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CORRECTIONS TO "ON CHANGING FIXED POINTS AND COINCIDENCES TO ROOTS"

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We are grateful to Professor Boju Jiang who pointed out to us after we sent the proofs to the editor that Theorem 2.3 of [1] is false in general. The main results, however, remain valid provided we give an alternate argument to replace the role of 2.3 played in Lemma 3.1 of [1].

Let $f, g : M \to M$ be two maps of a compact connected nilmanifold. Recall from the proof of [1, 3.2] that we may assume without loss of generality that f and g are fiber preserving maps of the principal T-bundle $T \to M \xrightarrow{p} N$ where T is a torus and N is a compact connected nilmanifold of dimension $\dim N < \dim M$. In order to complete the proof of [1, 3.1], it suffices to use induction on $\dim M$ together with the following

Lemma. *Let C and D be coincidence classes of f and g, and let $\overline{C} = p(C)$ and $\overline{D} = p(D)$ be the corresponding coincidence classes of $\overline{f}$ and $\overline{g}$. Then*

$$I(f, g; C) = I(f, g; D)$$

if and only if

$$I(\overline{f}, \overline{g}; \overline{C}) = I(\overline{f}, \overline{g}; \overline{D}).$$

Proof. First note that $M = G/\Gamma$, $T = G_k/\Gamma'$, and $N = (G/G_k)/(\Gamma/\Gamma')$ where G is a simply connected nilpotent Lie group with a uniform discrete subgroup Γ, G_k is the last nontrivial subgroup in the lower central series of G, and $\Gamma' = \Gamma \cap G_k$. Define

$$\varphi : G \to M, \qquad \varphi_T : G_k \to T, \qquad \overline{\varphi} : G/G_k \to N$$

by

$$\begin{aligned} \varphi(\sigma) &= \tilde{f}(\sigma)^{-1}\tilde{g}(\sigma)\Gamma, \\ \varphi_T(\sigma) &= f|T(\sigma\Gamma')^{-1}g|T(\sigma\Gamma'), \\ \overline{\varphi}(\sigma G_k) &= p(\varphi(\sigma)), \end{aligned}$$

respectively, where $\tilde{f}, \tilde{g} : G \to G$ are lifts of f and g.

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Note that the map ϕ defined in [1, 3.1] is simply the restriction of φ on an n-disk containing a root class from each fiber.

It follows that the diagram

$$(*)\qquad \begin{array}{ccc} G_k & \xrightarrow{\varphi_T} & T \\ \downarrow & & \downarrow \\ G & \xrightarrow{\varphi} & M \\ {\scriptstyle \tilde{p}}\downarrow & & \downarrow{\scriptstyle p} \\ G/G_k & \xrightarrow{\overline{\varphi}} & N \end{array}$$

commutes.

Using the same argument as in [1, 3.1], the representatives of the root classes (one from each fiber) of φ, φ_T, and $\overline{\varphi}$ are in one-to-one correspondence with the coincidence classes of $\{f, g\}$, $\{f|T, g|T\}$, and $\{\overline{f}, \overline{g}\}$, respectively. Furthermore, if γ is a root class of φ, and if $\overline{\gamma} = \tilde{p}(\gamma)$ and $\gamma_T = \gamma \cap G_k$ are the corresponding root classes of $\overline{\varphi}$ and φ_T, then

$$(**)\qquad \omega(\varphi; \gamma) = \omega(\varphi_T; \gamma_T) \cdot \omega(\overline{\varphi}; \overline{\gamma})$$

by the commutativity of $(*)$ and the fact that $\tilde{p}$ and p are locally trivial orientable bundles.

Since all coincidence classes on a torus have the same index and we proved in [1, 3.1] that $\omega(\varphi; \gamma) = I(f, g; C)$ where C is the corresponding coincidence class, it suffices to show that

$$\omega(\overline{\varphi}; \overline{\gamma}) = I(\overline{f}, \overline{g}; \overline{C}),$$

and the assertion will follow from $(**)$.

Let $\widetilde{N} \xrightarrow{q} N$ be the universal cover so that $\widetilde{N}$ is a simply connected nilpotent Lie group. It follows that $\eta : \widetilde{N} \to G/G_k$ is the universal cover. Define $\zeta : \widetilde{N} \to N$ by

$$\zeta(\sigma) = q[\tilde{\overline{f}}(\sigma)^{-1}\tilde{\overline{g}}(\sigma)]$$

where $\tilde{\overline{f}}$, $\tilde{\overline{g}}$ are lifts of $\overline{f}$, $\overline{g}$, respectively. By the definition of the root index [1, §2] and the fact that $\zeta = \overline{\varphi} \circ \eta$,

$$\omega(\overline{\varphi}; \eta(\delta)) = \omega(\zeta; \delta)$$

for any root class δ of ζ. It follows from the proof of [1, 3.1] that

$$\omega(\zeta; \delta) = I(\overline{f}, \overline{g}; q(\delta)).$$

Choosing δ to be the class so that $\eta(\delta) = \tilde{p}(\gamma) = \overline{\gamma}$ yields the equality

$$\omega(\overline{\varphi}; \overline{\gamma}) = I(\overline{f}, \overline{g}; \overline{C}). \quad \square$$

References

1. R. Brooks and P. Wong, *On changing fixed points and coincidences to roots*, Proc. Amer. Math. Soc. **115** (1992), 527–533.

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