

CRUMPLED LAMINATIONS AND MANIFOLDS OF NONFINITE TYPE

R. J. DAVERMAN AND F. C. TINSLEY

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ABSTRACT. Using a group-theoretic construction due to Bestvina and Brady, we build $(n + 1)$ -manifolds W which admit partitions into closed, connected n -manifolds but which do not have finite homotopy type.

At the heart of this note is an example due to Bestvina and Brady [1] of an almost finitely presented group which is not finitely presented. Specifically, they describe a finitely presented group G with a perfect normal subgroup P such that G/P is not finitely presented (i.e., P fails to be the normal closure in G of a finite set); furthermore, P itself is expressed as an infinite free product $*_i P_i$ of finitely presented groups P_i , which happen to be pairwise isomorphic.

For each positive integer m let Γ_m denote $P_1 * P_2 * \cdots * P_m \subset P$, and let N_m denote the normal closure of Γ_m in G . Then

$$N_1 \subset N_2 \subset \cdots \subset N_m \subset N_{m+1} \subset \cdots$$

and $P = \bigcup N_m$. Set $G'_m = G/N_m$. Note the existence of natural projections $\psi_m : G'_m \rightarrow G'_{m+1}$; the direct limit of $\{\psi_m\}$ is G/P . Since $P = \bigcup N_m$ fails to be finitely generated as a normal subgroup, infinitely many of $\{\psi_m\}$ must have nontrivial kernel. This answers Questions 3.4 and 3.5 of [2]. We use it here to describe crumpled laminations $p : W \rightarrow \mathbb{R}$ on manifolds W which do not have finite homotopy type, answering Question 3.1 of [2] negatively, and illustrating the sharpness of the main result (Theorem 1.1) there.

Recall that a *crumpled lamination* on an $(n + 1)$ -manifold W is a closed map p of W to an interval J (possibly noncompact) such that each $p^{-1}(t)$ ($t \in J$) is a closed, connected n -manifold.

Given a compact n -manifold M , $n \geq 5$, and a finitely generated, perfect subgroup H of $\pi_1(M)$, the mapping cylinder construction of [3] provides a map $f : M \rightarrow M'$ from M onto another n -manifold M' and a compact $(n + 1)$ -dimensional cobordism (W, M, M') , where the $(n + 1)$ -manifold W is obtained from the mapping cylinder of f by attaching a collar $M' \times [1, 2]$ to the obvious copy of M' ; in addition, here the inclusion $M' \rightarrow W$ is a homotopy equivalence, $\pi_1(M')$ is isomorphic to the quotient of G by $N(H)$, the normal closure of H , and inclusion $M \rightarrow W$ induces the obvious projection $G \rightarrow G/N(H)$. Since W is determined as the disjoint union

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of $M \times [0, 1)$ and $M' \times [1, 2]$, it possesses a crumpled lamination derived from an obvious map $p : W \rightarrow [0, 2]$ having n -manifolds as point preimages.

For $n \geq 5$ name a closed n -manifold M_0 for which $\pi_1(M_0) \cong G$. The mapping cylinder construction provides a laminated cobordism (W_1, M_0, M_1) such that $M_1 \rightarrow W_1$ is a homotopy equivalence, $\pi_1(M_1) \cong \pi_1(W_1) \cong G'_1$, and the inclusion $M_0 \rightarrow W_1$ induces the natural projection $\psi_0 : G \rightarrow G'_1 = G/N_1$. Applying the construction recursively, we obtain successive laminated cobordisms (W_m, M_{m-1}, M_m) such that $M_m \rightarrow W_m$ is a homotopy equivalence, $\pi_1(M_m) \cong \pi_1(W_m) \cong G'_m$, and the inclusion $M_{m-1} \rightarrow W_m$ induces $\psi_{m-1} : G'_{m-1} \rightarrow G'_m$. We regard distinct W_i, W_j as intersecting only if $i = j \pm 1$ and then $W_i \cap W_{i+1} = M_i$. Consequently,

$$W = (M_0 \times (-\infty, 0)) \cup \left(\bigcup_{i=1}^{\infty} W_i \right)$$

is an $(n+1)$ -manifold equipped with a lamination. It follows routinely that $\pi_1(W)$ is the direct limit of the inclusion-induced sequence

$$\left\{ \pi_1 \left((M_0 \times (-\infty, 0)) \cup \left(\bigcup_{i=1}^m W_i \right) \right) \rightarrow \pi_1 \left((M_0 \times (-\infty, 0)) \cup \left(\bigcup_{i=1}^{m+1} W_i \right) \right) \right\},$$

namely, G/P . Hence, W cannot be homotopy equivalent to a finite complex.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TENNESSEE-KNOXVILLE, KNOXVILLE, TENNESSEE 37996-1300

E-mail address: `daverman@novell.math.utk.edu`

DEPARTMENT OF MATHEMATICS, THE COLORADO COLLEGE, COLORADO SPRINGS, COLORADO 80903

E-mail address: `ftinsley@cc.colorado.edu`