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C^*-ALGEBRAS OF PROPER FOLIATIONS

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Abstract. We study the C^*-algebras of proper foliations. In case of finite depth they are described by a tower of exact sequences. We conclude with remarks about foliations almost without holonomy and AF-embeddability.

1. Introduction

Let $(M, \mathcal{F})$ be a closed n-manifold with a smooth codimension one foliation. Our purpose is to describe the C^*-algebra associated to $\mathcal{F}$ when the foliation is proper of finite depth: There is a closed subset C of M which is a finite union of leaves and such that each component V of $M \backslash C$ fibers over the circle with fiber the leaves of $\mathcal{F}|_V$. A result of Natsume [7] allows us to treat foliations almost without holonomy. We also study the embeddability in AF-algebras, after Fack and Wang [5].

We note that the case of foliations by Reeb components was treated by Torpe in [9].

In a certain sense, the structure of the C^*-algebra of a proper foliation is a reflection of its geometrical hierarchical structure, that is, that of a finite graph.

2. The graph of a foliation

Define the graph or holonomy groupoid $G(M, \mathcal{F})$ of the foliated manifold $(M, \mathcal{F})$ to be the collection of all triples $\gamma = (x, [\alpha], y)$ where x and y lie on the same leaf L, α is a path from x to y in L, and $[\alpha]$ is the holonomy equivalence class of α: α and β are equivalent if the (germinal) holonomy of $\alpha^{-1}\beta$ is trivial. Denote by r and s the range and source maps from $G = G(\mathcal{F})$ to M: $r(\gamma) = x$, $s(\gamma) = y$, and for A and B subsets of M, let G_A^B be the set of those elements of G with source in A and range in B. Similarly, $G_A = \{\gamma; s(\gamma) \in A\}$ and $G^B = \{\gamma; r(\gamma) \in B\}$. For instance, if L is the leaf of $\mathcal{F}$ through the point $x \in M$, then G_x is the holonomy covering of L and G_x^x is the holonomy group of L.

We describe a topology for G. Let $\gamma = (x, [\alpha], y)$ be a point in G. Choose a path α representing $[\alpha]$, and a chain $U_1, \ldots, U_k$ of Frobenius charts along α so the holonomy map h_α is defined from an open transversal T_x at x into a transversal T_y at y. Let f: $U_1 \to T_x$ be the projection along the plaques in U_1. An element of a subbase for the topology of G is

$$W = \{(v, [q_{vw}\alpha_v p_v], w); v \in U_1, w \in L_v \cap U_k\}$$

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where p_v is a path in the plaque of v from v to $f(v)$, α_v is a path from $f(v)$ to $h_\alpha(f(v))$ that shadows α, and q_{vw} is a path from $h_\alpha(f(v))$ to a point w in the same plaque of U_k. These paths are unique up to holonomy. This topology gives G the structure of a locally euclidean space of dimension $2n-1$, but not necessarily Hausdorff. Furthermore, the foliation $\mathcal{F}$ of M induces a codimension one foliation $\mathcal{G}$ on G: given a leaf L of $\mathcal{F}$, a leaf of $\mathcal{G}$ consists of all $\gamma \in G$ with range and source in L. In fact, the subbasic element W defined above is a Frobenius chart for $\mathcal{G}$ around γ.

3. Structure of proper foliations

Recall that a leaf is proper if its topology as a manifold coincides with that induced from M. Equivalently, for each point of the leaf there is a neighborhood which is a Frobenius chart and intersects the leaf in just one plaque.

Our description of the C^*-algebra of a proper foliation is based on the theory of levels. What follows is a brief summary, more details are to be found in the Cantwell and Conlon article [3].

A compact leaf is at depth zero. A leaf L is at depth $k > 0$ if $\overline{L}\backslash L$ consists of leaves at depths at most $k-1$, at least one of which is at depth $k-1$. The depth $d(\mathcal{F})$ of the foliation is the supremum of the depths of the leaves of $\mathcal{F}$.

The set M_k of leaves at depth at most k is a closed saturated subset of $(M, \mathcal{F})$ and there is a nest

$$\emptyset = M_{-1} \subset M_0 \subset M_1 \subset \cdots \subset M.$$

In general, there may be leaves at infinite depth, but for a proper foliation one has $\bigcup_{k\geq 0} M_k = M$. Let $V_k = M\backslash M_k$ and let N_k be the collection of leaves at depth k.

Next we recall the structure of connected open saturated subsets of M. Let U be such a set. Its metric completion $\widehat{U}$ is a manifold with boundary, but noncompact in general. The inclusion $i\colon\ U \to M$ extends to an immersion $\hat{i}\colon \widehat{U} \to M$ that carries each component of $\partial\widehat{U}$ diffeomorphically onto a leaf of $\mathcal{F}$, although it may identify some components pairwise.

To describe the structure of $\widehat{U}$, we fix a flow $\mathcal{L}$ transverse to $\mathcal{F}$. Let $\widehat{\mathcal{F}}$ and $\widehat{\mathcal{L}}$ be the foliations induced on $\widehat{U}$. There is a *Dippolito decomposition* $\widehat{U} = K \cup A_1 \cup \cdots \cup A_q$. Here K is a compact connected n-manifold called the *nucleus* of $\widehat{U}$, and each *arm* A_r is diffeomorphic to $B_r \times [0,1]$ where B_r is a complete noncompact connected submanifold of $\partial\widehat{U}$, each $\{x\} \times [0,1], x \in B_r$, is a leaf of $\widehat{\mathcal{L}}$, and $\widehat{\mathcal{F}}|_{A_r}$ is a $[0,1]$-bundle over B_r. We say that U is *trivial at infinity* if the nucleus K of $\widehat{U}$ can be chosen so that $\widehat{\mathcal{F}}$ restricts to the product foliation in each arm.

Finally we describe the way a proper leaf L winds in on another proper leaf N. Let J be a connected compact codimension one submanifold of N and let N_J be obtained from $N\backslash J$ by attaching two copies J_1 and J_2 of J as boundary. Let $f\colon\ [0,\varepsilon] \to [0,\delta], \delta = f(\varepsilon) < \varepsilon, f(0) = 0$, be a contracting diffeomorphism. Let C be the manifold with corner obtained from $N_J \times [0,\varepsilon]$ by identifying $(x,t) \in J_1 \times [0,\varepsilon]$ with $(x, f(t)) \in J_2 \times [0,\delta]$. The product foliations $\{N_J \times \{t\}; t \in [0,\varepsilon]\}$ and $\{\{x\} \times [0,\varepsilon]; x \in N_J\}$ of $N_J \times [0,\varepsilon]$ induce foliations $\mathcal{F}_C$ and $\mathcal{L}_C$ of C. This foliated manifold $(C, \mathcal{F}_C)$ is called a spiral collar of N with juncture J.

The Poincaré-Bendixson theory of Cantwell-Conlon [3] says that if $N \subset \overline{L}$, then there is a spiral collar $(C, \mathcal{F}_C)$ of N as above such that $L \cap C$ is a leaf of $\mathcal{F}_C$ and $\mathcal{L}|_C = \mathcal{L}_C$.

In general, the restriction of $\mathcal{F}$ to C does not coincide with $\mathcal{F}_C$. But it does if the foliation is of finite depth d and there are only finitely many leaves at level $< d$.

From now on assume $(M, \mathcal{F})$ is a proper foliation of finite depth d and there are finitely many leaves at depth $< d$. The Generalized Kopell Lemma ([3], Theorem 2.8) gives the following

Lemma 1. *Every connected open saturated subset of $(M, \mathcal{F})$ is trivial at infinity.*

By assumption, the union of leaves at level d is open and has only a finite number of connected components. The following is, therefore, a corollary of the lemma.

Proposition 2. *The one-sided holonomy group of every leaf L of $(M, \mathcal{F})$ is infinite cyclic.*

This is obvious if L is a compact leaf: if it has holonomy we can construct a spiral collar of L as above. If L is at depth one select a connected open saturated subset U such that L is a leaf of $\widehat{U}$. Since $\widehat{U}$ is trivial at infinity, all the holonomy of L in the side approached by U is contained in a suitable nucleus K of $\widehat{U}$. Now $L \cap K$ is a compact leaf of the foliated manifold (with corners) $(K, \mathcal{F}|_K)$, hence its one-sided holonomy is infinite cyclic. As there is no holonomy in the arms, the van Kampen theorem shows that the one-sided holonomy of L is infinite cyclic also. The leaves at depth d have no holonomy, so this process ends.

We should remark that this result does not hold if the foliation is only C^1. However, it does hold if the depth is one, since in this case the holonomy group of the compact leaves is totally ordered archimedian, although it is easy to construct examples of continuous proper foliations of depth two with nonabelian holonomy groups. Also, if the foliation is not known to be proper but it has no holonomy, one needs smoothness to show that it is either a fibration or has all leaves dense.

There is also a partial converse to this result, namely

Proposition 3. *If $(M, \mathcal{F})$ is a smooth codimension one proper foliation with all the holonomy groups abelian, then it has finite depth.*

One finds several obstructions when trying to extend the results of Section 5 to proper foliations of infinite depth or with holonomy groups that are not abelian.

4. The C^*-algebra of a foliation

When the graph G of $\mathcal{F}$ is Hausdorff the basic elements of $C^*(M, \mathcal{F})$ are smooth half-densities with compact support on $G, f \in C_c^\infty(G, \Omega^{1/2})$, where $\Omega^{1/2}$ is the complex line bundle over G whose fiber $\Omega_\gamma^{1/2}$ over $\gamma \in G$ is the vector space $\Omega_x^{1/2} \otimes \Omega_y^{1/2}, x = s(\gamma), y = r(\gamma)$, and $\Omega_x^{1/2}$ is the one-dimensional complex vector space of maps from the exterior power $\Lambda^2 \mathcal{F}_x$ to $\mathbf{C}$ such that $\rho(\lambda v) = |\lambda|^{1/2} \rho v$.

When G is not Hausdorff, one first defines the set $C_c(G)$ of 'continuous' functions with compact support on G using its locally euclidean structure: an element of $C_c(G)$ is a finite linear combination of functions of the form $f \circ \varphi$, where $\varphi\colon\ U \to \mathbf{R}^{2n-1}$ is a local chart on G and $f \in C_c(\mathbf{R}^{2n-1})$ with support contained in $\varphi(U)$.

When G is Hausdorff this gives $C_c(G)$ as the set of continuous functions with compact support. However, when G is not Hausdorff $C_c(G)$ may contain elements that are not continuous, although all of them have compact support. With this definition of $C_c(G)$ we get $C_c^\infty(G, \Omega^{1/2})$, the space of smooth half-densities on G.

For f, g in $C_c^\infty(G, \Omega^{1/2})$ the convolution product $f * g$ is defined by

$$f * g(\gamma) = \int f(\gamma\gamma'^{-1})g(\gamma').$$

The $*$ operation defined by $f^*(\gamma) = \overline{f(\gamma^{-1})}$ makes $C_c^\infty(G, \Omega^{1/2})$ into a $*$-algebra. For each leaf L of $\mathcal{F}$, this $*$-algebra has a natural representation on the L^2 space of the holonomy covering of L. Fixing a base point x in L, the holonomy covering is identified with G_x and the representation π_x is defined by:

$$[\pi_x(f)\eta](\gamma) = \int f(\gamma\gamma'^{-1})g(\gamma')$$

where η is in $L^2(G_x, \Omega^{1/2})$. An element γ of G with source x and range y induces a natural isometry $L^2(G_x) \to L^2(g_y)$ and an isomorphism between π_x and π_y.

By definition, $C^*(M, \mathcal{F})$ is the completion of $C_c^\infty(G, \Omega^{1/2})$ with respect to the norm $\|f\| = \sup_{x \in M} \|\pi_x(f)\|$.

The following two results are proved by Connes in [2].

Theorem 1. *Assume $(M, \mathcal{F})$ is given by the fibers of a fibration $p\colon\ M \to B$. Then $C^*(M, \mathcal{F})$ is isomorphic to $C_0(B) \otimes \mathcal{K}(L^2(\text{fiber}))$.*

Theorem 2. *Let V be an open subset of M and let G_V be the graph of $(V, \mathcal{F}|_V)$. Then the inclusion $C_c^\infty(G_V, \Omega^{1/2}) \hookrightarrow C_c^\infty(G, \Omega^{1/2})$ extends to an isometric $*$-monomorphism of $C^*(V, \mathcal{F}|_V)$ into $C^*(M, \mathcal{F})$.*

5. The C^*-algebra of a proper foliation of finite depth

To simplify notation, write $C_c^\infty(G_A^B)$ instead of $C_c^\infty(G_A^B, \Omega^{1/2})$.

Let N be a closed saturated subset of M and let $V = M \backslash N$. Then G_V is the graph of $(V, \mathcal{F}|_V)$ and is an open subgroupoid of G, so $G_N = G \backslash G_V$ is a closed subgroupoid of G. We can define, as above, the $*$-algebra $C_c^\infty(G_N)$ and representations $\pi_x, x \in N$, of it. Completing with respect to the corresponding norm we get a C^*-algebra that we call $C^*(N)$. In our case, the set N is going to be a finite union of leaves having pairwise saturated open neighborhoods. Also, as the foliations $\mathcal{F}$ and $\mathcal{G}$ are proper, we do not need to consider relative topologies. ($\mathcal{G}$ proper means that every point γ of G has a neighborhood intersecting the leaf through γ in just one plaque.)

The inclusion $G_N \hookrightarrow G$ defines, by restriction, a $*$-homomorphism

$$\rho'\colon\ C_c(G) \to C_c(G_N)$$

which extends, by leafwise definition of the C^*-norm, to a $*$-homomorphism

$$\rho\colon\ C^*(M, \mathcal{F}) \to C^*(N).$$

The same definitions apply when U is an open saturated subset and N is relatively closed in U.

Theorem 1. *For every $k = 0, \ldots, d-1$ there is an exact sequence*

$$0 \to C^*(V_k, \mathcal{F}|_{V_k}) \xrightarrow{\iota} C^*(V_{k-1}, \mathcal{F}|_{V_{k-1}}) \xrightarrow{\rho} C^*(N_k) \to 0.$$

Theorem 2. *If l_k is the number of leaves at depth k, then*

$$C^*(N_k) \cong \bigoplus_{i=1}^{l_k} C^*(H_i) \otimes \mathcal{K}$$

where $C^*(H_i)$ is the group C^*-algebra of the holonomy group of the i-th leaf in N_k. (These groups are free abelian of rank either one or two.)

To abbreviate, write $U = V_{k-1}$, $V = V_k$ and $N = N_k$. Clearly, the kernel of ρ contains $C^*(V, \mathcal{F}|_V)$, so the sequence makes sense. What is left to prove is that ρ is surjective and that its kernel is contained in $C^*(V, \mathcal{F}|_V)$. Since there are finitely many leaves in N and they are closed in U, we can select a countable family of Frobenius charts $\{Y_i\}$ for the foliation $\mathcal{F}|_U$ such that, for each $i, Y_i \cap N$ is either empty or a plaque P_i of N. Let $G(Y_i)$ be the graph of the foliation restricted to Y_i. If $f \in C_c^\infty(G(Y_i))$ is such that $\rho f = 0$, then $f|p_i = 0$ and we can approximate f by a sequence in $C_c^\infty(G(Y_i \backslash P_i))$. Hence $f \in C^*(Y_i \backslash P_i)$. Thus, $\rho f = 0$ for $f \in C^*(Y_i)$ implies $f \in C^*(Y_i \backslash P_i)$. Now, if $\rho(f * g) = 0$, with $f \in C^*(Y_i)$ and $g \in C^*(Y_j)$, then $\rho f = 0$ (or $\rho g = 0$) and so f can be viewed as a Cauchy sequence $[f_n]$ in $C_c^\infty(G(Y_i \backslash P_i))$. Then $[f_n * g]$ is a Cauchy sequence in $C^*(V)$ representing $f * g$. Since the algebras $C^*(Y_i)$ generate $C^*(U)$ and the $C^*(Y_i \backslash P_i)$'s generate $C^*(V)$, this shows that $C^*(V)$ contains the kernel of ρ.

To show that ρ is surjective, one observes that, since the foliation $\mathcal{G}$ is proper, we can always extend an element of $C_c^\infty(G_N)$ to $C_c^\infty(G_U)$, so the proof of the exactness of the sequence is complete.

Since there are finitely many leaves in N and they are closed in U, we can assume N is just one leaf. Fix a point a in N. The second statement consists of proving that $C^*(N)$ is isomorphic to $C^*(G_a^a) \otimes \mathcal{K}$, where $C^*(G_a^a)$ is the completion of $C_c(G_a^a)$ with respect to the norm $\|f\| = \|\pi_a(f)\|$ given by the representation $\pi_a\colon\ C_c(G_a^a) \to \mathcal{L}(L^2(G_a^a))$, i.e., $C^*(G_a^a)$ is the reduced group C^*-algebra of the holonomy group of the leaf N. This is suggested by the fact that G_N^N is a fibration over N with fiber G_a^N.

The isomorphism is obtained using the Hilbert modules of Kasparov (see [6] or [1], Chapter 13).

To start, note that $C_c^\infty(G_a^N)$ is a $C^*(G_a^a)$-module with the action given by convolution. It becomes a pre-Hilbert $C^*(G_a^a)$-module with the inner product $\langle f, g\rangle = f^* * g$. Let E be the Hilbert $C^*(G_a^a)$-module obtained by completion of $C_c^\infty(G_a^N)$ with respect to the norm $\|\cdot\| = \|\langle\cdot,\cdot\rangle\|_{C^*(G_a^a)}^{1/2}$. Clearly, the span of $\{\langle f, g\rangle; f, g \in E\}$ is dense in $C^*(G_a^a)$, i.e., E is a full module.

Given $f \in C_c^\infty(G_N^N)$ define $\tau(f)\colon\ C_c^\infty(G_a^N) \to C_c^\infty(G_a^N)$ by $\tau(f)(\eta) = f * \eta$. This extends to a map

$$\tau\colon C^*(G_N^N) \to \mathrm{End}(E)$$

by completion. Let $\mathcal{L}(E)$ be the C^*-algebra of module endomorphisms of E with adjoint. Since $\langle \tau(f)(\eta), \eta'\rangle = \langle \eta, \tau(f^*)(\eta')\rangle$, we get a $*$-homomorphism of C^*-algebras

$$\tau\colon C^*(G_N^N) \to \mathcal{L}(E).$$

One easily checks that τ is injective. We will show that the image of τ is the algebra $\mathcal{K}(E)$ of 'compact' operators on E. Recall that $\mathcal{K}(E)$ is the closure of the linear span of $\{\theta_{\xi,\eta}; \xi, \eta \in E\}$ where $\theta_{\xi,\eta}(\zeta) = \xi * \langle \eta, \zeta\rangle, \zeta \in E$.

Given $\xi, \eta \in C_c^\infty(G_a^N)$, then $\theta_{\xi,\eta} = \tau(f)$, where $f \in C_c^\infty(G_N^N)$ is given by

$$f(\gamma) = \int_{G_a^{r(\gamma)}} \xi(\delta)\overline{\eta(\gamma^{-1}\delta)}.$$

This implies that $\mathcal{K}(E)$ is contained in the image of τ. To show the reverse inclusion $\mathrm{Im}(\tau) \subset \mathcal{K}(E)$, it is enough to show that $\tau(f) \in \mathcal{K}(E)$ for $f \in C_c^\infty(G_N^N \cap G(Y_i))$. (Here $G(Y_i), Y_i$ and P_i are as above.) Since $G_N^N \cap G(Y_i) = P_i \times P_i$, we may assume that f is of the form $f(x,y) = g_1(x)g_2(y)$, with $g_j \in C_c^\infty(P_i)$. Hence, viewing $\overline{g}_1, g_2$ as elements of $C_c^\infty(G_a^{P_i})$ we have $\tau(f) = \theta_{g_1,g_2}$.

By [1], Chapter 13, all this shows that $C^*(G_N^N)$ and $C^*(G_a^a)$ are Morita equivalent, hence stable isomorphic.

On the other hand, we can view $C_c(G_a^a) \otimes C_c(N)$ as a subspace of $C_c^\infty(G_a^N)$: Fix a fundamental domain Δ for the action of G_a^a on G_a^N. Given $\gamma \in G_a^N$ there is a unique α_γ in G_a^a such that $\gamma \in \alpha_\gamma \Delta$. Now, if $\xi \in C_c(G_a^a)$ and $\varphi \in C_c(N)$, define

$$\xi \otimes \varphi(\gamma) = \xi(\alpha_\gamma)\varphi(r(\gamma)).$$

With the inner product

$$\langle \xi \otimes \varphi, \eta \otimes \psi \rangle = (\xi^* * \eta) \int_N \overline{\varphi}\psi$$

it becomes a $C^*(G_a^a)$-Hilbert module, which is the same module structure induced from $C_c^\infty(G_a^N)$. The completion of $C_c(G_a^a) \otimes C_c(N)$ with respect to the norm $\|\cdot\| = \|\langle\cdot,\cdot\rangle\|^{1/2}_{C^*(G_a^a)}$ is seen to be $C^*(G_a^a) \otimes L^2(N)$. But since $C_c(G_a^a) \otimes C_c(N)$ is dense in $C_c(G_a^N)$, the modules E and $C^*(G_a^a) \otimes L^2(N)$ are isomorphic.

Finally, as $L^2(N)$ is separable, we can choose a countable orthonormal base and exhibit an isomorphism of $C^*(G_a^a)$-Hilbert modules between $C^*(G_a^a) \otimes L^2(N)$ and $\mathcal{H}_{C^*(G_a^a)}$.

By Lemma 4 of [6], the C^*-algebra of 'compact' operators of $\mathcal{H}_{C^*(G_a^a)}$ is isomorphic to $C^*(G_a^a) \otimes \mathcal{K}$, and the proof of the theorem is complete.

6. Foliations almost without holonomy

One could obtain similar theorems for foliations almost without holonomy with a finite number of compact leaves. The holonomy groups of the compact leaves are abelian because they are totally ordered archimedian. The connected components of the complement of the compact leaves are of two types: open saturated sets that fiber over the circle (the leaves are proper) and open saturated sets with all leaves dense and without holonomy. The C^*-algebras of these foliations can be computed as in Natsume [7], because the group of periods is finitely generated.

7. Embeddings into AF-algebras

In [5], Fack and Wang show that C^*-algebras of Reeb foliations are not AF-embeddable. They prove that the algebra $C_0([0,\infty)) \times_\alpha Z$, where $\alpha\colon [0,\infty) \to [0,\infty)$ is a dilation with 0 as unique fixed point, is not AF, but it is embedded in those algebras of the Reeb foliation.

The algebra $C_0([0,\infty)) \times_\alpha Z$ is found by looking at a flow transverse to the foliation, picking a flow line and attaching one end to the boundary. The transformation α is induced by sliding along leaves.

In the general case of a proper foliation (with the same finiteness conditions as before) there is no canonical way to choose such an α. However, their same technique may be applied here. We briefly sketch how this is done.

We assume that the level is $d > 1$. Let U be a connected open saturated set of leaves at level d. The foliation $\mathcal{F}|_U$ has no holonomy and by Sacksteder's and Tischler's theorems (see [4] for details on these open sets) it can be described as $L \times [0,1]/\varphi$ where $\varphi\colon\ L \to L$ is a diffeomorphism which 'pushes away' the ends of L (that is, $\mathcal{F}|_U$ is a fiber bundle over the circle). By attaching one of the flowlines to one of the boundary leaves of U, near a juncture, we get an embedded copy of $[0,\infty)$. The holonomy near 0 is a contraction and extends to a contraction of $[0,\infty)$ by sliding along leaves. This sliding is well defined because there is no holonomy inside U. Then, by applying [5], we get

Theorem 1. *If $(M,\mathcal{F})$ is a proper smooth foliation of finite depth $d > 1$ and there are finitely many leaves at depth $< d$, then $C^*(M,\mathcal{F})$ is not embeddable in an AF-algebra.*

Apart from the algebraic interest of nonembeddability pointed out in [5], there is also some geometric curiosity. Namely, Pimsner's [8] geometric criterion for embedding transformation group C^*-algebras into AF-algebras is that the orbits be wandering. For foliations this is almost related to the leaves being wandering. In fact leaves at top level are, but they are tied up by the ones at lower level.

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