1} r_i^2 < p^2-q^2$, there are at least two r_i's which equal 1 (otherwise, $\sum_1^{m+l-1} r_i^2 \ge \sum_1^{m+l-2} r_i^2 + 1 \ge 4(m+l-2)+1 > p^2-q^2$, a contradiction!), i.e., we have $n \le m+l-3$.

If $z_i = r_i$, $i=1,\dots,m+l-1$, or $z_i=-r_i$, $i=1,\dots,m+l-1$, then

$$\begin{aligned}\delta_1 &= \left(\sum_1^{m+l-1} r_i^2\right)^2 - (p^2-q^2)\left(\sum_1^{m+l-1} r_i^2 - 1\right) - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right)\\ &= \left(\sum_1^{m+l-1} r_i^2\right)^2 - \left(\sum_1^{m+l-1} r_i^2 + 1\right)\left(\sum_1^{m+l-1} r_i^2 - 1\right) - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right)\\ &= 1 - (p^2-q^2)\left(\sum_{m+l}^{m+k+l-1} z_i^2\right) \le 1.\end{aligned}$$

We will show that if $\{z_i\}_{i=1}^{m+l-1} \ne \pm\{r_i\}_{i=1}^{m+l-1}$ then $\delta_1 < 0$, hence (2) has at most two solutions.

It is easy to see that if $\operatorname{sign}(z_i) \ne \operatorname{sign}(z_j)$ for some i, j, $i \le i$, $j \le m+l-1$, then the Cauchy-Schwarz inequality is strict inequality and $\delta_1 < 0$. So we

assume $\operatorname{sign}(z_i) = \operatorname{sign}(z_j)$, $1 \le i, j \le m+l-1$. Without loss of generality, we assume $z_i > 0$, $i = 1, \dots, m+l-1$.

(a) $z_i \ge 2$ for some i, $n+1 \le i \le m+l-1$, say, $z_{m+l-1} \ge 2$. As proved in Case 3 for $s = m+l-2$, we have $(\sum_1^{m+l-2} r_i z_i)^2 - (p^2-q^2)(\sum_1^{m+l-2} z_i^2 - 1) < 0$ from (7).

If

$$\sum_1^{m+l-2} r_i z_i - (p^2 - q^2) \ge -2,$$

then

$$\left(\sum_1^{m+l-2} r_i^2\right)\left(\sum_1^{m+l-2} z_i^2\right) \ge \left(\sum_1^{m+l-2} r_i z_i\right)^2 \ge (p^2 - q^2 - 2)^2.$$

Since

$$\sum_1^{m+l-2} r_i^2 = p^2 - q^2 - 2,$$

we have

$$\sum_1^{m+l-2} z_i^2 \ge p^2 - q^2 - 2,$$

thus

$$\sum_1^{m+l-1} z_i^2 = \sum_1^{m+l-2} z_i^2 + z_{m+l-1}^2 \ge p^2 - q^2 - 2 + z_{m+l-1}^2 \ge p^2 - q^2 + 2 > p^2 - q^2.$$

It follows that (7) is valid for $s = m+l-1$. Thus $\delta < 0$ and $\delta_1 < 0$.

If

$$\sum_1^{m+l-2} r_i z_i - (p^2 - q^2) \le -3,$$

then

$$\begin{aligned}
\delta &= \left(\sum_1^{m+l-2} r_i z_i + z_{m+l-1}\right)^2 - (p^2 - q^2)\left(\sum_1^{m+l-2} z_i^2 + z_{m+l-1}^2 - 1\right) \\
&= \left(\sum_1^{m+l-2} r_i z_i\right)^2 + 2z_{m+l-1}\left(\sum_1^{m+l-2} r_i z_i\right) + z_{m+l-1}^2 \\
&\quad - (p^2 - q^2)\left(\sum_1^{m+l-2} z_i^2 - 1\right) - z_{m+l-1}^2(p^2 - q^2) \\
&< 2z_{m+l-1}(p^2 - q^2 - 3) - z_{m+l-1}^2(p^2 - q^2 - 1) \\
&\le z_{m+l-1}^2(p^2 - q^2 - 3) - z_{m+l-1}^2(p^2 - q^2 - 1) < 0.
\end{aligned}$$

Thus $\delta_1 < 0$.

(b) $z_{n+1} = \cdots = z_{m+l-1} = 1$. Let $t_i = r_i - z_i$, $i = 1, \ldots, n$. Since $\{z_i\} \neq \{r_i\}$, we have $t_i \neq 0$ for some i. Thus

$$\begin{aligned}
\delta &= \left(\sum_1^{m+l-1} r_i z_i\right)^2 - (p^2 - q^2)\left(\sum_1^{m+l-1} z_i^2 - 1\right) \\
&= \left(\sum_1^{m+l-1} r_i^2 - \sum_1^n r_i t_i\right)^2 - (p^2 - q^2)\left(\sum_1^{m+l-1} r_i^2 - 1 - 2\sum_1^n r_i t_i + \sum_1^n t_i^2\right) \\
&= \left(\sum_1^{m+l-1} r_i^2\right)^2 - (p^2 - q^2)\left(\sum_1^{m+l-1} r_i^2 - 1\right) - 2\left(\sum_1^{m+l-1} r_i^2\right)\left(\sum_i^n r_i t_i\right) \\
&\quad + \left(\sum_i^n r_i t_i\right)^2 + 2(p^2 - q^2)\left(\sum_1^n r_i t_i\right) - (p^2 - q^2)\left(\sum_1^n t_i^2\right) \\
&= (p^2 - q^2 - 1)^2 - (p^2 - q^2)(p^2 - q^2 - 2) \\
&\quad - 2(p^2 - q^2 - 1)\left(\sum_1^n r_i t_i\right) + \left(\sum_i^n r_i t_i\right)^2 \\
&\quad + 2(p^2 - q^2)\left(\sum_i^n r_i t_i\right) - (p^2 - q^2)\left(\sum_1^n t_i^2\right) \\
&= 1 + 2\left(\sum_1^n r_i t_i\right) + \left(\sum_i^n r_i t_i\right)^2 - (p^2 - q^2)\left(\sum_1^n t_i^2\right) \\
&= \left(1 + \sum_1^n r_i t_i\right)^2 - (p^2 - q^2)\left(\sum_1^n t_i^2 + 1 - 1\right) < 0.
\end{aligned}$$

The last inequality is proved in Case 3. Therefore $\delta_1 < 0$.

By Cases 1, 2, 3, and 4, we have that the number of solutions of (2), hence of (1), are at most $2(m + k + l - 1)$. This proves Theorem 1.

3. Proof of Theorem 2

Suppose the theorem is false, i.e., there is a normal immersion $f: S^2 \to M$ representing ξ with $d = d_f \leq [(l - r - 1)/4]$.

We can remove each self-intersection by "blowing up a $-CP^2$" (cf. [G]). We add two copies of CP^1 with opposite orientations in a new $-CP^2$ to eliminate a negative self-intersection and we do not change the homology class of the immersed 2-sphere. This enlarges the value of k. Moreover, we add two copies of CP^1 with positive orientation in a new $-CP^2$ to eliminate a positive self-intersection and the homology class of the immersed 2-sphere is changed by twice the generator of $H_2(-CP^2; Z)$.

Eventually we can represent the homology class

$$\eta = ph + qg + \sum_1^m r_i g_i + \sum_{m+k+1}^{m+k+d} 2g_i$$

by a smoothly embedded 2-sphere in M', where

$$M' = CP^2\#(-CP^2)\#P_1\#\cdots\#P_{m+k}\#P_{m+k+1}\#\cdots\#P_{m+k+d}\,.$$

We can check that M' and η satisfy the hypothesis of Theorem 1. Setting $l' = \eta^2 = \xi^2 - 4d = l - 4d$, $m' = m + d$, we have

$$\begin{aligned} 2(m' + l' - 2) &= 2(m + d + l - 4d - 2) \\ &\geq 2\left(m + r + (l - r - 1) - 3\left[\frac{l - r - 1}{4}\right] - 1\right) \\ &\leq 2\left(m + r + \left[\frac{l - r - 1}{4}\right] - 1\right) \geq p^2 - q^2 \end{aligned}$$

by the hypothesis of Theorem 2. Theorem 1 implies that η cannot be represented by a smoothly embedded 2-sphere. The contradiction completes the proof.

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