PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 124, Number 7, July 1996

CHARACTERIZATIONS OF THE GELFAND-SHILOV SPACES VIA FOURIER TRANSFORMS

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(Communicated by Palle E. T. Jorgensen)

ABSTRACT. We give symmetric characterizations, with respect to the Fourier transformation, of the Gelfand-Shilov spaces of (generalized) type S and type W. These results explain more clearly the invariance of these spaces under the Fourier transformations.

1. INTRODUCTION

The purpose of this paper is to give new characterizations of the Gelfand–Shilov spaces of (generalized) type S and type W by means of the Fourier transformation. Gelfand and Shilov introduced the above spaces in [6] to study the uniqueness of the Cauchy problems of partial differential equations. In [1] the Schwartz space $\mathcal{S}$ is characterized by the estimates

$$\sup_x |x^\alpha \varphi(x)| < \infty, \quad \sup_x |\partial^\beta \varphi(x)| < \infty, \tag{1.1}$$

or by the estimates

$$\sup_x |x^\alpha \varphi(x)| < \infty, \quad \sup_\xi |\xi^\beta \hat{\varphi}(\xi)| < \infty, \tag{1.2}$$

where the Fourier transform $\hat{\varphi}$ is defined by $\hat{\varphi}(\xi) = \int e^{-ix\cdot\xi}\varphi(x)\,dx$.

In addition, the Sato space $\mathcal{F}$ of test functions for the Fourier hyperfunctions is characterized by the estimates

$$\sup_x |\varphi(x)| \exp k|x| < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp h|\xi| < \infty \tag{1.3}$$

for some $h, k > 0$ in [2].

Generalizing the above results in a similar manner we give more symmetric characterizations of the Gelfand-Shilov spaces in terms of the Fourier transformations as follows.

I. For the space S_r^s the following statements are equivalent.

(1) $\varphi \in S_r^s$;
(2) $\sup_x |\varphi(x)| \exp k|x|^{1/r} < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp h|\xi|^{1/s} < \infty$ for some $h,\, k > 0$.

Received by the editors November 3, 1994 and, in revised form, January 26, 1995.
1991 *Mathematics Subject Classification.* Primary 46F12, 46F15.
Key words and phrases. Gelfand-Shilov spaces, Fourier transform, associated function.
Partially supported by Korea Research Foundation and GARC.

II. For the space $S_{M_p}^{N_p}$ the following statements are equivalent.

(1) $\varphi \in S_{M_p}^{N_p}$;

(2) $\sup_x |\varphi(x)| \exp M(ax) < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp N(b\xi) < \infty$ for some a, $b > 0$, where $M(x)$ and $N(\xi)$ are associated functions of M_p and N_p, respectively (see (2.1) for the definition).

III. For the space W_M^Ω the following statements are equivalent.

(1) $\varphi \in W_M^\Omega$;

(2) $\sup_x |\varphi(x)| \exp M(a|x|) < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp \Omega^*(b|\xi|) < \infty$ for some a, $b > 0$, where Ω^* is the Young conjugate of Ω (see Definition 3.3).

2. CHARACTERIZATION OF GELFAND–SHILOV SPACES OF (GENERALIZED) TYPE S

In this section we characterize the Gelfand–Shilov spaces of type S and generalized type S in a more symmetric way by means of the Fourier transformation, which are generalizations of (1.2) and (1.3).

We first introduce the *Gelfand–Shilov spaces of generalized type* S and *type* S. Let M_p, $p = 0, 1, 2, \ldots$, be a sequence of positive numbers. We impose the following conditions on M_p:

(M.1) (logarithmic convexity) $M_p^2 \leq M_{p-1} M_{p+1}, \quad p = 1, 2, \ldots;$

(M.2) (stability under differential operators) there are constants A and H such that $M_{p+q} \leq A H^{p+q} M_p M_q$, $p, q = 0, 1, 2, \ldots$.

Definition 2.1. Let M_p and N_p, $p = 0, 1, 2, \ldots$, be sequences of positive numbers. Then the Gelfand-Shilov spaces S_{M_p}, S^{N_p} and $S_{M_p}^{N_p}$ consist of all infinitely differentiable functions $\varphi(x)$ on $\mathbb{R}^n$ satisfying the following estimates, respectively,

$$\sup_x |x^\alpha \partial^\beta \varphi(x)| \leq C_\beta A^{|\alpha|} M_{|\alpha|},$$

$$\sup_x |x^\alpha \partial^\beta \varphi(x)| \leq C_\alpha B^{|\beta|} N_{|\beta|},$$

$$\sup_x |x^\alpha \partial^\beta \varphi(x)| \leq C A^{|\alpha|} B^{|\beta|} M_{|\alpha|} N_{|\beta|}$$

for some positive constants A and B for all multi-indices α and β.

Remark. In particular, if $M_p = p!^r$ and $N_p = p!^s$, then we denote the spaces S_{M_p}, S^{N_p} and $S_{M_p}^{N_p}$ by S_r, S^s and S_r^s, respectively, and call these spaces the *Gelfand–Shilov spaces of type S*.

Definition 2.2. Let M_p and N_p be sequences of positive numbers satisfying (M.1). Then we write $M_p \subset N_p$ ($M_p \prec N_p$, respectively) if there are constants $L, C > 0$ (for any $L > 0$ there is a constant $C > 0$, respectively) such that $M_p \leq C L^p N_p, \quad p = 0, 1, 2, \ldots$.

Also, M_p and N_p are said to be equivalent if $M_p \subset N_p$ and $M_p \supset N_p$ hold.

Theorem 2.3. *If M_p and N_p satisfy* (M.1) *and* (M.2) *and $M_p N_p \supset p!$, then the following conditions are equivalent.*

(i) $\varphi \in S_{M_p}^{N_p}$.

(ii) *There exist positive constants A, B and C such that*

$$\sup_x |x^\alpha \varphi(x)| \leq C A^{|\alpha|} M_{|\alpha|}, \quad \sup_x |\partial^\beta \varphi(x)| \leq C B^{|\beta|} N_{|\beta|}$$

for all multi-indices α and β.

(iii) *There exist positive constants A, B and C such that*

$$\sup_x |x^\alpha \varphi(x)| \le CA^{|\alpha|} M_{|\alpha|}, \quad \sup_\xi |\xi^\beta \hat{\varphi}(\xi)| \le CB^{|\beta|} N_{|\beta|}$$

for all multi-indices α and β.

Proof. The implications (i) ⇒ (ii) and (i) ⇒ (iii) follow immediately from the equality $\widehat{S_{M_p}^{N_p}} = S_{N_p}^{M_p}$. We now prove the implication (ii) ⇒ (i). We can use the L^2-norm instead of the supremum norm. Using integration by parts, the Leibniz formula and the Schwarz inequality we obtain

$$\begin{aligned}
\|x^\alpha \partial^\beta \varphi(x)\|_{L^2}^2 &= \int_{\mathbb{R}^n} [x^{2\alpha} \partial^\beta \varphi(x)] \partial^\beta \varphi(x)\, dx \\
&\le \sum_{\substack{\gamma \le 2\alpha \\ \gamma \le \beta}} \binom{\beta}{\gamma} \binom{2\alpha}{\gamma} \gamma! \|\partial^{2\beta-\gamma} \varphi(x)\|_{L^2} \|x^{2\alpha-\gamma}\varphi(x)\|_{L^2} \\
&\le C^2 \sum_\gamma \binom{\beta}{\gamma} \binom{2\alpha}{\gamma} \gamma! A^{2(|\alpha|+|\beta|-|\gamma|)} M_{|2\alpha-\gamma|} N_{|2\beta-\gamma|}.
\end{aligned}$$

It follows from the conditions (M.1), (M.2) and $M_p N_p \supset p!$ that

$$\begin{aligned}
\|x^\alpha \partial^\beta \varphi(x)\|_{L^2}^2 &\le C^2 A^{2(|\alpha|+|\beta|)} M_{|2\alpha|} N_{|2\beta|} \sum_\gamma \binom{\beta}{\gamma} \binom{2\alpha}{\gamma} \gamma! / (M_{|\gamma|} N_{|\gamma|}) \\
&\le C^2 (2AH)^{2(|\alpha|+|\beta|)} M_{|\alpha|}^2 N_{|\beta|}^2,
\end{aligned}$$

which implies that $\varphi(x)$ belongs to $S_{M_p}^{N_p}$. Therefore, it remains to prove (iii) ⇒ (ii). The inequality $|\xi^\beta \hat{\varphi}(\xi)| \le CB^{|\beta|} N_{|\beta|}$ means that

$$|\hat{\varphi}(\xi)| \le C \exp[-N(|\xi|/B)],$$

where $N(\rho)$ is the associated function of N_p defined by

$$N(\rho) = \sup_p \log \frac{\rho^p}{N_p}. \tag{2.1}$$

Therefore, by the conditions (M.1) and (M.2) of N_p we have

$$\begin{aligned}
|\partial^\beta \varphi(x)| &\le (2\pi)^{-n} \int |e^{ix\cdot\xi} \xi^\beta \hat{\varphi}(\xi)|\, d\xi \\
&\le C_1 \int |\xi|^{|\beta|} \exp[-N(|\xi|/B)]\, d\xi \\
&\le C_1 \sup_\xi \left[|\xi|^{2|\beta|} \exp\big(-N(|\xi|/B)\big)\right]^{1/2} \int \exp\left[-N(|\xi|/B)/2\right] d\xi \\
&\le C_2 B^{|\beta|} N_{2|\beta|}^{1/2} \le C_2 B_1^{|\beta|} N_{|\beta|},
\end{aligned}$$

which completes the proof.

Using the associated function as in (2.1) we can characterize $S_{M_p}^{N_p}$ as follows.

Corollary 2.4. *If M_p and N_p satisfy* (M.1) *and* (M.2) *and $M_pN_p \supset p!$, then the following conditions are equivalent.*

(i) $\varphi \in S_{M_p}^{N_p}$.

(ii) *There exist positive constants a and b such that*

$$\sup_x |\varphi(x)| \exp M(ax) < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp N(b\xi) < \infty.$$

In particular, putting $M_p = p!^r$ and $N_p = p!^s$ we can give simple characterizations for the Gelfand–Shilov spaces of type S as corollaries.

Corollary 2.5. *If $r + s \geq 1$, then the following are equivalent*:

(i) $\varphi \in S_r^s$.

(ii) *There exist positive constants h and k such that*

$$\sup_x |\varphi(x)| \exp k|x|^{1/r} < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp h|\xi|^{1/s} < \infty.$$

Remark. We can easily prove the similar results on the characterization of S_{M_p} and S^{N_p}.

3. CHARACTERIZATION OF GELFAND–SHILOV SPACES OF TYPE W

In this section we characterize the Gelfand–Shilov spaces of type W in a more symmetric way by means of the Fourier transformation.

Let $M(x)$ and $\Omega(y)$ be differentiable functions on $[0, \infty)$ satisfying the condition (K): $M(0) = \Omega(0) = M'(0) = \Omega'(0) = 0$ and their derivatives are continuous, increasing and tending to infinity.

We now define the *Gelfand–Shilov spaces of type W* as in [6].

Definition 3.1. (i) The space W_M consists of all infinitely differentiable functions $\varphi(x)$ on $\mathbb{R}^n$ satisfying the estimate $|\partial^\beta \varphi(x)| \leq C_\beta \exp(-M(a|x|))$ for some $a > 0$.

(ii) The space W^Ω consists of all entire analytic functions $\varphi(\zeta)$ on $\mathbb{C}^n$ satisfying the estimate $|\zeta^\alpha \varphi(\zeta)| \leq C_\alpha \exp \Omega(b|\eta|)$ for some $b > 0$, where $\zeta = \xi + i\eta \in \mathbb{R}^n + i\mathbb{R}^n$.

(iii) The space W_M^Ω consists of all entire analytic functions $\varphi(\zeta)$ on $\mathbb{C}^n$ satisfying the estimate $|\varphi(\xi + i\eta)| \leq C \exp(-M(a|\xi|) + \Omega(b|\eta|))$ for some $a, b > 0$.

In order to relate the sequences M_p and the functions $M(x)$ we need the following definitions.

Definition 3.2. If $M(\rho)$ is an increasing convex function in $\log \rho$ and increases more rapidly than $\log \rho^p$ for any p as ρ tends to infinity, we define its *defining sequence* by

$$M_p = \sup_{\rho > 0} \rho^p / \exp M(\rho), \quad p = 0, 1, 2, \ldots.$$

Definition 3.3. Let $M : [0, \infty) \to [0, \infty)$ be a convex and increasing function with $M(0) = 0$ and $\lim_{x \to \infty} x/M(x) = 0$. Then we define its *Young conjugate* M^* by $M^*(\rho) = \sup_x (x\rho - M(x))$.

To prove the main theorem on the characterizations of the Gelfand–Shilov spaces of type W we need the following relations between the defining sequences and the associated functions as in [3, 5, 9].

Proposition 3.4 ([9]). *If M_p satisfies* (M.1)*, then M_p is the defining sequence of the associated function of itself.*

Proposition 3.5 ([5]). *Let $M : [0,\infty) \to [0,\infty)$ be a function satisfying the condition* (K). *Then $M(x)$ is equivalent to the associated function of the defining sequence of itself.*

Here, $M(x)$ and $N(x)$ are said to be *equivalent* if there exist constants A and B such that $M(Ax) \le N(x) \le M(Bx)$.

Proposition 3.6 ([9]). *Let $m_p = M_p/M_{p-1},\ p = 1, 2, \ldots,$ and let $m(\lambda)$ be the number of m_p such that $m_p \le \lambda$. Then we have $M(\rho) = \int_0^\rho m(\lambda)/\lambda \, d\lambda$.*

Lemma 3.7. *Let $M(\rho)$ be a function satisfying the condition* (K). *Then the defining sequence M_p^* of the Young conjugate $M^*(\rho)$ of $M(\rho)$ is equivalent to $p!/M_p$, where M_p is the defining sequence of $M(\rho)$. In fact, $M_p^* = (p/e)^p/M_p$. Consequently, M_p satisfies the following conditions:*

- (M.1)$'$ (*strong logarithmic convexity*) *$m_p = M_p/M_{p-1}$ is increasing and tends to infinity as $p \to \infty$;*
- (M.1)* (*duality*) *$p!/M_p$ satisfies* (M.1)$'$.

Conversely, if M_p satisfies (M.1)$'$ *and* (M.1)**, then the associated function $M^{\#}(\rho)$ of $p!/M_p$ and the Young conjugate $M^*(\rho)$ of the associated function $M(\rho)$ of M_p are equivalent.*

Proof. We may assume that $M'(\rho)$ is strictly increasing. Then it is easy to see that $M^*(\rho) = \int_0^\rho M'^{-1}(t)\, dt$ where M'^{-1} is the inverse function of M'. Let $g(t) = t^p/\exp M^*(t)$. To find the maximum of $g(t)$ for $t > 0$, taking logarithm, differentiating and equating the result to zero, we obtain

$$p/t = M'^{-1}(t). \tag{3.1}$$

Let t_0 be the root of the equation (3.1). Then there exists $\rho_0 > 0$ such that $M'(\rho_0) = t_0$. Putting $t = M'(\rho_0)$ in (3.1) we have $\rho_0 M'(\rho_0) = p$. Thus we have

$$M^*(M'(\rho_0)) = \sup_x (xM'(\rho_0) - M(x)) = \rho_0 M'(\rho_0) - M(\rho_0)$$

since the function $h(x) = xM'(\rho_0) - M(x)$ takes its maximum at $x = \rho_0$. Therefore we have

$$\begin{aligned} M_p^* &= \sup_{t>0} \frac{t^p}{\exp M^*(t)} = \frac{t_0^p}{\exp M^*(t_0)} = \frac{[M'(\rho_0)]^p}{M^*(M'(\rho_0))} \\ &= \frac{[M'(\rho_0)]^p}{\exp[\rho_0 M'(\rho_0) - M(\rho_0)]} = \frac{[M'(\rho_0)]^p}{\exp(p - M(\rho_0))} \\ &= \left(\frac{p}{e}\right)^p \frac{\exp M(\rho_0)}{\rho_0^p} = \left(\frac{p}{e}\right)^p \frac{1}{M_p}. \end{aligned}$$

For the converse, by Stirling's formula it is easy to see that $p!$ and $(p/e)^p$ are equivalent. So we have for any $t, \rho > 0$

$$M(t) + M^{\#}(\rho) = \sup_p \log \frac{t^p}{M_p} + \sup_p \log \frac{\rho^p M_p}{p!} \ge \sup_p \log \frac{(t\rho)^p}{p!} \ge At\rho$$

for some $A > 0$, where $M^{\#}(\rho)$ is the associated functions of $p!/M_p$.

Thus we have

$$M^{\#}(\rho) \geq At\rho - M(t). \tag{3.2}$$

Taking the supremum for t in the right-hand side of (3.2), we have $M^{\#}(\rho) \geq M^*(A\rho)$. Since M_p satisfies (M.1)$'$ and (M.1)*, we may assume that the sequences $m_p = M_p/M_{p-1}$ and p/m_p are strictly increasing.

We denote by $m(\lambda)$ the number of m_p such that $m_p \leq \lambda$. Then we have by Proposition 3.6

$$M^*(\rho) = \sup_x \int_0^x (\rho - m(\lambda)/\lambda)\, d\lambda.$$

Putting $\rho = p/m_p$ and $x = m_p$, we obtain

$$\begin{aligned} M^*(p/m_p) &\geq \int_0^{m_p} (p/m_p - m(\lambda)/\lambda)\, d\lambda \\ &= p - \int_0^{m_p} m(\lambda)/\lambda\, d\lambda \\ &= p - \sum_{j=1}^{p-1} \int_{m_j}^{m_{j+1}} j/\lambda\, d\lambda \\ &= p + \log \frac{m_1 \cdots m_{p-1}}{m_p^{p-1}}. \end{aligned}$$

On the other hand, let $m^{\#}(\lambda)$ be the number of p/m_p such that $p/m_p \leq \lambda$. Then we have

$$\begin{aligned} M^{\#}(p/m_p) &= \int_0^{p/m_p} m^{\#}(\lambda)/\lambda\, d\lambda \\ &= \sum_{j=1}^{p-1} \int_{j/m_j}^{(j+1)/m_{j+1}} j/\lambda\, d\lambda \\ &= \log \frac{p^p m_1 \cdots m_{p-1}}{p! \cdot m_p^{p-1}} \\ &\leq p + \log \frac{m_1 \cdots m_{p-1}}{m_p^{p-1}} \\ &\leq M^*(p/m_p). \end{aligned}$$

Now, for any $\rho > 0$ such that $p/m_p < \rho < (p+1)/m_{p+1}$ we have

$$\begin{aligned} M^*(\rho) &\geq M^*(p/m_p) \geq M^{\#}(p/m_p) \\ &\geq M^{\#}\left(\frac{1}{2}(p+1)/m_{p+1}\right) \geq M^{\#}\left(\frac{1}{2}\rho\right), \end{aligned}$$

which completes the proof.

We are now in a position to state and prove the main theorems on the characterizations of the Gelfand–Shilov spaces of type W.

Theorem 3.8. *If there is a constant* L *such that* $M(x) \le \Omega(Lx)$, *then the space* W_M^Ω *is characterized by the following estimates*

$$|\varphi(x)| \le C \exp(-M(a|x|)), \quad |\hat{\varphi}(\xi)| \le C \exp(-\Omega^*(b|\xi|)). \tag{3.3}$$

Proof. If $\varphi \in C^\infty(\mathbb{R}^n)$ satisfies (3.3), then $\varphi(x)$ satisfies

$$\sup_x |x^\alpha \varphi(x)| \le CA^{|\alpha|} M_{|\alpha|}, \quad \sup_\xi |\xi^\beta \hat{\varphi}(\xi)| \le CB^{|\beta|} N_{|\beta|}$$

for some $A, B > 0$, where M_p and N_p are the defining sequences of $M(x)$ and $\Omega^*(y)$, respectively. Then the sequences M_p and N_p satisfy (M.1)$'$ and (M.1)* by Lemma 3.7 and the condition $M(x) \le \Omega(Lx)$ implies $M_p N_p \supset p!$. Therefore $\varphi(x)$ belongs to $S_{M_p}^{N_p}$ by Theorem 2.3, since (M.1)$'$ and (M.1)* are stronger than (M.1) and (M.2), respectively. We now prove that $S_{M_p}^{N_p} \subset W_M^\Omega$. Let $\varphi \in S_{M_p}^{N_p}$. Then for every $\alpha, \beta \in \mathbb{N}_0^n$ we obtain

$$|\xi^\alpha \partial^\beta \varphi(\xi)| \le CA^{|\alpha|} B^{|\beta|} M_{|\alpha|} N_{|\beta|} \tag{3.4}$$

for some $A, B > 0$. Since N_p satisfies (M.1)* or $p!/N_p$ satisfies (M.1)$'$, it is easy to see that $N_p \prec p!$. Hence the function $\varphi(\xi)$ can be continued analytically into the complex domain as an entire analytic function. Applying the Taylor expansion and the inequality (3.4) we have

$$\begin{aligned} |\xi^\alpha \varphi(\xi + i\eta)| &\le \sum_{\gamma \in \mathbb{N}_0^n} \frac{|\xi^\alpha \partial^\gamma \varphi(\xi)|}{\gamma!} |\eta|^{|\gamma|} \\ &\le C \sum_{\gamma \in \mathbb{N}_0^n} A^{|\alpha|} M_{|\alpha|} N_{|\gamma|} |B\eta|^{|\gamma|}/\gamma! \\ &\le 2^n C A^{|\alpha|} M_{|\alpha|} \exp N^\#(2B|\eta|). \end{aligned} \tag{3.5}$$

Dividing $|\xi|^{|\alpha|}$ in both sides of the inequality (3.5) and taking infimum for $|\alpha|$ in the right-hand side of (3.5), we have

$$|\varphi(\xi + i\eta)| \le 2^n C \exp\big[-M(|\xi|/A) + N^\#(2B|\eta|)\big].$$

Note that we may use $|\xi|^{|\alpha|} \partial^\beta \varphi(\xi)|$ instead of $|\xi^\alpha \partial^\beta \varphi(\xi)|$ in (3.4). Also, Lemma 3.7 implies

$$N^\#(2B|\eta|) \le N^*(B'|\eta|) \le (\Omega^*)^*(B''|\eta|) = \Omega(B''|\eta|)$$

for some B', $B'' > 0$, where $N^\#$ is the associated function of $p!/M_p$ and N^* is the Young conjugate of the associated function of N_p. Thus, we have

$$|\varphi(\xi + i\eta)| \le C_1 \exp\big[-M(|\xi|/A) + \Omega(B''|\eta|)\big],$$

which completes the proof.

Remark. We can prove the characterization theorem for W_M and W^Ω similarly.

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