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AN ALGEBRAIC SL_2-VECTOR BUNDLE OVER R_2 AS A VARIETY

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ABSTRACT. We show the stable triviality of all the elements in $\mathrm{VEC}(R_2, R_n)$ concretely, and describe $\mathrm{VEC}(R_2, R_n)$ as surjection classes from a trivial bundle to another. The results also contain the explicit description of non-linearizable SL_2 actions on $\mathbb{C}^n$.

INTRODUCTION

Let G be a reductive complex algebraic group, and let B, F be G-modules over $\mathbb{C}$. Let $\mathrm{VEC}(B, F)$ be the set of isomorphism classes of algebraic G-vector bundles over B whose fiber over 0 is isomorphic to F. In [7], Schwarz found that there are non-trivial $\mathrm{VEC}(B, F)$'s when the ring of invariants $\mathcal{O}(B)^G$ of B is a polynomial ring in one variable (see Kraft-Schwarz [4] for the details), which were the counterexamples to the equivariant Serre conjecture. The G-actions on those non-trivial G-vector bundles provide non-linearizable G-actions on $\mathbb{C}^n$. As they treated an algebraic vector bundle as a family of transition functions in [4] and [7], the G-varieties which are determined by the total spaces of G-vector bundles were not explicitly described. On the other hand, Bass and Haboush [2] proved that all the elements in $\mathrm{VEC}(B, F)$ are stably trivial, and the stable triviality is essential in Masuda and Petrie [6]. For a G-module S, their object $\mathrm{VEC}(B, F; S)$ is the subset of $\mathrm{VEC}(B, F)$ consisting of the isomorphism classes of G-vector bundles over B whose Whitney sum with the trivial vector bundle with fiber S is trivial. In fact, they considered an algebraic G-vector bundle over B as a kernel of a surjection from one trivial bundle onto another. They gave an explicit description of the total space as a G-variety for some families of vector bundles. Each approach has different advantages, e.g. the first approach is aimed at classification of G-vector bundles over representations whose algebraic quotients by G are one dimensional, and the second at constructing families of distinct (non-trivial) G-vector bundles without assumption on the base. Therefore it is useful to compare them.

In the present paper, for $G = SL_2$ we realize all the elements in $\mathrm{VEC}(R_2, R_n)$ as a closed SL_2-subvariety of some SL_2-module. Here, R_n is the SL_2-module of binary forms of degree n. All isomorphism classes of irreducible SL_2-modules are represented by $\{R_n\}_{n\in\mathbf{N}}$. The main theorem is

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Theorem. *For any natural number n, we have*

$$\mathrm{VEC}(R_2, R_n) = \mathrm{VEC}(R_2, R_n; R_{n-2} \oplus \ldots \oplus R_{n-2[\frac{n}{2}]}).$$

In fact $\mathrm{VEC}(R_2, R_n)$ *is isomorphic to* $\mathbb{C}^k$ *where* $k = [\frac{(n-1)^2}{4}]$. *The specific isomorphism is given by sending* τ *to* $E(\tau)$ *where* τ *is a collection of* $\tau_{n-2i},\ i = 1, \ldots, [\frac{n}{2}]$ *(cf. Proposition 1.1), and ranges over the vector space* $(I + N^n)/(I + M^n)$ *isomorphic to* $\mathbb{C}^k$ *and* $E(\tau)$ *is the subvariety of* $R_2 \oplus R_n \oplus R_{n-2} \oplus \ldots \oplus R_{n-2[\frac{n}{2}]}$ *defined in §2.*

In §1, we review the results by Kraft and Schwarz [4],[7] and give a pleasing representation of $\mathrm{VEC}(R_2, R_n)$. In §2, we give a construction of the algebraic vector bundles. The proof of the theorem concludes the paper.

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1. A representation of $\mathrm{VEC}(R_2, R_n)$

We will start with the SL_2-action on $R_n = \{\sum_{i=0}^n a_i X^{n-i} Y^i | a_i \in \mathbb{C}\}$. For an element $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2$ and $f(X, Y) \in R_n$, let

$$f^g(X, Y) = f(aX + bY, cX + dY).$$

We will review the structure of $\mathrm{VEC}(R_2, R_n)$ in [4] and [7], according to [3] (cf. [5]). Let $\Delta : R_2 \longrightarrow \mathbb{C}$ be the discriminant. Because $\mathcal{O}(R_2)^{SL_2} = \mathbb{C}[\Delta]$, for arbitrary $[E]$ in $\mathrm{VEC}(R_2, R_n)$, E is trivial when it is restricted on the Zariski open subsets $\Delta^{-1}(\mathbb{C} - \{0\})$ and $\Delta^{-1}(\mathbb{C} - S)$ in R_2, for a finite subset S in $\mathbb{C} - \{0\}$. The transition function of E,

$$\Psi_E : \Delta^{-1}(\mathbb{C} - (\{0\} \cup S)) \longrightarrow GL(R_n),$$

is an SL_2-morphism, and the SL_2-action on $GL(R_n)$ is given by the conjugation. Let T be the maximal torus in SL_2, consisting of the diagonal matrices, and NT be the normalizer of T in SL_2. Since $\Delta^{-1}(\mathbb{C} - \{0\}) \cong (\Delta^{-1}(\mathbb{C} - \{0\}))^T \times^{NT} SL_2$, Ψ_E is determined by the restriction $\Psi_E|_{R_2^T}$ over R_2^T , which is a W-morphism

$$\Psi_E|_{R_2^T} : \Delta^{-1}(\mathbb{C} - (\{0\} \cup S)) \cap R_2^T \longrightarrow GL(R_n)^T$$

where $W = NT/T$ is the Weyl group of SL_2. The Weyl group W is an order 2 group generated by the class of $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. As a T-module, R_n decomposes to a sum of mutually non-isomorphic one-dimensional modules generated by $X^{n-i}Y^i$, $i = 0, \ldots, n$, so $GL(R_n)^T$ is a group of diagonal matrices by Schur's lemma. Then $\Psi_E|_{R_2^T}$ has the following form:

$$\Psi_E(\lambda XY) = \mathrm{diag}(q_0(\lambda), q_1(\lambda), \ldots, q_n(\lambda)) \quad for \quad \lambda XY \in R_2^T = \{\lambda XY | \lambda \in \mathbb{C}\},$$

with respect to the basis $\{X^{n-i}Y^i\}_{i=0,\ldots,n}$ of R_n, where $\{q_i(\lambda)\}$ are rational functions in λ with $q_i(\lambda) = q_{n-i}(-\lambda)$ for $i = 0, 1, \ldots, n$. The relations among $q_i(\lambda)$'s come from the compatibility with the W-action. Composing the isomorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$, we can normalize $\Psi_E|_{R_2^T}$ such that $\Psi_E(0) = I$, and expand it into a formal power series (cf. [3]). Since SL_2-orbit of R_2^T is dense in R_2, the inclusion $i : R_2^T \longrightarrow R_2$ induces an injection

$$i^* : \mathrm{Mor}(R_2, \mathrm{End}\, R_n)^{SL_2} \longrightarrow \mathrm{Mor}(R_2^T, (\mathrm{End}\, R_n)^T)^W.$$

Denote by M_k^n (resp. N_k^n) the linear space of the homogeneous elements of degree k in $i^* \operatorname{Mor}(R_2, \operatorname{End} R_n)^{SL_2}$ (resp. $\operatorname{Mor}(R_2^T, (\operatorname{End} R_n)^T)^W$) and set $M^n = \prod_{k\geq 1} M_k^n$ (resp. $N^n = \prod_{k\geq 1} N_k^n$). It is known (cf. [5]) that an arbitrary element in $\operatorname{Mor}(R_2^T, (\operatorname{End} R_n)^T)^W$ is of the following form:

$$\rho(\lambda XY) = \operatorname{diag}(f_0(\lambda), ..., f_n(\lambda))$$

where $f_i(\lambda)$'s are polynomials in λ, such that $f_i(-\lambda) = f_{n-i}(\lambda)$ for $i = 0, ..., n$. Then $\Psi_E|_{R_2^T}$ belongs to $I + N^n$. If $\Psi_E|_{R_2^T}$ belongs to $I + M^n$, it turns out that Ψ_E can be extended to an isomorphism over $\Delta^{-1}(\mathbb{C} - S)$ and E is isomorphic to a trivial bundle. By Schwarz [4], [7] (cf. [5]), it is proved that the correspondence $E \longmapsto \Psi_E|_{R_2^T}$ induces a bijection

$$\operatorname{VEC}(R_2, R_n) \longrightarrow (I + N^n)/(I + M^n).$$

We will introduce some notation to explain our representative. Let φ be the element in $\operatorname{Mor}(R_2, \operatorname{Hom}(R_{m-2}, R_m))^{SL_2}$ such that $\varphi(u) : R_{m-2} \longrightarrow R_m$ is multiplication by $u \in R_2$. Since $\varphi(u)$ is an injection unless $u = 0$, its dual $\varphi(u)^* : R_m^* \longrightarrow R_{m-2}^*$ is a surjection unless $u = 0$. Since R_m is self-dual for any m, $\varphi(u)^*$ defines an element ψ in $\operatorname{Mor}(R_2, \operatorname{Hom}(R_m, R_{m-2}))^{SL_2}$. The composition $\psi \circ \varphi$ is an automorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$. Let σ be the inverse of $\psi \circ \varphi$ over $\Delta^{-1}(\mathbb{C} - \{0\})$; then

$$\sigma \circ \psi \circ \varphi = I_{R_{m-2}} = \psi \circ \varphi \circ \sigma$$

over $\Delta^{-1}(\mathbb{C} - \{0\})$. Let ϕ be $\varphi \circ \sigma$; then $\psi \circ \phi = I_{R_{m-2}}$ over $\Delta^{-1}(\mathbb{C} - \{0\})$. The elements ψ and ϕ actually depend on m, but to simplify the notation, we did not give ψ and ϕ any suffixes. We write $\psi^i(u) : R_m \longrightarrow R_{m-2i}$ and $\phi^i(u) : R_{m-2i} \longrightarrow R_m$ as the i-fold compositions of the ψ's and ϕ's respectively. We remark that ψ and ϕ are represented by the following matrices respectively on R_2^T:

$$\psi = \begin{pmatrix} 0 & * & & & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & * & 0 \end{pmatrix}, \qquad \phi = \begin{pmatrix} 0 & \dots & 0 \\ * & & \\ & \ddots & \\ & & * \\ 0 & \dots & 0 \end{pmatrix}.$$

Then, for $\tau_{n-2i} \in \operatorname{Mor}(R_2, \operatorname{End} R_{n-2i})^{SL_2}$ with $\tau_{n-2i}(0) = 0$, $i = 1, ..., [\frac{n}{2}]$, we have

$$(\phi^i \tau_{n-2i} \psi^i)(\lambda XY) = \operatorname{diag}(0, ..., 0, q_0^i(\lambda), ..., q_i^i(\lambda), 0, ..., 0)$$

and $\operatorname{diag}(q_0^i(\lambda), ..., q_i^i(\lambda)) = \tau_{n-2i}(\lambda XY)$ has polynomials as diagonal entries. Hence the restriction of $I + \phi^i \tau_{n-2i} \psi^i$ to R_2^T belongs to $I + N^n$. We remark that $I + N^n$ is commutative with respect to the composition.

Proposition 1.1. $(I+N^n)/(I+M^n)$ *is multiplicatively generated by the restriction of the following morphisms to* R_2^T*:*

$$\{(I + \phi^i \tau_{n-2i} \psi^i) | \tau_{n-2i} \in \operatorname{Mor}(R_2, \operatorname{End} R_{n-2i})^{SL_2} \\ \textit{with} \quad \tau_{n-2i}(0) = 0, \quad i = 1, ..., [\frac{n}{2}]\}.$$

To prove Proposition 1.1 we use the fact that $\operatorname{Mor}(R_2, \operatorname{End} R_m)^{SL_2}$ is an algebra over $\mathcal{O}(R_2)^{SL_2} = \mathbb{C}[\Delta]$ with one generator, for every m. The following Lemma is in Masuda-Nagase [5].

Lemma 1.2. $\mathrm{Mor}(R_2, \mathrm{End}\, R_m)^{SL_2}$ *is isomorphic to*

$$\mathbb{C}[\Delta][\gamma_m]/\prod_{i=0}^{m}(\gamma_m - (m-2i)\sqrt{\Delta}),$$

as an $\mathcal{O}(R_2)^{SL_2} = \mathbb{C}[\Delta]$*-algebra where* γ_m *is a homogeneous element of degree one such that*

$$\gamma_m(\lambda XY) = \mathrm{diag}(m\lambda, (m-2)\lambda, (m-4)\lambda, ..., -m\lambda)$$

for $\lambda XY \in R_2^T$ *with respect to the basis* $\{X^{m-i}Y^i\}_{i=0,1,...,m}$ *of* R_m.

Proof of Proposition 1.1. Take an arbitrary element $\rho \in I + N^n$. Then ρ is of the following form:

$$\rho(\lambda XY) = \mathrm{diag}(f_0(\lambda), f_1(\lambda), ..., f_n(\lambda)) \quad for \quad \lambda XY \in R_2^T$$

where $f_i(\lambda)$ are power series such that $f_i(-\lambda) = f_{n-i}(\lambda)$ for all $i = 0, ..., n$. So ρ has $1 + [\frac{n}{2}]$ independent power series in its entries. On the other hand for each natural number k, $\{\phi^i \gamma_{n-2i}^k \psi^i,\ i = 0, ..., [\frac{n}{2}]\}$ are linearly independent on R_2^T. Here γ_m is the degree 1 generator in $\mathrm{Mor}(R_2, \mathrm{End}\, R_m)^{SL_2}$. Since the restriction to R_2^T is injective, although our discussion is on R_2^T, we may use the same notation. Now we can choose complex numbers $\alpha_0, \alpha_1, ..., \alpha_{[\frac{n}{2}]}$ such that $\sum_{i=0}^{[\frac{n}{2}]}(\phi^i \alpha_i \gamma_{n-2i} \psi^i)$ is the degree 1 term of ρ. Then the degree 1 term vanishes in $\rho \prod_{i=0}^{[\frac{n}{2}]}(I - \phi^i \alpha_i \gamma_{n-2i} \psi^i)$. Repeating the same method for $k = 2, ..., n-2$, we have

$$\rho \prod_{i=0}^{[\frac{n}{2}]}(I + \phi^i g_{n-2i} \psi^i) = I + \lambda^{n-1}\eta,$$

where $g_{n-2i}, i = 0, 1, ..., [\frac{n}{2}]$, are polynomials of γ_{n-2i} with no constant term, and η is a diagonal matrix with formal power series of λ as its entries. Since $I + g_m$ has its inverse in $I + M^m$, let $I + \hat{\tau}_m$ be the inverse of $I + g_m$, where $\hat{\tau}_m(0) = 0$. Then,

$$\rho = (\prod_{i=0}^{[\frac{n}{2}]}(I + \phi^i \hat{\tau}_{n-2i} \psi^i))(I + \lambda^{n-1}\eta).$$

Since $N_k^n = M_k^n$ for $k \geq n-1$ (cf. [5]),

$$\rho = \prod_{i=1}^{[\frac{n}{2}]}(I + \phi^i \tau_{n-2i} \psi^i) \quad in \quad (I + N^n)/(I + M^n),$$

where τ_m is $\hat{\tau}_m$ with the terms of degree greater than $n-2$ discarded. □

2. The construction of the algebraic SL_2-vector bundles

In this section we construct an SL_2-vector bundle with the transition function mentioned in Proposition 1.1.

Let τ_m be an element in $\mathrm{Mor}(R_2, \mathrm{End}\, R_m)^{SL_2}$ such that $\tau_m(0) = 0$, and let

$$L : R_2 \times (R_n \oplus R_{n-2} \oplus ... \oplus R_{n-2[\frac{n}{2}]}) \longrightarrow R_2 \times (R_{n-2} \oplus ... \oplus R_{n-2[\frac{n}{2}]})$$

be an SL_2-bundle morphism over R_2 given by

$$L = \begin{pmatrix} \psi & -(I+\tau_{n-2}) & & & \\ & \psi & -(I+\tau_{n-4}) & & \\ & & \ddots & \ddots & \\ & & & \psi & -(I+\tau_{n-2[\frac{n}{2}]}) \end{pmatrix}.$$

Here the ψ's are elements in $\mathrm{Mor}(R_2, \mathrm{Hom}(R_m, R_{m-2}))^{SL_2}$ defined in §1. They are surjections at non-zero u in R_2, and $(I+\tau_j)(0) = I$ for any j, so L is a surjection over R_2. Its kernel

$$\ker L = \{(u, (z_n, z_{n-2}, ..., z_{n-2[\frac{n}{2}]})) | \psi(u) z_{n-2i} = (I+\tau_{n-2i-2})(u) z_{n-2i-2},$$
$$u \in R_2, z_{n-2i} \in R_{n-2i}, \quad i = 1, ..., [\frac{n}{2}]\}$$

is an algebraic SL_2-vector bundle over R_2 with 0-fiber R_n. In fact, since L has a splitting (cf. [1]), $\ker L$ becomes trivial when the trivial bundle with fiber $R_{n-2} \oplus ... \oplus R_{n-2[\frac{n}{2}]}$ is added.

We will show that $\ker L$ is trivial when it is restricted to Zariski open subsets $\Delta^{-1}(\mathbb{C} - \{0\})$ and $\Delta^{-1}(\mathbb{C} - S)$ in R_2, where

$$S = \{\Delta(u) \in \mathbb{C} | \prod_{i=1}^{[\frac{n}{2}]} \det(I + \tau_{n-2i})(u) = 0\}$$

is a finite subset in $\mathbb{C} - \{0\}$. Trivializations of $\ker L$ on each open subset are given in the following.

Over $\Delta^{-1}(\mathbb{C} - S)$, define

$$t_S : \ker L|_{\Delta^{-1}(\mathbb{C}-S)} \longrightarrow \Delta^{-1}(\mathbb{C} - S) \times R_n \tag{2.1}$$

by

$$t_S(u, (z_n, z_{n-2}, ..., z_{n-2[\frac{n}{2}]})) = (u, z_n).$$

Proposition 2.1. *t_S is an SL_2-isomorphism over $\Delta^{-1}(\mathbb{C} - S)$.*

Proof. Since $\psi(u) z_{n-2j} = (I + \tau_{n-2j-2})(u) z_{n-2j-2}$ for $j = 0, 1, ..., [\frac{n}{2}] - 1$ and $(I + \tau_{n-2j-2})(u)$ are isomorphisms over $\Delta^{-1}(\mathbb{C} - S)$, t_S is an injection, so t_S is an SL_2-isomorphism over $\Delta^{-1}(\mathbb{C} - S)$. □

Define

$$t_0 : \ker L|_{\Delta^{-1}(\mathbb{C}-\{0\})} \longrightarrow \Delta^{-1}(\mathbb{C} - \{0\}) \times R_n \tag{2.2}$$

by

$$t_0(u, (z_n, z_{n-2}, ..., z_{n-2[\frac{n}{2}]})) = (u, \sum_{i=0}^{[\frac{n}{2}]} \phi(u)^i (I - \phi(u)\psi(u)) z_{n-2i})$$

where $\phi(u)\psi(u)$ is understood to be zero for $i = [\frac{n}{2}]$.

Proposition 2.2. *t_0 is an SL_2-isomorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$. Its inverse is given by the following formula. For $(u, w) \in \Delta^{-1}(\mathbb{C} - \{0\}) \times R_n$, if we write*

$$t_0^{-1}(u, w) = (u, (z'_n, z'_{n-2}, ..., z'_{n-2[\frac{n}{2}]}))$$

where $z'_m \in R_m$, *then*

$$z'_{n-2j} = (\prod_{i=1}^{[\frac{n}{2}]-j} (I + \phi^i \tau_{n-2j-2i} \psi^i) \psi^j)(u) w \quad for \quad j = 0, ..., [\frac{n}{2}] - 1,$$
$$z'_{n-2[\frac{n}{2}]} = \psi(u)^{[\frac{n}{2}]} w.$$

Proof. t_0^{-1} is well defined. Indeed, by using the fact that $\psi\phi = I$ over $\Delta^{-1}(\mathbb{C} - \{0\})$, we have

$$\psi(I + \phi^i \tau_{n-2j-2i} \psi^i) = (I + \phi^{i-1} \tau_{n-2(j+1)-2(i-1)} \psi^{(i-1)}) \psi.$$

This relation implies $\psi z'_{n-2j} = (I + \tau_{n-2(j+1)}) z'_{n-2(j+1)}$. The proof of $t_0 \circ t_0^{-1} = I$ follows easily below:

Since $\psi\phi = I$ over $\Delta^{-1}(\mathbb{C} - \{0\})$, $(I - \phi\psi)(I + \phi^k \tau_{n-2i-2k} \psi^k) = (I - \phi\psi)$; so we have

$$\begin{aligned} t_0(t_0^{-1}(u, w)) &= (u, (\sum_{i=0}^{[\frac{n}{2}]} \phi^i (I - \phi\psi)(\prod_{k=1}^{[\frac{n}{2}]-i} (I + \phi^k \tau_{n-2i-2k} \psi^k) \psi^i)(u) w)) \\ &= (u, (\sum_{i=0}^{[\frac{n}{2}]} \phi^i (I - \phi\psi) \psi^i)(u) w) \\ &= (u, w). \end{aligned}$$

Hence t_0 is a surjection and an isomorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$. □

The transition function of $\ker L$,

$$\Psi_{\ker L} : \Delta^{-1}(\mathbb{C} - (\{0\} \cup S)) \longrightarrow GL(R_n),$$

is, in the above notation,

$$\begin{aligned} \Psi_{\ker L}(u)(w) &= t_S \circ t_0^{-1}(u, w) \\ &= (u, \prod_{i=1}^{[\frac{n}{2}]} (I + \phi^i \tau_{n-2i} \psi^i)(u) w). \end{aligned}$$

That is,

$$\Psi_{\ker L}(u) = \prod_{i=1}^{[\frac{n}{2}]} (I + \phi^i \tau_{n-2i} \psi^i)(u).$$

Proof of the Theorem. Let $E(\tau_{n-2}, ..., \tau_{n-2[\frac{n}{2}]})$ be the SL_2-vector bundle $\ker L$ which is constructed above. The transition function of $E(\tau_{n-2}, ..., \tau_{n-2[\frac{n}{2}]})$ was $\prod_{i=1}^{[\frac{n}{2}]} (I + \phi^i \tau_{n-2i} \psi^i)$. It follows from Proposition 1.1 that the inclusion

$$\mathrm{VEC}(R_2, R_n : R_{n-2} \oplus ... \oplus R_{n-2[\frac{n}{2}]}) \hookrightarrow \mathrm{VEC}(R_2, R_n)$$

is surjective. Thus the theorem has been proved. □

The decomposition of the transition function is not unique. Using the results of Schwarz mentioned in §1, one can show that

Corollary 2.3.

$$E(\tau_{n-2}, ..., \tau_{n-2[\frac{n}{2}]}) \cong E(\tau'_{n-2}, ..., \tau'_{n-2[\frac{n}{2}]})$$

if and only if

$$\prod_{i=1}^{[\frac{n}{2}]}(I + \phi^i \tau_{n-2i} \psi^i) \equiv \prod_{i=1}^{[\frac{n}{2}]}(I + \phi^i \tau'_{n-2i} \psi^i) \qquad \mathrm{mod} \qquad j^* \operatorname{Mor}(R_2, \operatorname{End} R_n)^{SL_2}$$

where j is the inclusion $\Delta^{-1}(\mathbb{C} - \{0\}) \hookrightarrow R_2$.

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