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FIXED SETS OF UNITARY G-MANIFOLDS

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ABSTRACT. Using K-theory characteristic numbers we generalize results of Conner and Floyd on the orders of spheres in free unitary $\mathbb{Z}/p^s$-bordism. These calculations imply results on the possible fixed sets in actions of $\mathbb{Z}/p^s$ on unitary manifolds. This in turn generalizes to abelian p-groups.

INTRODUCTION

Conner and Floyd showed in early work on cyclic p-group actions on manifolds that there are severe restrictions on the possible fixed sets, depending on the local geometry. Specifically, they proved the following in [CF]: Let p be prime, and let $G = \mathbb{Z}/p$ act on the unitary manifold M so that the normal bundle in M of the fixed set M^G is a product $M^G \times W$ for some G-module W on which G acts freely away from 0. Then $[M^G] \in p^{1+a(n)}U_*$, where $n = \dim_{\mathbb{C}} W$ and $a(n) = [(n-1)/(p-1)]$. Here, U_* denotes (nonequivariant) unitary bordism. Notice that $a(n)$ grows with the codimension of the fixed set and that this result implies that there is no unitary $\mathbb{Z}/p$-manifold of dimension > 0 with a single fixed point.

Connor and Floyd's work in [CF] also implies a partial generalization of the above result to semifree $\mathbb{Z}/p^s$-actions with discrete fixed sets. Let $G = \mathbb{Z}/p^s$ act semifreely on the unitary manifold M so that M^G is discrete with normal bundle $M^G \times nV$ for some irreducible G-module V on which G acts freely away from 0. Then $[M^G] \in p^{s+a(n)}U_*$, where $a(n)$ is as above.

One of the goals of this paper is to complete the generalization of the result for $\mathbb{Z}/p$-actions to $\mathbb{Z}/p^s$-actions. Thus we remove the three restrictions in the partial generalization just mentioned: that M be semifree, that M^G be discrete, and that $W = nV$. All of the results of Conner and Floyd come from the calculation of the order of $Y \times SW$ in free G-bordism, where Y is unitary with trivial G-action and SW is the unit sphere in the G-module W. In order to carry out our goal, we first calculate in §2 the order of SW in free $\mathbb{Z}/p^s$-bordism, generalizing the results of Conner and Floyd for SnV. This calculation is based on K-theory characteristic numbers for G-bordism, and the theory is set up in §1. We then go on in §3 to replace $Y \times SW$ by a slightly more complicated free G-manifold, this calculation being needed to deal with $\mathbb{Z}/p^s$-manifolds that are

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not semifree. In §4 we give our main results on the fixed points of $\mathbb{Z}/p^s$-actions on unitary manifolds. We also give a generalization to abelian p-group actions and conjecture a result for arbitrary p-groups.

1. $K_G(-;\mathbb{Q}/\ )$-CHARACTERISTIC NUMBERS

Let G be a finite group, and let V be a finite-dimensional unitary representation of G. The interesting cases will be those in which V has no trivial summands. Let SnV denote the unit sphere in nV; $\operatorname{colim}_n SnV$ is then a model for the universal G-space $E\mathscr{F}V[P]$, where $\mathscr{F}V$ is the family $\{H \subset G \mid V^H \neq 0\}$. Here $E\mathscr{F}V$ is characterized as a G-CW complex with $E\mathscr{F}V^H$ contractible if $H \in \mathscr{F}V$ and empty otherwise.

Definition 1.1. The stably almost complex G-manifold M will be called *V-free* if M^H is empty unless $H \in \mathscr{F}V$.

By the universal property of $E\mathscr{F}V$, this is equivalent to saying that M maps equivariantly into $E\mathscr{F}V$.

Let K_G^* and K_*^G denote equivariant unitary K-theory and K-homology theory [A1]. The object of this section is to describe characteristic numbers in $K_0^G(*;\mathbb{Q}/\mathbb{Z}) = R(G)\otimes\mathbb{Q}/\mathbb{Z}$ for unitary G-bordism of V-free manifolds. These characteristic numbers are essentially those of Wilson in [Wi] in the case of free bordism of groups with periodic cohomology (space form groups). On the other hand, the present formulation lends itself directly to computation without using Atiyah-Singer type fixed-point index calculations [Wi, G].

We introduce $\mathbb{Q}/\mathbb{Z}$-coefficients by smashing the representing spectrum of K_G^* with a nonequivariant Moore space of type $\mathbb{Q}/\mathbb{Z}$. Characteristic numbers (of odd-dimensional manifolds) will be obtained from characteristic classes in $K_G^1(E\mathscr{F}V \times B_GU;\mathbb{Q}/\mathbb{Z})$, where B_GU is the classifying space for unitary G-vector bundles. Explicitly, if $c \in K_G^1(E\mathscr{F}V \times B_GU;\mathbb{Q}/\mathbb{Z})$, then the characteristic number of $c(M) \in K_0^G(*;\mathbb{Q}/\mathbb{Z})$ of the unitary V-free G-manifold M is $\langle p^*c,\ [M]\rangle$, where $p: M \to E\mathscr{F}V \times B_GU$ classifies M and its normal bundle and $[M] \in K_1^G(M)$ is the fundamental class given by Atiyah's K-theory orientation [A2]. By the usual argument such characteristic numbers are V-free unitary bordism invariants.

To describe the possibilities for these numbers we begin by looking at $K_G^1(E\mathscr{F}V) = K_G^1(\operatorname{colim}_n SnV)$. The pair (DnV, SnV) gives the exact sequences

$$0 \to K_G^1(SnV) \to R(G) \xrightarrow{e_V^n} R(G) \to K_G^0(SnV) \to 0$$

and

$$0 \to K_0^G(SnV) \to R(G) \xrightarrow{e_V^n} R(G) \to K_1^G(SnV) \to 0,$$

where e_V is the Euler class $\lambda_{-1}(V) = \sum_i(-1)^i\lambda^i(V)$. Thus

$$K_G^1(SnV) \cong K_0^G(SnV) \cong \ker e_V^n$$

and

$$K_G^0(SnV) \cong K_1^G(SnV) \cong R(G)/\langle e_V\rangle^n,$$

where $\langle e_V\rangle$ is the ideal generated by e_V. (Here and elsewhere we use the same letter to denote an element of $R(G)$ and the map given by multiplication by that element.)

The inclusion $\imath\colon SnV \to S(n+1)V$ induces the following commutative diagram:

$$\begin{array}{ccccccccc}
0 & \to & K_G^1(S(n+1)V) & \to & R(G) & \xrightarrow{e_V^{n+1}} & R(G) & \to & K_G^0(S(n+1)V) & \to & 0 \\
 & & \downarrow & & \downarrow{\scriptstyle e_V} & & \| & & \downarrow & & \\
0 & \to & K_G^1(SnV) & \to & R(G) & \xrightarrow{e_V^{n}} & R(G) & \to & K_G^0(SnV) & \to & 0
\end{array}$$

Thus the induced map $K_G^1(S(n+1)V) \to K_G^1(SnV)$ is $e_V\colon \ker e_V^{n+1} \to \ker e_V^n$, while the map $K_G^0(S(n+1)V) \to K_G^0(SnV)$ is the projection $R(G)/\langle e_V^{n+1}\rangle \to R(G)/\langle e_V^n\rangle$.

In order to say more about these maps, we consider the structure of the representation ring. For the cyclic group $C = \langle\omega\rangle \cong \mathbb{Z}/k$ let $\zeta_C = e^{2\pi i/k}$ and let η_C be the one-dimensional representation given by $\eta_C(\omega) = \zeta_C$. The following result follows from [tD2, §IV.10].

Lemma 1.2. *For any finite group G*

$$R(G)\otimes\mathbb{Q} \cong \bigoplus_{(C)} \mathbb{Q}[\zeta_C]^{WC},$$

where the sum runs over conjugacy classes of cyclic subgroups of G and WC permutes the powers of ζ_C according to its action on C. Thus the summand $\mathbb{Q}[\zeta_C]^{WC}$ is the subfield of $\mathbb{Q}[\zeta_C]$ corresponding to the subgroup of its Galois group $\operatorname{Aut} C$ *given by the image of WC in* $\operatorname{Aut} C$. □

Lemma 1.3. *If $x \in R(G)$, then $\ker x^n = \ker x$ for all $n \geq 1$.*

Proof. In $R(G)\otimes\mathbb{Q}$ multiplication by x is either an isomorphism or zero in the (C)-component. This gives the result in $R(G)\otimes\mathbb{Q}$, and the result for $R(G)$ follows from the fact that $R(G) \subset R(G)\otimes\mathbb{Q}$. □

Lemma 1.4. *If $x \in R(G)$, then the projection $R(G)/\langle x^{n+1}\rangle \to R(G)/\langle x^n\rangle$ restricts to an epimorphism on torsion subgroups.*

Proof. By Lemma 1.2, $\operatorname{Im}(x^n\otimes\mathbb{Q}) = \operatorname{Im}(x^{n+1}\otimes\mathbb{Q})$. It follows that $x^n = qyx^{n+1}$ for some $q \in \mathbb{Q}$ and $y \in R(G)$. Thus $kx^n \in \langle x^{n+1}\rangle$ for some nonzero $k \in \mathbb{Z}$. The result follows. □

The naturality of the universal coefficient exact sequence

$$0 \to K_G^1(SnV)\otimes\mathbb{Q}/\mathbb{Z} \to K_G^1(SnV;\mathbb{Q}/\mathbb{Z}) \to \operatorname{Tor}(K_G^0(SnV),\mathbb{Q}/\mathbb{Z}) \to 0$$

gives us the commutative diagram

$$\begin{array}{ccccccccc}
0 & \to & K_G^1(S(n+1)V)\otimes\mathbb{Q}/\mathbb{Z} & \to & K_G^1(S(n+1)V;\mathbb{Q}/\mathbb{Z}) & \to & \operatorname{Tor}(K_G^0(S(n+1)V),\mathbb{Q}/\mathbb{Z}) & \to & 0 \\
 & & \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle j} & & \downarrow{\scriptstyle k} & & \\
0 & \to & K_G^1(SnV)\otimes\mathbb{Q}/\mathbb{Z} & \to & K_G^1(SnV;\mathbb{Q}/\mathbb{Z}) & \to & \operatorname{Tor}(K_G^0(SnV),\mathbb{Q}/\mathbb{Z}) & \to & 0
\end{array}$$

where i is multiplication by e_V. Since $K_G^1(SnV) \cong \ker e_V^n \cong \ker e_V$ by Lemma 1.3, it follows that $i = 0$. On the other hand, k is induced by a projection which, by Lemma 1.4, induces an epimorphism of torsion subgroups. Since $\operatorname{Tor}(-,\mathbb{Q}/\mathbb{Z})$ gives the torsion subgroup, it follows that k is an epimorphism. This gives $\lim K_G^1(SnV; \mathbb{Q}/\mathbb{Z}) \cong \lim \operatorname{tor}(R(G)/\langle e_V^n \rangle)$, where tor denotes the torsion subgroup. A similar calculation gives $\lim K_G^0(SnV; \mathbb{Q}/\mathbb{Z}) \cong \lim(R(G)/\langle e_V^n \rangle \otimes \mathbb{Q}/\mathbb{Z})$ and $\lim^1 K_G^0(SnV; \mathbb{Q}/\mathbb{Z}) = 0$. We conclude that

$$K_G^1(E\mathscr{F}V; \mathbb{Q}/\mathbb{Z}) \cong \lim \operatorname{tor}(R(G)/\langle e_V^n \rangle).$$

We now turn to $K_G^*(B_G U)$. We may take $B_G U = \operatorname{colim}_n B_G U(n)$, where $B_G U(n)$ is the Grassmannian of complex n-planes in $\mathbb{C}[G]^\infty$. Following the comment preceding [S, 3.9], standard calculations from nonequivariant K-theory generalize to give

Theorem 1.5. *$K_G^0(B_G U) \cong R(G)[[c_1, c_2, \ldots]]$, where the c_i are equivariant Chern classes; $K_G^1(B_G U) \cong 0$. Further, the c_i restrict to the nonequivariant Chern classes in $K^0(B_G U) \cong K^0(BU)$.* □

Corollary 1.6. *The natural map*

$$K_G^1(E\mathscr{F}V; \mathbb{Q}/\mathbb{Z}) \otimes_{R(G)} K_G^0(B_G U) \to K_G^1(E\mathscr{F}V \times B_G U; \mathbb{Q}/\mathbb{Z})$$

is an isomorphism.

Proof. Since $K_G^0(B_G U)$ is a free $R(G)$-module, the cohomology theories $K_G^1(- \times B_G U; \mathbb{Q}/\mathbb{Z})$ and $K_G^1(-; \mathbb{Q}/\mathbb{Z}) \otimes_{R(G)} K_G^0(B_G U)$ are isomorphic. □

The cases of interest to us will be the manifolds of the form $SnV \times M$, where G acts trivially on M. If $x \in K_G^1(E\mathscr{F}V; \mathbb{Q}/\mathbb{Z})$ and c_I is a monomial in the c_i, then the associated characteristic class $(x \otimes c_I)(SnV \times M) = x(SnV)c_I(M)$, where $x(SnV) \in R(G) \otimes \mathbb{Q}/\mathbb{Z}$ and $c_I(M) \in \mathbb{Z}$ agrees with the associated nonequivariant K-theory Chern class of M.

We now relate our construction to the constructions given by tom Dieck and Wilson in [tD1] and [Wi] in the special case of free space form group actions. Here we let G act freely on V so that $E\mathscr{F}V = EG$ and $K_G^*(E\mathscr{F}V) \cong K^*(BG)$. In view of the above calculations, we can express the evaluation of characteristic numbers as a map

$$\theta : U_*(BG) \to \operatorname{Hom}(K^*(BG; \mathbb{Q}/\mathbb{Z}) \otimes \mathbb{Z}[[c_1, c_2, \ldots]], \mathbb{Q}/\mathbb{Z}).$$

There is then a commutative diagram

$$\begin{array}{ccc} U_*(BG) & \xrightarrow{\theta} & \operatorname{Hom}(K^*(BG; \mathbb{Q}/\mathbb{Z}) \otimes \mathbb{Z}[[c_1, c_2, \ldots]], \mathbb{Q}/\mathbb{Z}) \\ {\scriptstyle\mu}\big\downarrow & & \big\downarrow{\scriptstyle r} \\ \operatorname{Hom}(\mathbb{Z}[c_1, c_2, \ldots], K_*(BG)) & \xrightarrow[\epsilon]{} & \operatorname{Hom}(\mathbb{Z}[c_1, c_2, \ldots], \operatorname{Hom}(K^*(BG; \mathbb{Q}/\mathbb{Z}), \mathbb{Q}/\mathbb{Z})) \end{array}$$

where r is given by adjunction and restriction, μ is the tom Dieck-Wilson formulation of characteristic numbers, and ϵ is induced by evaluation. The following lemma implies that ϵ is an isomorphism in odd grading, while the source is known to be zero in even grading.

Lemma 1.7. *The evaluation $e: K_1(BG) \to \operatorname{Hom}(K^1(BG; \mathbb{Q}/\mathbb{Z}), \mathbb{Q}/\mathbb{Z})$ is an isomorphism.*

Proof. Let I be the augmentation ideal of $R(G)$. One has, by [AS],

$$\begin{aligned} \operatorname{Hom}(K^1(BG; \mathbb{Q}/\mathbb{Z}), \mathbb{Q}/\mathbb{Z}) &\cong \operatorname{Hom}(\lim \operatorname{tor} R(G)/I^n, \mathbb{Q}/\mathbb{Z}) \\ &\cong \operatorname{Hom}(\lim I/I^n, \mathbb{Q}/\mathbb{Z}) \cong K_1(BG). \quad \square \end{aligned}$$

We now obtain the following restatement of a well-known result (cf. [Wi, 1.3]).

Theorem 1.8. *$K_G(-;\mathbb{Q}/\mathbb{Z})$-characteristic numbers detect unitary free bordism of space form groups.*

Proof. Since μ is injective [tD1, Wi], it now follows that θ is injective and also that the characteristic numbers we define are essentially the same as those of tom Dieck and Wilson. □

2. Orders of spheres

Let $G = \mathbb{Z}/p^s$, let W be an n-dimensional complex representation on whose unit sphere G acts freely $(n \geq 1)$, and let V be the one-dimensional complex representation on which a generator of G acts by multiplication by $\exp(2\pi i/p^s)$. Both Conner and Floyd [CF] and Wilson [Wi] calculated the order of the free sphere $[SnV]$ in unitary free G-bordism; Conner and Floyd also calculated the order of $[SW]$ when $G = \mathbb{Z}/p$. We now use the characteristic numbers we developed in the previous section to calculate the order of $[SW]$ for any s.

Let $a(n) = [(n-1)/(p-1)]$, where $[-]$ denotes the greatest integer function. Then our main result is

Theorem 2.1. *The order of $[SW]$ in $U_*^G(E\mathbb{Z}/p^s)$ is $p^{s+a(n)}$, where $n = \dim_{\mathbb{C}} W$.* □

If M is a unitary manifold, all of whose nonequivariant K-theory characteristic classes are divisible by p, then $[M] \in U_*$ is also divisible by p, by Hattori-Stong. This fact, together with the remarks after Corollary 1.6, now gives

Corollary 2.2. *Let M be a unitary manifold with trivial G-action, and assume that $[M]$ is not divisible by p in U_*. Then the order of $[SW \times M]$ in $U_*^G(E\mathbb{Z}/p^s)$ is $p^{s+a(n)}$, where $n = \dim_{\mathbb{C}} W$.* □

The remainder of this section constitutes the proof of Theorem 2.1.

Since the normal bundle to SW is trivial, the only characteristic classes of interest are those that come from the inclusion $i : SW \to EG$. Here $K_G^1(EG;\mathbb{Q}/\mathbb{Z}) \cong \lim \operatorname{tor}(R(G)/\langle e^n\rangle)$, where $e = e_V$ is the Euler class of the irreducible representation V given above. Now $R(G) = \mathbb{Z}[x]/\langle x^{p^s}-1\rangle$, where $x = [V]$, and $e = 1-x$. From this we get $R(G) \cong \mathbb{Z}\oplus\langle e\rangle$. Since $(1-e)^{p^s} = 1$, it follows that $pe \in \langle e^2\rangle$, from which it follows that $\langle e\rangle/\langle e^n\rangle$ is a finite p-group and that $\operatorname{tor}(R(G)/\langle e^n\rangle) = \langle e\rangle/\langle e^n\rangle$. (This is a special case of the general fact that I/I^n is torsion.)

We now give an alternate description of $K_G^1(EG;\mathbb{Q}/\mathbb{Z})$ using the exact sequence of the pair (DnV, SnV)

$$0 \to K_G^1(SnV;\mathbb{Q}/\mathbb{Z}) \to R(G)\otimes\mathbb{Q}/\mathbb{Z} \xrightarrow{e^n\otimes\mathbb{Q}/\mathbb{Z}} R(G)\otimes\mathbb{Q}/\mathbb{Z} \to K_G^0(SnV;\mathbb{Q}/\mathbb{Z}) \to 0,$$

from which it follows that

$$K_G^1(SnV;\mathbb{Q}/\mathbb{Z}) \cong \ker(e^n\otimes\mathbb{Q}/\mathbb{Z}) \cong [(e^n\otimes\mathbb{Q})^{-1}R(G)]/R(G).$$

As in §1, the inclusion $SnV \to S(n+1)V$ gives the following diagram:

$$\begin{tikzcd}
0 \arrow[r] & K_G^1(S(n+1)V;\mathbb{Q}/\mathbb{Z}) \arrow[r] \arrow[d] & R(G)\otimes\mathbb{Q}/\mathbb{Z} \arrow[r, "e^{n+1}\otimes\mathbb{Q}/\mathbb{Z}"] \arrow[d, "e\otimes\mathbb{Q}/\mathbb{Z}"] & R(G)\otimes\mathbb{Q}/\mathbb{Z} \arrow[r] \arrow[d, "\|"] & K_G^0(S(n+1)V;\mathbb{Q}/\mathbb{Z}) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & K_G^1(SnV;\mathbb{Q}/\mathbb{Z}) \arrow[r] & R(G)\otimes\mathbb{Q}/\mathbb{Z} \arrow[r, "e^{n}\otimes\mathbb{Q}/\mathbb{Z}"] & R(G)\otimes\mathbb{Q}/\mathbb{Z} \arrow[r] & K_G^0(SnV;\mathbb{Q}/\mathbb{Z}) \arrow[r] & 0
\end{tikzcd}$$

It follows that $K_G^1(EG;\mathbb{Q}/\mathbb{Z}) \cong \lim[(e^n\otimes\mathbb{Q})^{-1}R(G)]/R(G)$, where the maps in the limit are given by multiplication by e. It also follows that the image of $K_G^1(EG;\mathbb{Q}/\mathbb{Z})$ in $K_G^1(SnV;\mathbb{Q}/\mathbb{Z})$ is contained in

$$[\mathrm{Im}(e\otimes\mathbb{Q})\cap(e^n\otimes\mathbb{Q})^{-1}R(G)]/[\mathrm{Im}(e\otimes\mathbb{Q})\cap R(G)].$$

Similarly we have that $K_G^1(SW;\mathbb{Q}/\mathbb{Z}) = [(e_W\otimes\mathbb{Q})^{-1}R(G)]/R(G)$, where e_W is the Euler class of W. Now, if $c\in K_G^1(EG;\mathbb{Q}/\mathbb{Z})$ is any characteristic class, we have $c(sW) = \langle i^*c, [SW]\rangle = \langle i^*c, \partial[DW, SW]\rangle = \langle \delta i^*c, [DW, SW]\rangle$. Thus the characteristic number is the image of i^*c under the inclusion

$$0 \to K_G^1(SW;\mathbb{Q}/\mathbb{Z}) \xrightarrow{\delta} R(G)\otimes\mathbb{Q}/\mathbb{Z}$$

in the long exact sequence of the pair (DW, SW). Hence we need only find the order of i^*c in $K_G^1(SW;\mathbb{Q}/\mathbb{Z})$. We shall concentrate on one particular characteristic class. Since e_W is invertible in the ideal $\mathrm{Im}(e_V\otimes\mathbb{Q})$, we can form the element $e_V/e_W \in [(e_W\otimes\mathbb{Q})^{-1}R(G)]/R(G)$. The collection of these elements for varying W clearly forms an inverse system, i.e., an element in $K_G^1(EG;\mathbb{Q}/\mathbb{Z})$. We may concentrate on this element because of

Lemma 2.3. $[\mathrm{Im}(e_V\otimes\mathbb{Q})\cap(e_W\otimes\mathbb{Q})^{-1}R(G)]/[\mathrm{Im}(e_V\otimes\mathbb{Q})\cap R(G)] \cong \langle e_V\rangle/\langle e_We_V\rangle = \langle e\rangle/\langle e^{n+1}\rangle$, *with* e_V/e_W *corresponding to* $e=e_V$. *Moreover,* e_V/e_W *has the largest order of any element of this group.*

Proof. The first isomorphism is given by multiplication by e_W. Notice that $y \in \mathrm{Im}(e_V\otimes\mathbb{Q})\cap(e_W\otimes\mathbb{Q})^{-1}R(G)$ iff $e_Wy \in \mathrm{Im}(e_W\otimes\mathbb{Q})\cap R(G) = \mathrm{Im}(e_V\otimes\mathbb{Q})\cap R(G) = \langle e_V\rangle$, this group being the augmentation ideal of $R(G)$ in which e_W is invertible. Similarly, $y\in\mathrm{Im}(e_V\otimes\mathbb{Q})\cap R(G) = \langle e_V\rangle$ iff $e_Wy\in\langle e_We_V\rangle$.

Now $\langle e_We_V\rangle = \langle e_V^{n+1}\rangle$, for, if U and V are any two one-dimensional representations on whose spheres G acts freely, then $\langle e_U\rangle = \langle e_V\rangle =$ the augmentation ideal of $R(G)$, so e_U and e_V are $R(G)$-multiples of one another. Induction on the dimension of W, which is a direct sum of such one-dimensional representations, gives the claim.

Finally, it is clear that e has the highest order of any element of $\langle e\rangle/\langle e^{n+1}\rangle$, since all the elements are $R(G)$-multiples of e. □

We now calculate the order of e_V/e_W in $K_G^1(SW;\mathbb{Q}/\mathbb{Z})$.

Proposition 2.4. *The order of* e *in* $\langle e\rangle/\langle e^{n+1}\rangle$ *is* $p^{s+a(n)}$.

Proof. Let t_n be the exponent of the order of e in $\langle e\rangle/\langle e^n\rangle$ so that t_n is the smallest integer for which $p^{t_n}e$ is a linear combination with integer coefficients of e^i; $n \le i \le n+p^s-2$. Write

$$e = \sum_{i=n}^{n+p^s-2} x_{n,i}e^i, \qquad x_{n,i} \in \mathbb{Q}.$$

If $x \in \mathbb{Q}$ is expressed as $p^k(u/v)$ with u and v integers prime to p, write $\operatorname{ord}_p x = k$. Then t_n is the maximum of the negatives of the $\operatorname{ord}_p x_{n,i}$, so we concern ourselves with establishing bounds for these inductively on n.

When $n=2$, the relation $(1-e)^{p^s} = 1$ gives

$$x_{2,i} = \frac{(-1)^i}{p^s}\binom{p^s}{i}.$$

Since

$$\operatorname{ord}_p \binom{p^s}{i} = s - \operatorname{ord}_p i,$$

it follows that $\operatorname{ord}_p x_{2,i} = -\operatorname{ord}_p i$ for all i with $2 \le i \le p^s$ so that $t_2 = s$.

For larger n we do not attempt to calculate $\operatorname{ord}_p x_{n,i}$ explicitly but instead show

Lemma 2.5. *For all* $n \ge 2$ *and* $n \le i \le n+p^s-2$, $\operatorname{ord}_p x_{n,i} \ge -[(i-1)/(p-1)]$ *with strict inequality if* $i \ge n+p-1$ *and equality if* $i < n+p-1$ *and* $(p-1) \mid (i-1)$.

Proof. The case $n=2$ follows from the following sequence of inequalities, where $i = tp^k$ with $k = \operatorname{ord}_p i$:

$$\begin{aligned} \operatorname{ord}_p i &\le 1 + p + p^2 + \cdots + p^{k-1} \quad \text{(with equality iff } k \le 1)\\ &= (p^k-1)/(p-1)\\ &\le [tp^k - 1/p-1] \quad \text{(with equality iff } t=1, \text{ or } k=0 \text{ and } t<p)\\ &= [i-1/p-1]. \end{aligned}$$

The case $n > 2$ requires the recursive formulas

$$x = \begin{cases} x_{n,i} + x_{n,n}x_{2,i-n+1} & \text{for } n+1 \le i < n+p^s-1,\\ x_{n,n}x_{2,i-n+1} & \text{for } i = n+p^s-1. \end{cases} \tag{2.6}$$

To see this, write $e = x_{n,n}e^n + x_{n,n+1}e^{n+1} + \cdots$ and substitute

$$x_{n,n}e^n = x_{n,n}e^{n-1}e = x_{n,n}e^{n-1}(x_{2,2}e^2 + x_{2,3}e^3 + \cdots).$$

Comparing the resulting expansion of e in terms of $e^{n+1}, \ldots$ with the expansion defining $x_{n+1,i}$ gives the formulas.

Now assume inductively that the lemma is true for n. The case $n=2$ and the induction hypothesis give us

$$\operatorname{ord}_p(x_{n,n}x_{2,i-n+1}) \ge -\left[\frac{n-1}{p-1}\right] - \left[\frac{i-n}{p-1}\right] \ge -\left[\frac{i-1}{p-1}\right]$$

and

$$\operatorname{ord}_p x_{n,i} \ge -\left[\frac{i-1}{p-1}\right].$$

These inequalities, together with (2.6), give the inequality of the lemma. In the case $i \geq (n+1)+p-1$, the inductive hypothesis implies that both the above inequalities are strict, giving the next claim of the lemma. Finally, in the case $i < (n+1)+p-1$ and $(p-1) \mid (i-1)$, one of the above inequalities will be an equality and the other strict, depending on whether or not $i = n+p-1$. □

Proof of Proposition 2.4 *continued*. In order to obtain the result for $n > 2$, we again use induction on n. First note that $t_{n+1} \geq t_n$. Multiply the recursive formula (2.6) by p^t. If $t \geq t_n$, then the term $p^t x_{n,i}$ (if it occurs at all) is an integer, so $p^t x_{n+1,i}$ is an integer iff $p^t x_{n,n} x_{2,i-n+1}$ is an integer. Since $\operatorname{ord}_p(x_{n,n}x_{2,i-n+1}) = \operatorname{ord}_p x_{n,n} + \operatorname{ord}_p x_{2,i-n+1}$, taking the minimum over i with $n+1 \leq i \leq n+1+p^s-2$ gives $\min_i \operatorname{ord}_p(x_{n,n}x_{2,i-n+1}) = \operatorname{ord}_p x_{n,n} - s$ by the case $n = 2$. By the lemma $\operatorname{ord}_p x_{n,n} \geq -[(n-1)/(p-1)]$ with equality when $(p-1) \mid (n-1)$.

Now

$$\begin{aligned} t_{n+1} &= \max\{t_n, -\min_i \operatorname{ord}_p(x_{n,n}x_{2,i-n+1})\} = \max\{t_n, -\operatorname{ord}_p x_{n,n} + s\} \\ &= \max\{s + [(n-2)/(p-1)], s - \operatorname{ord}_p x_{n,n}\}. \end{aligned}$$

Since $s - \operatorname{ord}_p x_{n,n} \leq s + [(n-1)/(p-1)]$ with equality when $(p-1) \mid (n-1)$, it follows that $t_{n+1} = s + [(n-1)/(p-1)]$. □

Since the characteristic class e_V/e_W has the highest order of any element in $K_G^1(SW; \mathbb{Q}/\mathbb{Z})$ and these characteristic numbers detect free bordism, we have now proved Theorem 2.1.

3. Orders of some other elements

Here we do a second calculation that we shall need in the next section. Let $G = \mathbb{Z}/p^s$ as before, and let $A(G)$ be the Burnside ring of G. Take $\sigma = p - [G/H] \in A(G)$, where $H \subset G$ has index p; let W be an n-dimensional unitary representation on whose unit sphere G acts freely. Conner and Floyd [CF] found an upper bound for the order of $\sigma[SW]$ in unitary free G-bordism; Waner and Wu in [WW] used the eta invariant to show that the order goes to infinity as $n \to \infty$; here we shall calculate the exact order.

Let φ denote the Euler φ-function so that $\varphi(p^s) = p^s - p^{s-1}$. We shall show

Theorem 3.1. *The order of $\sigma[SW]$ in $U_*(B\mathbb{Z}/p^s)$ is $p^{b(n)}$, where $b(n) = [(n+p^{s-1}-2)/\varphi(p^s)]$.*

As in Corollary 2.2, we now obtain

Corollary 3.2. *Let M be a unitary manifold with trivial G-action, and assume that $[M]$ is not divisible by p in U_*. Then the order of $\sigma[SW \times M]$ in $U_*^G(E\mathbb{Z}/p^s)$ is $p^{b(n)}$.* □

As in the previous section, write $R(G) = \mathbb{Z}[x]/\langle x^{p^s} - 1\rangle$ and write e for the Euler class of the one-dimensional representation V on which a generator of G acts by multiplication by $\exp(2\pi i/p^s)$; $e = 1 - x$. If e_W is the Euler class of W, then as in Lemma 2.1 we have that $\langle e_W\rangle = \langle e^n\rangle$, i.e., e_W and e^n are $R(G)$-multiples of one another. It follows (writing $(e_W)^{-1}$ for $(e_W \otimes \mathbb{Q})^{-1}$) that $(e_W)^{-1}R(G) = (e^n)^{-1}R(G)$, so $K_G^1(SW; \mathbb{Q}/\mathbb{Z}) \cong (e^n)^{-1}R(G)/R(G)$.

Theorem 3.1 will follow from

Lemma 3.3. *$R(G)\otimes\mathbb{Q}\cong R(H)\otimes\mathbb{Q}\oplus\mathbb{Q}[\xi]$, where $\xi=e^{2\pi i/p^s}$. Further, under this decomposition,*

$$R(G)=\left\{\left(\sum_{k=0}^{p^{s-1}-1} b_k x^k,\ \sum_{k=0}^{\varphi(p^s)-1} c_k\xi^k\right)\in R(H)\oplus\mathbb{Z}[\xi] \,\middle|\, \sum_{l=0}^{p-2} c_{k+lp^{s-1}}\equiv b_k(\operatorname{mod} p)\ \textit{for all } k\right\}.$$

Proof. The first statement follows from Lemma 1.2. For the second statement we are identifying $R(G)$ with $\mathbb{Z}[x]/(x^{p^s}-1)$ and $R(H)$ with $\mathbb{Z}[x]/(x^{p^{s-1}}-1)$; recall also that

$$\xi^{\varphi(p^s)}=-1-\xi^{p^{s-1}}-\xi^{2p^{s-1}}-\cdots-\xi^{(p-2)p^{s-1}}.$$

The result now follows by considering the image of an arbitrary element $\sum_{k=0}^{p^s-1} a_k x^k$ of $R(G)$ in $R(H)\otimes\mathbb{Q}\oplus\mathbb{Q}[\xi]$, ξ being the image of x in $\mathbb{Q}[\xi]$. □

Lemma 3.4. *In $R(G)\otimes\mathbb{Q}$ we have*

(a) $R(G)\cap\mathbb{Q}[\xi]=\langle(1-\xi)^{p^{s-1}}\rangle$, *the $\mathbb{Z}[\xi]$-module generated by $(1-\xi)^{p^{s-1}}$; and*

(b) *the projection of* $\operatorname{Im}(e\otimes\mathbb{Q})\cap R(G)$ *into $\mathbb{Q}[\xi]$ is $\langle 1-\xi\rangle$.*

Proof. For (a) it follows from the previous lemma that the elements

$$p,\, p(1-\xi),\, p(1-\xi)^2,\, \dots,\, p(1-\xi)^{p^{s-1}-1},\, (1-\xi)^{p^{s-1}},\, \dots,\, (1-\xi)^{\varphi(p^s)-1}$$

form an integral basis for $R(G)\cap\mathbb{Q}[\xi]$. However, since $(1-\xi)^{\varphi(p^s)}$ and p are associates in $\mathbb{Z}[\xi]$, we can replace the terms $p(1-\xi)^i$ by $(1-\xi)^{\varphi(p^s)+i}$, proving the claim.

As for (b), it follows from the previous lemma that the elements

$$p,\, (1-\xi),\, (1-\xi)^2,\, \dots,\, (1-\xi)^{\varphi(p^s)-1}$$

form an integral basis for the projection of $\operatorname{Im}(e\otimes\mathbb{Q})\cap R(G)$ in $\mathbb{Q}[\xi]$. To see this, we note that an element $(\sum b_k x^k, \sum c_k\xi^k)$ is a multiple of e iff $\sum b_k=0$ so that by the lemma $\sum c_k\equiv 0\ (\operatorname{mod} p)$. The result now follows by the same argument as for part (a). □

Lemma 3.5. *For $n\geq 1$ the projection of $(e^n)^{-1}R(G)$ into $\mathbb{Q}[\xi]$ is $\langle(1-\xi)^{-n+1}\rangle$.*

Proof. Since $z\in(e^n)^{-1}R(G)$ iff $e^n z\in\operatorname{Im}(e\otimes\mathbb{Q})\cap R(G)$, it follows from the previous lemma that the projection $\pi(z)$ of z is in $(1-\xi)^{-n}\langle 1-\xi\rangle=\langle(1-\xi)^{-n+1}\rangle$. Conversely, if $w\in\langle(1-\xi)^{-n+1}\rangle$, then there is a $z'\in\operatorname{Im}(e\otimes\mathbb{Q})\cap R(G)$ with $(1-\xi)^n w=\pi(z')$. Since e is invertible in $\operatorname{Im}(e\otimes\mathbb{Q})$, we obtain $w=\pi(z'/e^n)$, with $z'/e^n\in(e^n)^{-1}R(G)$. □

Note that $A(G)$ acts on $R(G)$ through the ring map $\tau: A(G)\to R(G)$ which assigns to a G-set the associated permutation module. Since $\tau(\sigma)=(0,p)$ under the decomposition in Lemma 3.3, we have the following consequence of Lemma 3.5.

Lemma 3.6. *$\sigma(e^n)^{-1}R(G) = \langle(1-\xi)^{\varphi(p^s)-n+1}\rangle$ as a $\mathbb{Z}[\xi]$-submodule of $\mathbb{Q}[\xi]$.*

Proof. The result follows from the fact that multiplication by σ is projection into $\mathbb{Q}[\xi]$ followed by multiplication by p and the fact (used in Lemma 3.4) that $(1-\xi)^{\varphi(p^s)}$ and p are associates in $\mathbb{Z}[\xi]$. □

Corollary 3.7.

$$\begin{aligned}&\sigma(e^n)^{-1}R(G)/\sigma(e^n)^{-1}R(G)\cap R(G)\\ &\qquad = \langle(1-\xi)^{\varphi(p^s)-n+1}\rangle/\langle(1-\xi)^k\rangle, \qquad k = \max\{\varphi(p^s)-n+1, p^{s-1}\},\\ &\qquad \cong \mathbb{Z}[\xi]/\langle(1-\xi)^m\rangle, \qquad m = \max\{0, p^{s-1}-\varphi(p^s)+n-1\}.\end{aligned}$$

Proof of Theorem 3.1. First, if $c \in K_G^1(EG; \mathbb{Q}/\mathbb{Z})$, then $c(\sigma SW) = \sigma c(SW)$. We can choose c so that $\sigma c(SW)$ corresponds to the generator $1 \in \mathbb{Z}[\xi]/\langle(1-\xi)^m\rangle$ under the isomorphism of Corollary 3.7. It thus remains to calculate the torsion of 1 in $\mathbb{Z}[\xi]/\langle(1-\xi)^m\rangle$. If $m = q\varphi(p^s)$, then $\mathbb{Z}[\xi]/\langle(1-\xi)^m\rangle \cong \mathbb{Z}[\xi]/p^q$ so that 1 has torsion p^q. In general, 1 has the highest order, which must be p to the power $[m+\varphi(p^s)-1/\varphi(p^s)]$. Rewriting this in terms of n gives the result. □

4. Fixed sets of unitary G-manifolds

Here we state and prove the results on G-actions on unitary manifolds mentioned in the introduction. In the first several results, when we say that a fixed set M^H is framed in M we mean that there is an NH/H-vector bundle isomorphism between the normal bundle of M^H in M and a trivial bundle $M^H \times W$.

We first prove a simple result about semifree actions. Recall that $a(n) = [(n-1)/(p-1)]$ and $b(n) = [(n+p^{s-1}-2)/\varphi(p^s)]$. We write $|W| = \dim_{\mathbb{C}} W$.

Theorem 4.1. *Let $G = \mathbb{Z}/p^s$ with p a prime, let W be a unitary representation of G on whose unit sphere G acts freely, and let M be a closed semifree unitary G-manifold of dimension $W+k$ such that M^G is framed in M. Then $[M^G] \in p^{s+a(|W|)}U_k$. Further, if $[Y] \in p^{s+a(|W|)}U_k$, then there exists a closed unitary $(nV+k)$-dimensional G-manifold M with $M^G = Y$.*

Proof. Since $M = (DW \times M^G) \cup_\partial F$, where F is a free G-manifold, F is a free null bordism of $SW \times M^G$, so, by Corollary 2.2, $[M^G] \in p^{s+a(|W|)}U_k$. Conversely, if $Y \in p^{s+a(|W|)}U_k$, then, again by Corollary 2.2, $SW \times Y$ is null bordant so that we can use a null bordism F to construct $M = (DW \times Y) \cup_\partial F$. □

Turning to the general case, we first need a lemma. With $G = \mathbb{Z}/p^s$, let $\sigma \in A(G)$ be as in §3, $\sigma = p - [G/H]$, where $H = \mathbb{Z}/p^{s-1}$.

Lemma 4.2. *The map $\sigma U_*^G(EG) \to U_*^G$ is trivial.*

Proof. By [CF] $U_*^G(EG) \cong U_* \oplus \tilde{U}_*^G(EG)$, where the summand U_* consists of the manifolds of the form $G \times M$, M a nonequivariant manifold, and the summand $\tilde{U}_*^G(EG)$ is generated by spheres SW on which G acts freely. The second summand clearly vanishes in U_*^G; the first is annihilated by σ since $\sigma G = 0$ in $A(G)$. □

Theorem 4.3. *Let $G = \mathbb{Z}/p^s$ with p a prime, let W be a unitary representation of G on whose unit sphere G acts freely, and let M be a closed unitary G-manifold of dimension $W + k$ such that each fixed set is framed in M. Then $[M^G] \in p^{b(|W|)}U_k$. Further, if $[Y] \in p^{b(|W|)}U_k$, then there exists a closed unitary $(W + k)$-dimensional G-manifold M with $M^G = pY$.*

Proof. Let M be a closed unitary G-manifold of dimension $W+k$ that frames its fixed sets. We claim that σM is G-bordant to a G-manifold of the form

$$N = (DW \times \sigma M^G) \cup_\partial F,$$

where F is a free G-manifold. To see this, let $L = \mathbb{Z}/p$ be the smallest nontrivial subgroup of G. Since M is $(W + k)$-dimensional and $W^L = 0$, it follows that $M^L = M^G \coprod \coprod P_i$, where each P_i is a k-dimensional free G/K_i-manifold for some proper nontrivial subgroup K_i of G. Therefore,

$$(\sigma M)^L = \sigma M^G \coprod \coprod \sigma P_i.$$

By the lemma (applied to each G/K_i) and the fact that the P_i are framed in M, σM is G-bordant to a G-manifold of the form $N = (DW \times \sigma M^G) \cup_\partial F$ with F a free G-manifold as claimed. Thus F is a free null G-bordism of $\sigma(SW \times M^G)$. It follows by Corollary 3.2 that $M^G \in p^{b(|W|)}U_*$, as desired.

Conversely, suppose that $Y \in p^{b(|W|)}U_*$. Then $\sigma(SW \times Y)$ is freely null bordant by Corollary 3.2. If F is a free null G-bordism, then $M = (DW \times \sigma Y) \cup_\partial F$ is a G-manifold with $M^G = pY$. □

We can also show the following variant of Theorem 4.3 for any abelian p-group.

Theorem 4.4. *Let G be an abelian p-group, let V be a unitary representation of G with $V^G = 0$, and let M be a closed unitary G-manifold of dimension $nV+k$ that frames its fixed sets. Then $[M^G] \in p^{s(n)}U_k$, where $s(n) \to \infty$ as $n \to \infty$. Further, there is a sequence $t(n) \geq s(n)$ such that if $[Y] \in U_k$, then there exists a closed unitary $(nV + k)$-dimensional G-manifold M with $M^G = p^{t(n)}Y$.*

Proof. We do induction on the order of G, the case G cyclic and acting freely on SV having been settled above. If G is a noncyclic abelian p-group or if G does not act freely on SV, then there exists a nontrivial subgroup H such that $V^H \neq 0$. Thus, if M is an $(nV+k)$-manifold which frames its fixed sets, M^H is in turn an (nV^H+k)-dimensional G/H-manifold which frames its fixed sets. The first statement of the theorem now follows by induction on $|G|$.

For the converse we outline a construction similar to one given in [W] in the context of framed G-manifolds. Let $\alpha \in A(G)$ be such that α^G is a power of p and $\alpha^H = 0$ for all proper subgroups $H \subset G$. Thus α restricts to zero under the forgetful homomorphisms $A(G) \to A(H)$. For a G-module W, let S^W denote the one-point compactification of W. We can realize α as a based G-map $\tilde{\alpha} : S^W \to S^W$ for some W because $A(G)$ is isomorphic to the 0th stable homotopy group. Let TU be the Thom spectrum representing stably almost complex bordism. Now, starting with a stable map $y : S^k \to TU$ representing $[Y]$, we try to extend to a G-map $S^{nV+k} \to T_G U$, the equivariant spectrum representing $(nV+*)$-dimensional bordism (which contains TU in its G-fixed set). The obstructions we encounter will be on G-cells of the form $G/H \times D^r$, with H a proper subgroup of G. We can kill these obstructions by multiplying by α, i.e., by suspending by the map $\tilde{\alpha}$. The end result is that we can construct a based G-map $f : S^{nV+mW+k} \to \Sigma^{mW} T_G U$ for some large m so that f is an extension of $\alpha^m y$. This implies that $f^G : S^{mW^G+k} \to \Sigma^{mW^G} TU$ represents $p^t T$ for some t. Thus f represents an $(nV + k)$-manifold M with $M^G = p^t Y$. Note that here t depends only on the G-cell structure of S^{nV+k} and not on Y. This gives the result. $\square$

The following counterexamples support the hypotheses of Theorem 4.3.

Example 4.5. (a) *Fixed sets not framed in M.* Let $G = \mathbb{Z}/p$ with p any odd prime, let R be the complex regular representation of G, let $n \geq 1$, and let $M_n = \mathbb{C}P(R^n)$. Then M_n^G is a disjoint union of p copies of $\mathbb{C}P^{n-1}$ with normal bundles isomorphic and of the form $\xi \otimes V^n$, where ξ is the canonical line bundle and V is the reduced regular representation. It follows that M_n is a unitary G-manifold of dimension $nV + (2n - 2)$ with $M_n^G \notin p^2 U_*$ for infinitely many n.

(b) *G not a p-group.* (This example is taken from [W].) Let $G = \mathbb{Z}/pq$ with p and q distinct primes, let V be a semifree one-dimensional complex representation of G, and let $\alpha = 1 - r[\mathbb{Z}/p] - s[\mathbb{Z}/q] \in A(G)$ where $rp + sq = 1$. Then $\alpha^G = 1$, and α restricts to zero under the forgetful homomorphism $A(G) \to A(e) = \mathbb{Z}$. Let S^W denote the one-point compactification of the representation W, and realize α as a based G-map $\tilde{\alpha} : S^W \to S^W$ for some W as in the proof of Theorem 4.4. Now, starting with the identity map $S^0 \to S^0$, we try to extend to a map $S^{nV} \to S^0$. The obstructions we encounter will be on free cells; we can kill these obstructions by multiplying by α, i.e., by suspending by the map $\tilde{\alpha}$. The end result is that we can construct a based G-map $f : S^{nV+mW} \to S^{mW}$ for some large m so that the restriction of f to S^{mW} is α^m. This implies that $f^G : S^{mW^G} \to S^{mW^G}$ has degree 1. If we now make f transverse to 0 we can form the nV-manifold $M = f^{-1}(0)$. Since f^G has degree 1, it follows that M^G consists, up to cancellation of oppositely oriented points, of a single fixed point; i.e., $[M^G] = 1 \in U_0$.

Note. We would like to propose a result similar to Theorem 4.4 for nonabelian p-groups. However, if G is nonabelian, then the condition that M frame its proper fixed subsets need not ensure that this is inherited by the NH/H-manifold M^H. What is needed is that M frame its proper fixed sets "consistently". We now make this more precise.

Definition 4.6. The W-dimensional Riemannian G-manifold M *frames its proper fixed sets consistently* if for every $K \subset G$ there exists a bundle isomorphism $\varepsilon_K : V_K \times M^K \to \nu_K = \nu(M^K, M)$. These trivializations are compatible in the following sense: if $g \in G$ and $H \subset gKg^{-1}$, then the diagram

$$\begin{tikzcd}
 & \nu_K \arrow[r, "\pi g"] \arrow[dd] & \nu_H \arrow[dd] & \\
V_K \times M^K \arrow[ur, "\varepsilon_K"] \arrow[dr] \arrow[rrr, "\pi g \times g"] & & & V_H \times M^H \arrow[ul, "\varepsilon_H"'] \arrow[dl] \\
 & M^K \arrow[r, "g"] & M^H &
\end{tikzcd}$$

commutes. Here g denotes translation by g and π denotes orthogonal projection.

We suggest

Conjecture 4.7. Let G be a finite p-group, and let V be any G-module with $V^G = 0$. Let M be a closed unitary G-manifold of dimension $nV + k$ that consistently frames its fixed sets. Then $[M^G] \in p^{s(n)} U_k$, where $s(n) \to \infty$ as $n \to \infty$. Further, there is a sequence $t(n) \geq s(n)$ such that if $[Y] \in U_k$, then there exists a closed unitary $(nV + k)$-dimensional G-manifold M with $M^G = p^{t(n)} Y$.

The version of this conjecture for framed G-manifolds was shown in [W]. The following result gives a special case of Conjecture 4.7.

Proposition 4.8. *Let G be a p-group, and assume that V is a unitary G-module with $V^G = 0$ and $V^H \neq 0$ for some normal subgroup H with G/H abelian. Then the conclusion of Conjecture* 4.7 *holds for $(nV + k)$-dimensional G-manifolds M.*

Proof. Let M be an $(nV + k)$-dimensional manifold that consistently frames its fixed sets. Then the G/H-manifold M^H (consistently) frames its fixed sets. The result follows by Theorem 4.4. □

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