DIFFEOMORPHISMS WITH PERSISTENCY

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Abstract. The $C^1$ interior of the set of all diffeomorphisms satisfying Lewowicz’s persistency is characterized as the set of all diffeomorphisms satisfying Axiom A and the strong transversality condition.

In [5], Lewowicz introduced a notion of persistency for a homeomorphism of a compact metric space $X$, and it is remarked that persistence is a weaker property than topological stability when $X$ is a manifold. It is also proved there that every pseudo-Anosov map (on a surface) is persistent. The purpose of this paper is to analyze the dynamics of diffeomorphisms having persistency. More precisely we shall prove the following theorem.

Let $M$ be a $C^\infty$ closed manifold and let $\text{Diff}(M)$ be the space of $C^1$ diffeomorphisms of $M$ endowed with $C^1$ topology. We denote by $P(M)$ the set of all $f \in \text{Diff}(M)$ having persistency.

Theorem. The $C^1$ interior of $P(M)$ in $\text{Diff}(M)$, $\text{int}P(M)$, is characterized as the set of all diffeomorphisms satisfying Axiom A and the strong transversality condition.

It was proved in [7] and [9] respectively that the $C^1$ interior of the set of all $f \in \text{Diff}(M)$ having topological stability and the $C^1$ interior of the set of all $f \in \text{Diff}(M)$ having the pseudo-orbit tracing property were characterized as the set of all diffeomorphisms satisfying Axiom A and the strong transversality condition. Therefore, if the theorem is established, then these two open sets and $\text{int}P(M)$ are equal.

Let $E(M)$ be the set of all expansive diffeomorphisms of $M$. Gerber and Katok [3] proved that if $f$ is a pseudo-Anosov map on a surface $M$ and if $N^0(f)$ is a $C^0$ neighborhood of $f$, then there exists a smooth diffeomorphism $g \in N^0(f)$ conjugating to $f$. Thus it can be checked that $g \in E(M) \cap P(M)$, and more precisely, the following corollary implies that $g$ belongs to $E(M) \cap \partial P(M)$. Here $\partial P(M)$ denotes the boundary of $P(M)$ in $\text{Diff}(M)$.

Corollary. $E(M) \cap \text{int}P(M)$ is characterized as the set of all Anosov diffeomorphisms.

The corollary is an easy consequence of our theorem. Indeed, since every $f \in \text{Diff}(M)$ satisfying Axiom A and the strong transversality condition is structurally...
stable, if \( f \in \mathcal{E}(M) \), then it is Anosov (by [6]). Conversely, if \( f \in \text{Diff}(M) \) is Anosov, then \( f \) is persistent since \( f \) is topologically stable.

Let \( d \) be a metric on \( M \) which is induced from a Riemannian metric \( \| \cdot \| \) on \( TM \), and let us denote by \( \mathcal{H}(M) \) the set of all homeomorphisms of \( M \). We say that \( f \in \mathcal{H}(M) \) is persistent if for each \( \varepsilon > 0 \), there is \( \delta > 0 \) such that for every \( x \in M \) and \( g \in \mathcal{H}(M) \) with \( d(f, g) < \delta \), there is \( y \in M \) satisfying \( d(f^n(x), g^n(y)) < \varepsilon \) \((\forall n \in \mathbb{Z})\). The notion is independent of a metric for \( M \) and is conjugacy invariant.

Let \( \Lambda(f) \) be a hyperbolic set of \( f \in \text{Diff}(M) \). For any \( \varepsilon > 0 \) and \( x \in \Lambda(f) \), the local stable manifold and the local unstable manifold are denoted by \( W^s(x, f) \) and \( W^u(x, f) \) respectively. The stable manifold \( W^s(x, f) \) and the unstable manifold \( W^u(x, f) \) of \( x \in \Lambda(f) \) are defined by a usual way. Let \( f \in \text{Diff}(M) \) satisfy Axiom A. Then the non-wandering set of \( f \), \( \Omega(f) \), is a disjoint union of basic sets \( \Lambda_1(f) \cup \cdots \cup \Lambda_\ell(f) \). Recall that the periodic points of \( f_{|\Lambda_\ell(f)} \) are dense in \( \Lambda_\ell(f) \) \((1 \leq i \leq \ell)\) and that for every \( x \in M \) there are \( p \in \Lambda_\ell(f) \) and \( q \in \Lambda_i(f) \) \((1 \leq i \neq j \leq \ell)\) such that \( x \in W^s(p, f) \cap W^u(q, f) \). We say that \( f \) satisfies the strong transversality condition if for every \( x \in M, T_x W^s(p, f) + T_x W^u(q, f) = T_x M \) for some \( p, q \in \Omega(f) \).

Let \( P(f) \) denote the set of all periodic points of \( f \in \text{Diff}(M) \), and let \( \mathcal{F}(M) \) be the set of all \( f \in \text{Diff}(M) \) having a \( C^1 \)-neighborhood \( \mathcal{U}(f) \subset \text{Diff}(M) \) such that every \( p \in P(g) \) \((\forall g \in \mathcal{U}(f)) \) is hyperbolic. Then such a set was characterized as the set of all diffeomorphisms satisfying Axiom A with no-cycles (see [1, 4]). It is well known that every \( f \in \text{Diff}(M) \) satisfying Axiom A and the strong transversality condition is persistent (because \( f \) is topologically stable (see [8])). Therefore our theorem follows from the following two propositions.

**Proposition A.** The \( C^1 \) interior of \( \mathcal{P}(M) \), \( \text{int} \mathcal{P}(M) \), is a subset of \( \mathcal{F}(M) \).

**Proposition B.** Let \( f \in \text{Diff}(M) \) satisfy Axiom A with no-cycles. If \( f \in \text{int} \mathcal{P}(M) \), then \( f \) satisfies the strong transversality condition.

### 1. Proof of Proposition A

Let \( f \in \text{int} \mathcal{P}(M) \). To get the conclusion, it is enough to show that every \( p \in P(f) \) is hyperbolic. Indeed, if this is established, then for every \( C^1 \) neighborhood \( \mathcal{V}(f) \subset \text{int} \mathcal{P}(M) \) of \( f \), every \( q \in P(g) \) \((\forall g \in \mathcal{V}(f)) \) is hyperbolic because \( g \in \text{int} \mathcal{P}(M) \). Thus \( f \in \mathcal{F}(M) \) is obtained.

Fix a neighborhood \( \mathcal{U}(f) \subset \text{int} \mathcal{P}(M) \) of \( f \), and by assuming that there is a non-hyperbolic periodic point \( p = f^n(p) \), we shall derive a contradiction. Here \( n > 0 \) is the prime period of \( p \). The tangent space \( T_p M \) splits into the direct sum \( T_p M = E^u_p \oplus E^s_p \oplus E^c_p \) where \( E^u_p, E^s_p \) and \( E^c_p \) are \( D_p f^n \)-invariant subspaces corresponding to the absolute values of the eigenvalues of \( D_p f^n \) greater than one, less than one and equal to one, and suppose \( E^c_p \neq 0 \). Then, for every \( \varepsilon > 0 \) there exists a linear automorphism \( \mathcal{O} : T_p M \to T_p M \) such that

\[
\begin{cases}
\| \mathcal{O} - I \| \leq \varepsilon, \\
\mathcal{O}(E^\sigma_p) = E^\sigma_p \\
\text{all eigenvalues of } \mathcal{O} \circ D_p f^n|_{E^\sigma_p} \text{ are of a root of unity,}
\end{cases}
\]

where \( I : T_p M \to T_p M \) is an identity map. By making use of Franks's lemma (see [2, Lemma 1.1]), we can find \( \delta_0 > 0 \) and \( g \in \mathcal{U}(f) \) such that

\[
(i) \quad B_{4\delta_0}(f^i(p)) \cap B_{4\delta_0}(f^j(p)) = \emptyset \text{ for } 0 \leq i \neq j \leq n - 1,
\]
(ii) \( g(x) = f(x) \) for \( x \in \{p, f(p), \ldots, f^{n-1}(p)\} \cup \{M \setminus \cup_{i=0}^{n-1} B_{\delta_0}(f^i(p))\} \),

(iii) \( g(x) = \exp_{f^{i+1}(p)} \circ D_{f^i(p)} f \circ \exp_{f^i(p)}^{-1}(x) \) for \( x \in B_{\delta_0}(f^i(p)) \) \( (0 \leq i \leq n-2) \),

(iv) \( g(x) = \exp_p \circ O \circ D_{f^{n-1}(p)} f \circ \exp_{f^{n-1}(p)}^{-1}(x) \) for \( x \in B_{\delta_0}(f^{n-1}(p)) \),

where \( B_\varepsilon(x) = \{y \in M \|d(x, y) \leq \varepsilon\} \) for \( \varepsilon > 0 \).

Define \( G = O \circ D_p f^n \). Then there exists \( m > 0 \) such that \( G^m \mid_{E^c_p} \) is an identity map. For a sufficiently small \( 0 < \delta_1 < \delta_0 \), we have

\[
 g^m_{\exp_p, T_p M(\delta_1)} = \exp_p \circ G^m \circ \exp_p^{-1}
\]

where \( T_p M(\delta_1) = \{v \in T_p M, \|v\| \leq \delta_1\} \). Put \( E^c_p(\delta_1) = E^c_p \cap T_p M(\delta_1) \). Then it is clear that

\[
 g^m_{\exp_p, E^c_p(\delta_1)} = \text{id}^M_{\exp_p, E^c_p(\delta_1)} \cdot \exp_p^{-1} \exp_p(\delta_1).
\]

Let \( v = (v_1, v_2, \ldots, v_r) \) \( (r = \dim E^c_p) \) be the representation by components with respect to the fundamental vectors of \( R^r = E^c_p \). Put \( \varepsilon = \delta_1/8 \) and fix any \( 0 < \delta < \varepsilon \).

Let \( \varphi : R^r \rightarrow R^r \) be the time-one map given by the vector field

\[
 \dot{v}_i = \delta' \chi(v_1) \cdots \chi(v_r) v_i
\]

for \( 1 \leq i \leq r \). Here \( \chi : R \rightarrow R \) is a \( C^\infty \) function \( (0 \leq \chi(t) \leq 1) \) such that

\[
 \chi(t) = \begin{cases} 
 1 & \text{if } |t| \leq \delta_1/2, \\
 0 & \text{if } |t| \geq 2\delta_1/3,
\end{cases}
\]

and \( \delta' > 0 \) is a number chosen so that \( \|\varphi(v) - v\| < \delta \) for \( v \in R^r \) and \( \|D_v \varphi - \text{id}_{R^r}\| \leq |e^{\delta'} - 1| < \delta \) for \( v \in T_p M(\delta_1) \cap R^r \). We shall denote by \( \tilde{\varphi} : T_p M(\delta_1) \rightarrow T_p M(\delta_1) \) the extension of \( \varphi \) such that \( \tilde{\varphi}(v) = \varphi(v) \) for \( v \in E^c_p(\delta_1) \) and \( \|\tilde{\varphi}(v) - v\| < \delta \) for \( v \in T_p M(\delta_1) \). Put

\[
 \psi(x) = \begin{cases} 
 \exp_p \circ \tilde{\varphi} \circ \exp_p^{-1}(x) & \text{if } x \in \exp_p(T_p M(\delta_1)), \\
 x & \text{otherwise},
\end{cases}
\]

and define \( \tilde{g} = \psi \circ g \). Let \( \hat{\delta} = \delta(\tilde{g}, \hat{\varepsilon}) > 0 \) be a number as in the definition of persistency of \( g \). Then \( d(\tilde{g}, g) < \hat{\delta} \) \( (\tilde{g} \in \mathcal{H}(M)) \) for a sufficiently small \( \hat{\delta} \). Take and fix \( v \in E^c_p(\delta_1) \) such that \( d(\exp_p(v), p) = 2\hat{\varepsilon} \). Clearly \( g^m(\exp_p(v)) = \exp_p(v) \) for all \( m \in Z \). On the other hand, it is easy to see that for every \( y \in B_{\hat{\varepsilon}}(\exp_p(v)) \), there is \( m(y) \in Z \) such that

\[
 d(\hat{g}^m(y), g^m(y)(\exp_p(v))) = d(\hat{g}^m(y), \exp_p(v)) > \hat{\varepsilon}.
\]

This is a contradiction.

2. Proof of Proposition B

Before starting the proof of this proposition, we need some preparation. Throughout this section let \( f \in \text{Diff}(M) \) satisfy Axiom A with no-cycles. Take a basic set \( \Lambda(f) \) of \( f \) and fix \( \varepsilon_0 > 0 \) sufficiently small. Since \( \dim W^s_{\varepsilon_0}(x, f) = \dim W^s_{\varepsilon_0}(y, f) \) for \( x, y \in \Lambda(f) \), we denote by \( \text{Ind}(f) \) the dimension of \( W^s_{\varepsilon_0}(x, f) \) for \( x \in \Lambda(f) \). If \( g \in \text{Diff}(M) \) is \( C^1 \) close to \( f \), then the number of basic sets \( \{\Lambda_i(f)\} \) of \( g \) coincides with that of basic sets \( \{\Lambda_i(\hat{f})\} \) because of \( \Omega \)-stability of \( f \).

The following lemma is induced by Franks’s lemma (see [9, Lemma 3] for details).
Lemma 1. Let $\Lambda_1(f)$ and $\Lambda_2(f)$ be basic sets for $f$. Suppose that there are $p = f^n(p) \in \Lambda_1(f)$ ($n > 0$), $q \in \Lambda_2(f)$ and $x \in M \setminus \Omega(f)$ satisfying $x \in W^s(p, f) \cap W^u(q, f)$. Then, for a neighborhood $\mathcal{U}(f) \subset \operatorname{Diff}(M)$ of $f$, there are $0 < \epsilon_1 < \epsilon_0/2$, $g \in \mathcal{U}(f)$ and two distinct basic sets $\Lambda_1(g)$ and $\Lambda_2(g)$ such that

(i) $B_{4\epsilon_1}(f^i(p)) \cap B_{4\epsilon_1}(f^j(p)) = \emptyset$ for $0 \leq i \neq j \leq n - 1$,

(ii) $g(x) = \begin{cases} \exp_{f^{i+1}(p)} \circ D_{f^i(p)}f \circ \exp_{f^i(p)}^{-1}(x) & \text{if } x \in B_{\varepsilon_1}(f^i(p)) \text{ for } 0 \leq i \leq n - 1, \\ f(x) & \text{if } x \notin \cup_{i=0}^{n-1} B_{4\epsilon_1}(f^i(p)), \end{cases}$

(iii) $g^n(p) = p \in \Lambda_1(g)$ and $q \in \Lambda_2(g),
\quad x \in W^s(p, g) \cap W^u(q, g),
\quad T_x W^s(p, g) = T_x W^s(p, f) \text{ and } T_x W^u(q, g) = T_x W^u(q, f)$.

Since $f$ satisfies Axiom A, there exist a $Df$-invariant continuous splitting $T_{\Omega(f)}M = E^s \oplus E^u$ and a constant $0 < \lambda < 1$ such that

$$\|Df_{|E^s}\| < \lambda^m, \|Df_{|E^u}\| < \lambda^m$$

for all $m \geq 0$. We denote by $E^s_\sigma$ a fiber of $E^s$ at $x \in \Omega(f)$ and put $E^s_\sigma(\varepsilon) = \{ v \in E^s_\sigma \| v \| \leq \varepsilon \}$ for $\varepsilon > 0$ ($\sigma = s, u$). Let $g \in \operatorname{Diff}(M), p = g^n(p) \in \Lambda_1(g)$ and $\varepsilon_1 > 0$ be given as in Lemma 1. Then it is easily checked that for $0 < \varepsilon < \varepsilon_1$, we have

$$\exp_p(E^s_\sigma(\varepsilon)) \subset W^s_{\varepsilon_1}(p, g) \text{ and } \dim \exp_p(E^s_\sigma(\varepsilon)) = \dim W^s_{\varepsilon_1}(p, g)$$

for $\sigma = s, u$.

Now we shall prove Proposition B. Fix $x \in M \setminus \Omega(f)$ and let $\Lambda_i(f)$ and $\Lambda_j(f)$ be basic sets of $f$ such that $x \in W^s(\Lambda_i(f), f) \cap W^u(\Lambda_j(f), f)$. To simplify the proof we assume $i = 1$ and $j = 2$. If $\operatorname{Ind} \Lambda_1(f) = \dim M$ or $\operatorname{Ind} \Lambda_2(f) = 0$, then the conclusion of this proposition is clear. Thus we shall prove $T_x M = T_x W^s(x, f) + T_x W^u(x, f)$ when $\operatorname{Ind} \Lambda_1(f) \leq \dim M - 1$ and $\operatorname{Ind} \Lambda_2(f) \geq 1$.

Since $f \in \operatorname{int} \mathcal{P}(M)$ and $\Omega(f) = \overline{\mathcal{P}(f)}$, there is $f' \in \operatorname{int} \mathcal{P}(M)$ arbitrarily near to $f$ in the $C^1$ topology such that

(a) $f(y) = f'(y)$ for all $y$ outside of a small neighborhood of $x$,

(b) there are $p = f'^n(p) \in \Lambda_1(f)$ for some $n > 0$ and $q \in \Lambda_2(f)$ such that $x \in W^s(p, f') \cap W^u(q, f'), T_x W^s(p, f') = T_x W^s(x, f)$ and $W^u(q, f') = W^u(x, f)$.

By (a), there are basic sets $\Lambda_i(f')$ ($i = 1, 2$) for $f'$ such that $\Lambda_i(f') = \Lambda_i(f)$ since $f$ is $\Omega$-stable. Let us prove $T_x M = T_x W^s(p, f') + T_x W^u(q, f')$. We identify $f'$ with $f$ for simplicity, and let $\mathcal{U}(f)$ be a small neighborhood of $f$ such that $\mathcal{U}(f) \subset \mathcal{P}(M)$.

Then, by Lemma 1 there are $g \in \mathcal{U}(f)$ and basic sets $\Lambda_i(f)$ ($i = 1, 2$) satisfying Lemma 1 (i), (ii) and (iii). Thus $T_x W^s(p, g) = T_x W^s(x, f)$ and $W^u(q, g) = W^u(x, f)$. Pick $\ell > 0$ such that $g^\ell(x) \in W^s_{\varepsilon_{1/2}}(p, g)$ and $g^{-\ell}(x) \in W^u_{\varepsilon_{1/2}}(g^{-\ell}(q), g)$, and define

$$C^u(g^\ell(x)) = \text{the connected component of } g^\ell(x) \text{ in } W^u(g^\ell(q), g) \cap B_{\varepsilon_1}(p).$$

Clearly, $\exp_p^{-1}(C^u(g^\ell(x))) \subset T_p M$. 

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Proof of Lemma 2. Then there exist 0 \leq \varepsilon_1 such that

\begin{equation*}
T_{g^f(x)} \exp_p(E'_p(g^f(x))) = T_{g^f(x)} C^u(g^f(x))
\end{equation*}

and \( \exp_p(E'_p(g^f(x))) \subset B_{\varepsilon_1}(p) \) for 0 < \nu < \nu_0. Since \( g^f(x) \notin \Omega(g) \), there is 0 < \nu_1 such that \( B_{\nu_1}(g^f(x)) \) is \( \Omega \)-stable, and such that \( g^f(x) = \phi \) for all \( i \in \mathbb{Z} \setminus \{0\} \). If \( U(g) \subset U(f) \) is a neighborhood of \( g \), then there are 0 < \nu_2 < \nu_1/4 and \( \varphi \in \text{Diff}(M) \) such that

\begin{equation*}
\begin{aligned}
\varphi_{(B_{\nu_2}(g^f(x)))} &= \text{id}, \\
\varphi(g^f(x)) &= g^f(x), \\
\varphi(\exp_p(E'_p(g^f(x)))) &\subset C^u(g^f(x)), \\
\dim \varphi(\exp_p(E'_p(g^f(x)))) &= \dim C^u(g^f(x)), \\
g^f \in U(g) \text{ where } g^f = \varphi^{-1} \circ g.
\end{aligned}
\end{equation*}

We denote \( \exp_p(E'_p(g^f(x))) \) by \( \exp_p(E'_p(g^f(x))) \) because of \( g^f(x) = g^f(x) \). It is clear that there are two distinct basic sets \( \Lambda_i(g') \) \( (i = 1, 2) \) such that \( \Lambda_i(g') = \Lambda_i(g) \) since \( g \) is \( \Omega \)-stable, and such that

\begin{equation*}
T_{x'} W^\sigma(x, g') = T_{x'} W^\sigma(x, g) \quad (\sigma = s, u),
\end{equation*}

\begin{equation*}
\exp_p(E'_p(g^f(x))) \subset W^u(g^f(q), g') \cap B_{\nu_1}(p),
\end{equation*}

\begin{equation*}
\dim \exp_p(E'_p(g^f(x))) = \dim W^u(q, g') = \dim C^u(g^f(x)).
\end{equation*}

**Lemma 2.** Under the above notation, \( \exp_p(E'_p(g^f(x))) \) meets transversely \( W^s_{\varepsilon_1}(p, g') \) at \( g^f(x) \).

If this lemma is established, then we have \( T_{x'} M = T_{x'} W^s(x, f) + T_{x'} W^u(x, f) \) since \( T_{x'} W^\sigma(x, g') = T_{x'} W^\sigma(x, g) = T_{x'} W^\sigma(x, f) \) for \( \sigma = s, u \).

**Proof of Lemma 2.** Put \( C^u_{\varepsilon}(g^f(x)) = B_{\varepsilon}(g^f(x)) \cap g^{2\ell}(W^u_{\varepsilon}(g^{\ell-\varepsilon}(q), g')) \) for \( \varepsilon > 0 \). Take 0 < \varepsilon < \nu_2 such that \( C^u_{\varepsilon}(g^f(x)) \) is the connected component of \( g^f(x) \) in \( B_{\varepsilon}(g^f(x)) \cap g^{2\ell}(W^u_{\varepsilon}(g^{\ell-\varepsilon}(q), g')) \) for 0 < \varepsilon < \varepsilon_1, and such that \( B_{\varepsilon}(g^f(x)) \) meets transversely \( W^u_{\varepsilon_1}(g^f(x), g') \).

**Claim.** For every 0 < \varepsilon < \varepsilon_1, if \( d(g^{\ell-i}(g^f(x)), g^f(w)) < \varepsilon \) for all \( i \geq 0 \), then \( w \in C^u_\varepsilon(g^f(x)) \).

Indeed, it is clear that \( d(g^{\ell-i}(g^f(x)), g^{\ell-2\ell-i}(w)) < \varepsilon \) for all \( i \geq 0 \) on the other hand, since \( d(g^{\ell-i}(g^f(x)), g^{\ell-i}(q)) < \varepsilon_0/2 \) for all \( i \geq 0 \),

\begin{equation*}
\begin{aligned}
d(g^{\ell-2\ell-i}(w), g^{\ell-i}(q)) &\leq d(g^{\ell-2\ell-i}(w), g^{\ell-\ell-i}(x)) + d(g^{\ell-\ell-i}(x), g^{\ell-\ell-i}(q)) < \varepsilon_0 \\
&\leq \varepsilon \end{aligned}
\end{equation*}

for all \( i \geq 0 \) and so \( g^{\ell-2\ell}(w) \in W^u_{\varepsilon_0}(g^{\ell-\ell}(q), g') \). Thus \( w \in C^u_{\varepsilon_0}(g^f(x)) = B_{\varepsilon_0}(g^f(x)) \cap g^{2\ell}(W^u_{\varepsilon_0}(g^{\ell-\ell}(q), g')) \). Thus \( d(g^f(x), w) < \varepsilon \). The claim is proved.

Suppose that \( \exp_p(E'_p(g^f(x))) \) does not meet transversely \( W^s_{\varepsilon}(p, g') \) at \( g^f(x) \). Then there exist 0 < \nu_3 < \min\{\varepsilon_1, \nu_2/2\} such that for every \( \delta > 0 \) \( (\delta \ll \nu_3) \) there is \( \psi_\delta \in \text{Diff}(M) \) satisfying

\begin{equation*}
\begin{aligned}
\psi_\delta_{(B_{\varepsilon_1}(g^f(x)))} &= \text{id}, \\
d(\psi_\delta, \text{id}) &< \delta, \\
(\psi_\delta(\exp_p(E'_p(g^f(x)))) \cap W^s_{\varepsilon}(p, g') &= \phi.
\end{aligned}
\end{equation*}
Fix $0 < \varepsilon' < \nu_3/2$ and let $\delta' = \delta'(g', \varepsilon') > 0$ be a number as in the definition of persistency of $g'$. Take $\delta > 0$ such that $\tilde{g} = g' \circ \psi_\delta \in \mathcal{H}(M)$ and $d(\tilde{g}, g') < \delta'$. Then, for $g'$-orbit $\{g'^i(x)\}_{i \in \mathbb{Z}}$ of $x$, there is $y \in B_{\varepsilon'}(g'^i(x))$ such that $d(\tilde{g}^i(y), g'^i(g'^i(x))) < \varepsilon'$ for all $i \in \mathbb{Z}$. By the claim $y \in \exp_p(E'_{\nu_3/2}(g'^i(x)))$, from which $\tilde{g}(y) = g \circ \psi_\delta(y) \notin W^s_{\varepsilon'}(p, g')$. Thus, by the hyperbolicity, $d(\tilde{g}^i(\tilde{g}(y)), g'^{i+1}(g'^i(x))) > \varepsilon'$ for some $i \geq 0$. This is a contradiction.

References

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