CONTINUA AS POSITIVE WHITNEY LEVELS

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Abstract. It is shown that every continuum is a positive Whitney level of some continuum.

Krasinkiewicz and Nadler have asked (independently) if every continuum is a positive Whitney level of some continuum. The question has never been published, but it is known for people working in continuum theory. Here we present a very simple proof of it, which is in fact a compilation of known results.

For a given continuum $X$ we denote by $C(X)$ the hyperspace of all subcontinua of $X$ with the Hausdorff metric. A Whitney map is a map $\omega: C(X) \to [0, \infty)$ that satisfies the following two conditions:

1. $\omega(\{x\}) = 0$ for all $x \in X$, and
2. if $A, B \in C(X)$ satisfy $A \subseteq B$ and $A \neq B$ then $\omega(A) < \omega(B)$.

We will also use the notion of an atomic continuum. A subcontinuum $A$ of a continuum $X$ is called atomic if for any subcontinuum $B$ of $X$ we have $A \subseteq B$ or $B \subseteq A$. In newer papers atomic continua are also called terminal.

Theorem. Let $X$ be any continuum. Then there exist a continuum $M$, a Whitney map $\omega: C(M) \to [0, \infty)$, and a number $t \in (0, \omega(M))$ such that $X$ is homeomorphic to $\omega^{-1}(t)$.

Proof. By [1, Theorem, p. 507] there exist a continuum $M$ and a monotone open map $f: M \to X$ such that all point inverses $f^{-1}(x)$ for $x \in X$ are nondegenerate atomic subcontinua of $M$. Let

$$\mathcal{M} = \{\{m\}: m \in M\} \cup \{f^{-1}(x): x \in X\} \cup \{M\}.$$

Thus $\mathcal{M}$ is a compact subset of $C(M)$. Define $w: \mathcal{M} \to [0, \infty)$ by $w(\{m\}) = 0$, for $m \in M$, $w(f^{-1}(x)) = 1$, and $w(M) = 2$. Then $w$ is continuous, and by [3, Corollary 3.4, p. 468] it can be extended to a Whitney map $\omega: C(M) \to [0, \infty)$. Because all point inverses $f^{-1}(x)$ for $x \in X$ are atomic continua, we have $\omega^{-1}(1) = \{f^{-1}(x): x \in X\}$ and therefore $\omega^{-1}(1)$ is homeomorphic to $X$. The proof is complete.
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REFERENCES


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