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ON THE UNIQUE RANGE SET OF MEROMORPHIC FUNCTIONS

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ABSTRACT. This paper studies the unique range set of meromorphic functions and shows that there exists a finite set S such that for any two nonconstant meromorphic functions f and g the condition $E_f(S) = E_g(S)$ implies $f \equiv g$. As a special case this also answers an open question posed by Gross (1977) about entire functions and improves some results obtained recently by Yi.

1. INTRODUCTION

Let f be a nonconstant meromorphic function on the complex plane C and S be a subset of distinct elements in C. Define

$$E_f(S) = \bigcup_{a \in S} \{z | f(z) - a = 0\},$$

here a zero of $f(z) - a$ of multiplicity m appears m times in $E_f(S)$. In 1976 Gross proved [1] that there exist three finite sets S_j $(j = 1, 2, 3)$ such that for any two nonconstant entire functions f and g if $E_f(S_j) = E_g(S_j)$ $(j = 1, 2, 3)$, then $f \equiv g$. In the same paper Gross posed the following problem: Can one find two (or possibly even one) finite sets S_j $(j = 1, 2)$ such that any two entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ $(j = 1, 2)$ must be identical? In 1982, F. Gross and C. C. Yang proved the following result.

Theorem A ([2])**.** *Let $T = \{z | e^z + z = 0\}$. Let f and g be two nonconstant entire functions. If $E_f(T) = E_g(T)$, then $f \equiv g$.*

In [2] the set S such that for any two nonconstant entire functions f and g the condition $E_f(S) = E_g(S)$ implies $f \equiv g$ is called a unique range set (URS, in brief) of entire functions. A similar definition for meromorphic functions can be defined. Note that the set $T = \{z | e^z + z = 0\}$ contains an infinite number of elements. Recently, Yi [6] exhibited a finite unique range set of entire functions which gave a positive answer to Gross's problem. He proved

Theorem B. *Let $n \geq 15$, $n > m \geq 5$ with n and m having no common factors. Let a and b be two nonzero constants such that the algebraic equation $z^n + az^m + b = 0$ has no multiple roots. Then the set $S = \{z | z^n + az^m + b = 0\}$ is a URS of entire functions.*

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In this paper, we shall exhibit, among other results, a finite URS of meromorphic functions with 19 elements and a URS of entire functions with nine elements.

Theorem 1. *Let $m \geq 2$, $n > 4m + 10$ with n and $n - m$ having no common factors. Let a and b be two nonzero constants such that the algebraic equation $z^n + az^{n-m} + b = 0$ has no multiple roots. Let $S = \{z | z^n + az^{n-m} + b = 0\}$. Then for any two nonconstant meromorphic functions f and g, the condition $E_f(S) = E_g(S)$ implies $f \equiv g$.*

Theorem 2. *Let $m \geq 2$, $n > 4m + 6$ with n and $n - m$ having no common factors. Let a, b and S be as in Theorem 1. Then for any two nonconstant meromorphic functions f and g, the conditions $E_f(S) = E_g(S)$ and $E_f\{\infty\} = E_g\{\infty\}$ imply $f \equiv g$.*

Theorem 3. *Let $m \geq 1$, $n > 4m + 4$ with n and $n - m$ having no common factors. Let a, b and S be as in Theorem 1. Then for any two nonconstant entire functions f and g, the condition $E_f(S) = E_g(S)$ implies $f \equiv g$.*

The main tool will be Nevanlinna's theory of meromorphic functions, and it is assumed that the reader is familiar with its basic notation and results (see Hayman [4]). In the sequel the letter E will be used to denote a set of r values of finite linear measure.

2. Some lemmas

The following lemmas will be needed in the proof of our theorems.

Lemma 1 ([7])**.** *Let f and g be two nonconstant meromorphic functions, and c_1, c_2, and c_3 be nonzero constants. If $c_1 f + c_2 g \equiv c_3$, then*

$$T(r, f) < \overline{N}\left(r, \frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}(r, f) + S(r, f).$$

Here and in the sequel $S(r, f)$ denotes the quantity $o(T(r, f)), r \to \infty, r \notin E$.

Lemma 2. *Let f_1, f_2, and f_3 be nonconstant meromorphic functions and $f_1 + f_2 + f_3 \equiv 1$. If f_1, f_2, f_3 are linearly independent, then*

$$T(r, f_1) < 2\sum_{i=1}^{3} \overline{N}\left(r, \frac{1}{f_i}\right) + \sum_{i=1}^{3} \overline{N}(r, f_i) + o(T(r))$$

where $T(r) = \max_{1 \leq i \leq 3}\{T(r, f_i)\}$ and $r \notin E$.

Proof. By the proof of a generalization of Borel's theorem (a generalization of Picard's theorem) by Nevanlinna [3] (page 70), we have

$$T(r, f_1) < \sum_{i=1}^{3} N\left(r, \frac{1}{f_i}\right) - \sum_{i=2}^{3} N(r, f_i) + N(r, D) - N\left(r, \frac{1}{D}\right) + o(T(r)),$$

where D is the Wronskian of f_1, f_2, and f_3, i.e.,

$$D = \begin{vmatrix} f_1 & f_2 & f_3 \\ f'_1 & f'_2 & f'_3 \\ f''_1 & f''_2 & f''_3 \end{vmatrix}.$$

Since $f_1 + f_2 + f_3 \equiv 1$, we have

(1) $$D = f'_2 f''_3 - f''_2 f'_3 = -(f'_1 f''_3 - f''_1 f'_3) = f'_1 f''_2 - f''_1 f'_2.$$

Write

$$N(r) = \sum_{i=1}^{3} N\left(r, \frac{1}{f_i}\right) - \sum_{i=2}^{3} N(r, f_i) + N(r, D) - N\left(r, \frac{1}{D}\right)$$

and

$$N^*(r) = 2\sum_{i=1}^{3} \overline{N}\left(r, \frac{1}{f_i}\right) + \sum_{i=1}^{3} \overline{N}(r, f_i).$$

Thus clearly Lemma 2 follows immediately from the inequality

$$N(r) \le N^*(r), \tag{2}$$

which is to be shown next.

For a given meromorphic function f and a complex number $a \in \overline{C}$, we define

$$\mu_f^a(z) = \begin{cases} m, & z \text{ is an } a\text{-point of } f \text{ with multiplicity } m \ge 1, \\ 0, & z \text{ is not an } a\text{-point of } f \end{cases}$$

and

$$\overline{\mu}_f^a(z) = \begin{cases} 1, & z \text{ is an } a\text{-point of } f \text{ with multiplicity } m \ge 1, \\ 0, & z \text{ is not an } a\text{-point of } f. \end{cases}$$

Let

$$\mu = \mu_{f_1}^0 + \mu_{f_2}^0 + \mu_{f_3}^0 - \mu_{f_2}^\infty - \mu_{f_3}^\infty + \mu_D^\infty - \mu_D^0$$

and

$$\mu^* = 2\overline{\mu}_{f_1}^0 + 2\overline{\mu}_{f_2}^0 + 2\overline{\mu}_{f_3}^0 + \overline{\mu}_{f_1}^\infty + \overline{\mu}_{f_2}^\infty + \overline{\mu}_{f_3}^\infty.$$

Thus inequality (2) follows from $\mu(z) \le \mu^*(z)$ for any z. To prove this, we consider the following five cases for an arbitrary point $z \in C$.

Case 1. z is a zero-point of f_i $(i = 1, 2, 3)$ with multiplicity $m_i \ge 0$.

Case 2. z is a zero-point of f_1 with multiplicity $m \ge 1$ and a pole of f_2 and f_3 with multiplicity $k \ge 1$.

Case 3. z is a zero-point of f_2 with multiplicity $m \ge 1$ and a pole of f_1 and f_3 with multiplicity $k \ge 1$.

Case 4. z is a zero-point of f_3 with multiplicity $m \ge 1$ and a pole of f_1 and f_2 with multiplicity $k \ge 1$.

Case 5. z is a pole of D but not a zero of f_1, f_2, and f_3.

In each case we can verify that the inequality $\mu(z) \le \mu^*(z)$ holds. For instance, take Case 2; then we have $\mu_{f_1}^0(z) = m$, $\mu_{f_2}^0 = \mu_{f_3}^0(z) = 0$, $\mu_{f_2}^\infty(z) = \mu_{f_3}^\infty(z) = k$. Thus $\mu^*(z) = 4$.

If $k - m + 3 > 0$, then from (1), z is a pole of D with multiplicity at most $k - m + 3$. This means that $\mu_D^\infty \le k - m + 3$. It follows that

$$\mu(z) \le m - 2k + (k - m + 3) = 3 - k \le 2 < \mu^*(z).$$

If $k - m + 3 \le 0$, then from (1) z is a zero of D with multiplicity at least $m - k - 3$. This means that $\mu_D^\infty(z) = 0$ and $\mu_D^0(z) \ge m - k - 3$. Hence

$$\mu(z) \le m - 2k - (m - k - 3) = 3 - k \le 2 < \mu^*(z).$$

The remaining cases can be proved in a similar manner. This also completes the proof of the lemma.

Lemma 3 ([5])**.** *Let f be a meromorphic function, and*

$$P(f) = a_0 f^n + a_1 f^{n-1} + \cdots + a_n$$

be a polynomial in f of degree n, where a_0 $(\neq 0), a_1, \ldots, a_n$ are finite complex numbers. Then

$$T(r, P(f)) = nT(r, f) + S(r, f).$$

3. Proof of Theorem 1

Let $r_1, r_2, \ldots, r_n$ be the roots of equation $z^n + az^{n-m} + b = 0$. Since $E_f(S) = E_g(S)$, we have from Nevanlinna's second fundamental theorem

$$\begin{aligned}(n-2)T(r,g) &< \sum_{k=1}^{n} \overline{N}\left(r, \frac{1}{g - r_k}\right) + S(r,g) \\ &= \sum_{k=1}^{n} \overline{N}\left(r, \frac{1}{f - r_k}\right) + S(r,g) \\ &\leq nT(r,f) + S(r,g).\end{aligned}$$

It follows that

$$T(r,g) \leq \frac{n}{n-2} T(r,f) + S(r,g). \tag{3}$$

Similarly the following inequality holds:

$$T(r,f) \leq \frac{n}{n-2} T(r,g) + S(r,f). \tag{4}$$

In the sequel we use $S(r)$ to express either $S(r,f)$ or $S(r,g)$.

Consider now the following meromorphic function

$$\psi = \frac{f^n + af^{n-m} + b}{g^n + ag^{n-m} + b}. \tag{5}$$

The condition $E_f(S) = E_g(S)$ ensures that the zeros of ψ come from the poles of g, and the poles of ψ come from the poles of f. This means that the following inequalities hold:

$$\overline{N}\left(r, \frac{1}{\psi}\right) \leq \overline{N}(r,g) \tag{6}$$

and

$$\overline{N}(r,\psi) \leq \overline{N}(r,f). \tag{7}$$

Let

$$f_1 = -\frac{1}{b} f^{n-m}(f^m + a), \quad f_2 = \frac{1}{b}\psi g^{n-m}(g^m + a), \quad f_3 = \psi. \tag{8}$$

Then f_1, f_2, and f_3 are meromorphic functions and f_1 is not a constant. From (3), we have

$$f_1 + f_2 + f_3 \equiv 1. \tag{9}$$

Now we distinguish two cases.

Case 1: *f_3 is not a constant.* If f_1 and f_2 are linearly dependent, then $f_2 = cf_1, c \neq -1$. From (9) we have

$$(1+c)f_1 + f_3 \equiv 1.$$

By using Lemma 1 and Lemma 3 together with the inequalities (3) and (6), we deduce

$$\begin{aligned} nT(r,f) &= T(r,f_1) + S(r) \\ &< \overline{N}\left(r,\frac{1}{f_1}\right) + \overline{N}\left(r,\frac{1}{f_3}\right) + \overline{N}(r,f_1) + S(r) \\ &\leq \overline{N}\left(r,\frac{1}{f}\right) + \overline{N}\left(r,\frac{1}{f^m+a}\right) + \overline{N}(r,g) + \overline{N}(r,f) + S(r) \\ &\leq (m+2)T(r,f) + T(r,g) + S(r) \\ &\leq \left(m+2+\frac{n}{n-2}\right)T(r,f) + S(r) \\ &= \left(m+3+\frac{2}{n-2}\right)T(r,f) + S(r), \end{aligned}$$

which is contradictory to $n > 4m+10$. Hence f_1 and f_2 must be linearly independent.

If f_1, f_2, and f_3 are linearly independent and f_2 is not a constant, then by using Lemma 2 we have

$$\begin{aligned} T(r,f_1) < 2\overline{N}\left(r,\frac{1}{f_1}\right) + 2\overline{N}\left(r,\frac{1}{f_2}\right) + 2\overline{N}\left(r,\frac{1}{f_3}\right) \\ + \overline{N}(r,f_1) + \overline{N}(r,f_2) + \overline{N}(r,f_3) + S(r). \end{aligned}$$

From the identities (5) and (8), we can easily see that the zeros of f_2 cannot come from the zeros of ψ, and the poles of f_2 must come from the poles of f. By the above inequality and Lemma 3 together with (6), (7) and (8), we deduce that

$$\begin{aligned} nT(r,f) &< 2\overline{N}\left(r,\frac{1}{f}\right) + 2\overline{N}\left(r,\frac{1}{f^m+a}\right) + 2\overline{N}\left(r,\frac{1}{g}\right) + 2\overline{N}\left(r,\frac{1}{g^m+a}\right) \\ &\quad + 2\overline{N}(r,g) + \overline{N}(r,f) + \overline{N}(r,f) + \overline{N}(r,f) + S(r) \\ &\leq (2m+5)T(r,f) + 2(m+2)T(r,g) + S(r) \\ &\leq \left[(2m+5) + 2(m+2)\frac{n}{n-2}\right]T(r,f) + S(r) \\ &= \left(4m+9+\frac{4m+8}{n-2}\right)T(r,f) + S(r). \end{aligned}$$

This contradicts the assumption $n > 4m+10$. It follows that when f_1, f_2, and f_3 are linearly independent, f_2 must be constant and $f_2 \neq -1$, i.e. $f_1 + f_3 = 1 - f_2$ is a nonzero constant. By Lemma 1,

$$T(r,f_1) < \overline{N}\left(r,\frac{1}{f_1}\right) + \overline{N}\left(r,\frac{1}{f_3}\right) + \overline{N}(r,f_1) + S(r).$$

This leads to

$$nT(r,f) \leq \left(m+3+\frac{2}{n-2}\right)T(r,f) + S(r),$$

which is a contradiction to $n > 4m + 10$.

If f_1, f_2, and f_3 are linearly dependent, then there exist three constants c_1, c_2, and c_3, at least one of them is not zero, such that

$$c_1 f_1 + c_2 f_2 + c_3 f_3 \equiv 0. \tag{10}$$

This and the fact that f_1, f_2 are linearly independent imply $c_3 \neq 0$. So

$$c_1 \frac{f_1}{\psi} + c_2 \frac{f_2}{\psi} = -c_3. \tag{11}$$

If $c_1 = 0$, then $g^{n-m}(g^m + a)$; hence g is a constant. This is impossible.

If $c_2 = 0$, then

$$\frac{c_1}{b} f^{\,n-m}(f^{\,m} + a) = c_3 \psi. \tag{12}$$

Let $s_0 = 0, s_1, \ldots, s_m$ be the distinct roots of the equation $z^n + az^{n-m} = 0$. Then (12) shows that any s_j-*point* of f must be a zero of ψ and hence a pole of g. But from (5) and (12) one can see that the multiplicity of any zero of ψ is at least n, so the multiplicity of an s_j-*point* $(j \neq 0)$ of f is at least n and at least m for an s_0-*point* of f. Hence, we have

$$\Theta(s_j, f\,) = 1 - \varlimsup_{r\to\infty} \frac{\overline{N}(r, \frac{1}{f-s_j})}{T(r, f\,)} \geq 1 - \varlimsup_{r\to\infty} \frac{\overline{N}(r, \frac{1}{f-s_j})}{N(r, \frac{1}{f-s_j})} \geq 1 - \frac{1}{n},$$

$j = 1, 2, \ldots, m$, and

$$\Theta(s_0, f\,) \geq 1 - \frac{1}{m}.$$

Again by the second fundamental theorem about the deficiencies of meromorphic functions, we have

$$1 - \frac{1}{m} + m\left(1 - \frac{1}{n}\right) \leq \sum_{j=0}^{m} \Theta(s_j, f\,) \leq 2.$$

This is impossible because $m \geq 2, n > 4m + 10$.

Now that we have obtained $c_1 \neq 0, c_2 \neq 0, c_3 \neq 0$, by Lemma 1 and (11)

$$\begin{aligned} T\left(r, \frac{f_2}{\psi}\right) &< \overline{N}\left(r, \frac{\psi}{f_2}\right) + \overline{N}\left(r, \frac{\psi}{f_1}\right) + \overline{N}\left(r, \frac{f_2}{\psi}\right) + S(r) \\ &< \overline{N}\left(r, \frac{\psi}{f_2}\right) + \overline{N}(r, \psi) + \overline{N}\left(r, \frac{1}{f_1}\right) + \overline{N}\left(r, \frac{f_2}{\psi}\right) + S(r). \end{aligned}$$

Hence from Lemma 3 and (7) and (8), we have

$$\begin{aligned} nT(r, g) &< \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}\left(r, \frac{1}{g^m + a}\right) + \overline{N}(r, f\,) \\ &\quad + \overline{N}\left(r, \frac{1}{f^{\,m} + a}\right) + \overline{N}(r, g) + S(r) \\ &\leq (m+2)T(r, g) + (m+2)T(r, f\,) + S(r) \\ &\leq (m+2)\left(1 + \frac{n}{n-2}\right)T(r, f\,) + S(r), \end{aligned}$$

which is a contradiction to $n > 4m + 10$.

We can rule out Case 1.

Case 2: f_3 *is a constant.* In this case, f_2 cannot be a constant. From (5), we have

$$T(r,f) = T(r,g) + S(r). \tag{13}$$

If $f_3 \neq 1$, then $f_1 + f_2 = 1 - f_3 \neq 0$. By Lemma 1

$$T(r,f_1) < \overline{N}\left(r,\frac{1}{f_1}\right) + \overline{N}\left(r,\frac{1}{f_2}\right) + \overline{N}(r,f_1) + S(r).$$

That is,

$$\begin{aligned} nT(r,f) &< \overline{N}\left(r,\frac{1}{f_1}\right) + \overline{N}\left(r,\frac{1}{f^m+a}\right) + \overline{N}\left(r,\frac{1}{g}\right) \\ &\quad + \overline{N}\left(r,\frac{1}{g^m+a}\right) + \overline{N}(r,f) + S(r) \\ &< (m+2)T(r,f) + (m+1)T(r,g) + S(r) \\ &= (2m+3)T(r,f) + S(r). \end{aligned}$$

This contradicts the assumption that $n > 4m+10$.

If $f_3 = 1$, then from (5) we get

$$g^m(h^n - 1) = -a(h^{n-m} - 1) \tag{14}$$

where $h = f/g$ is a meromorphic function. Further (14) can be rewritten as

$$g^m(h-u_1)(h-u_2)\cdots(h-u_n) = -a(h^{n-m} - 1) \tag{15}$$

where $u_j = e^{i2j\pi/n}$, $j = 1,2,\ldots,n$. Since n and $n-m$ have no common factors, we see that $u_j^{n-m} - 1 \neq 0, j = 1,\ldots,n-1$. Hence from (15) the multiplicity of a u_j-*point* of h is at least m. Suppose that h is not a constant, then we have

$$\begin{aligned} \Theta(u_j,h) &= 1 - \overline{\lim_{r\to\infty}} \frac{\overline{N}(r,\frac{1}{h-u_j})}{T(r,h)} \\ &\geq 1 - \overline{\lim_{r\to\infty}} \frac{\overline{N}(r,\frac{1}{h-u_j})}{N(r,\frac{1}{h-u_j})} \geq 1 - \frac{1}{m}, \qquad j = 1,\ldots,n-1. \end{aligned}$$

Thus

$$(n-1)\left(1-\frac{1}{m}\right) \leq \sum_{j=1}^{n-1}\Theta(u_j,h) \leq 2,$$

which contradicts $m \geq 2$ and $n > 4m+10$. This shows that h must be a constant. Furthermore from (14) we can see that h must be equal to 1. Otherwise we will deduce that g is a constant. Hence $f \equiv g$. This completes the proof of Theorem 1.

Note that the function ψ in (5) will assume the form e^{α} with α being an entire function under the assumptions of Theorem 2 and Theorem 3. Furthermore under the assumption of Theorem 3 the inequalities (3) and (4) will be replaced by

$$T(r,g) \leq \frac{n}{n-1}T(r,f) + S(r,g)$$

and

$$T(r,f) \leq \frac{n}{n-1}T(r,g) + S(r,f)$$

respectively, and we can then prove these two theorems immediately following the same procedure of the proof of Theorem 1.

Example 1. The set $S = \{z|z^{19} - z^{17} + 1 = 0\}$ is a URS of meromorphic functions with 19 elements.

Example 2. The set $S = \{z|z^9 - z^8 + 1 = 0\}$ is a URS of entire functions with nine elements.

4. Concluding remarks

We would like to pose the following problems about the unique range set of meromorphic functions and entire functions for further investigations.

Problem 1. Can one find a URS of entire functions with fewer than nine elements? What is the smallest cardinality for a URS of entire functions?

Problem 2. Can one find a URS of meromorphic functions with fewer than 19 elements? What is the smallest cardinality for a URS of meromorphic functions?

Now we introduce the following notation:

$$\begin{aligned} U_M &= \{S|S \text{ is a URS of meromorphic functions}\}, \\ U_E &= \{S|S \text{ is a URS of entire functions}\}, \\ \lambda_M &= \min\{n(S)|S \in U_M\}, \\ \lambda_E &= \min\{n(S)|S \in U_E\}, \end{aligned}$$

where $n(S)$ denotes the cardinal number of the set S. Obviously,

$$\lambda_E \leq \lambda_M.$$

Example 1 and Example 2 show that $\lambda_E \leq 9$ and $\lambda_M \leq 19$, respectively. We claim

Theorem 4. $\lambda_E \geq 4$.

Proof. Let us consider the two entire functions

$$f = \frac{\omega_2 e^h}{\omega_2 - \omega_1} + \frac{t\omega_1 e^{-h}}{\omega_2 - \omega_1} + \frac{1}{3}(a_1 + a_2 + a_3)$$

and

$$g = \frac{e^h}{\omega_2 - \omega_1} + \frac{t e^{-h}}{\omega_2 - \omega_1} + \frac{1}{3}(a_1 + a_2 + a_3)$$

where h is any nonconstant entire function, a_1, a_2, and a_3 are three finite distinct complex numbers, and

$$t = a_1 a_2 + a_1 a_2 + a_2 a_3 - \frac{1}{3}(a_1 + a_2 + a_3)^2,$$
$$\omega_1 = e^{i2\pi/3}, \qquad \omega_2 = e^{i4\pi/3}.$$

It is easy to verify that

$$(f - a_1)(f - a_2)(f - a_3) \equiv (g - a_1)(g - a_2)(g - a_3),$$

which shows $E_f\{a_1, a_2, a_3\} = E_g\{a_1, a_2, a_3\}$, but obviously f is not identically equal to g. Hence $\lambda_E \geq 4$.

We conjecture that $\lambda_E = 4$ is the answer to Problem 1.

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