

AN ALGEBRAIC SL_2 -VECTOR BUNDLE
OVER R_2 AS A VARIETY

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ABSTRACT. We show the stable triviality of all the elements in $\text{VEC}(R_2, R_n)$ concretely, and describe $\text{VEC}(R_2, R_n)$ as surjection classes from a trivial bundle to another. The results also contain the explicit description of non-linearizable SL_2 actions on \mathbb{C}^n .

INTRODUCTION

Let G be a reductive complex algebraic group, and let B, F be G -modules over \mathbb{C} . Let $\text{VEC}(B, F)$ be the set of isomorphism classes of algebraic G -vector bundles over B whose fiber over 0 is isomorphic to F . In [7], Schwarz found that there are non-trivial $\text{VEC}(B, F)$'s when the ring of invariants $\mathcal{O}(B)^G$ of B is a polynomial ring in one variable (see Kraft-Schwarz [4] for the details), which were the counterexamples to the equivariant Serre conjecture. The G -actions on those non-trivial G -vector bundles provide non-linearizable G -actions on \mathbb{C}^n . As they treated an algebraic vector bundle as a family of transition functions in [4] and [7], the G -varieties which are determined by the total spaces of G -vector bundles were not explicitly described. On the other hand, Bass and Haboush [2] proved that all the elements in $\text{VEC}(B, F)$ are stably trivial, and the stable triviality is essential in Masuda and Petrie [6]. For a G -module S , their object $\text{VEC}(B, F; S)$ is the subset of $\text{VEC}(B, F)$ consisting of the isomorphism classes of G -vector bundles over B whose Whitney sum with the trivial vector bundle with fiber S is trivial. In fact, they considered an algebraic G -vector bundle over B as a kernel of a surjection from one trivial bundle onto another. They gave an explicit description of the total space as a G -variety for some families of vector bundles. Each approach has different advantages, e.g. the first approach is aimed at classification of G -vector bundles over representations whose algebraic quotients by G are one dimensional, and the second at constructing families of distinct (non-trivial) G -vector bundles without assumption on the base. Therefore it is useful to compare them.

In the present paper, for $G = SL_2$ we realize all the elements in $\text{VEC}(R_2, R_n)$ as a closed SL_2 -subvariety of some SL_2 -module. Here, R_n is the SL_2 -module of binary forms of degree n . All isomorphism classes of irreducible SL_2 -modules are represented by $\{R_n\}_{n \in \mathbb{N}}$. The main theorem is

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Theorem. For any natural number n , we have

$$\text{VEC}(R_2, R_n) = \text{VEC}(R_2, R_n; R_{n-2} \oplus \dots \oplus R_{n-2[\frac{n}{2}]}) .$$

In fact $\text{VEC}(R_2, R_n)$ is isomorphic to \mathbb{C}^k where $k = [\frac{(n-1)^2}{4}]$. The specific isomorphism is given by sending τ to $E(\tau)$ where τ is a collection of τ_{n-2i} , $i = 1, \dots, [\frac{n}{2}]$ (cf. Proposition 1.1), and ranges over the vector space $(I + N^n)/(I + M^n)$ isomorphic to \mathbb{C}^k and $E(\tau)$ is the subvariety of $R_2 \oplus R_n \oplus R_{n-2} \oplus \dots \oplus R_{n-2[\frac{n}{2}]}$ defined in §2.

In §1, we review the results by Kraft and Schwarz [4],[7] and give a pleasing representation of $\text{VEC}(R_2, R_n)$. In §2, we give a construction of the algebraic vector bundles. The proof of the theorem concludes the paper.

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1. A REPRESENTATION OF $\text{VEC}(R_2, R_n)$

We will start with the SL_2 -action on $R_n = \{\sum_{i=0}^n a_i X^{n-i} Y^i \mid a_i \in \mathbb{C}\}$. For an element $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2$ and $f(X, Y) \in R_n$, let

$$f^g(X, Y) = f(aX + bY, cX + dY).$$

We will review the structure of $\text{VEC}(R_2, R_n)$ in [4] and [7], according to [3] (cf. [5]). Let $\Delta : R_2 \rightarrow \mathbb{C}$ be the discriminant. Because $\mathcal{O}(R_2)^{SL_2} = \mathbb{C}[\Delta]$, for arbitrary $[E]$ in $\text{VEC}(R_2, R_n)$, E is trivial when it is restricted on the Zariski open subsets $\Delta^{-1}(\mathbb{C} - \{0\})$ and $\Delta^{-1}(\mathbb{C} - S)$ in R_2 , for a finite subset S in $\mathbb{C} - \{0\}$. The transition function of E ,

$$\Psi_E : \Delta^{-1}(\mathbb{C} - (\{0\} \cup S)) \rightarrow GL(R_n),$$

is an SL_2 -morphism, and the SL_2 -action on $GL(R_n)$ is given by the conjugation. Let T be the maximal torus in SL_2 , consisting of the diagonal matrices, and NT be the normalizer of T in SL_2 . Since $\Delta^{-1}(\mathbb{C} - \{0\}) \cong (\Delta^{-1}(\mathbb{C} - \{0\}))^T \times {}^{NT}SL_2$, Ψ_E is determined by the restriction $\Psi_E|_{R_2^T}$ over R_2^T , which is a W -morphism

$$\Psi_E|_{R_2^T} : \Delta^{-1}(\mathbb{C} - (\{0\} \cup S)) \cap R_2^T \rightarrow GL(R_n)^T$$

where $W = NT/T$ is the Weyl group of SL_2 . The Weyl group W is an order 2 group generated by the class of $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. As a T -module, R_n decomposes to a sum of mutually non-isomorphic one-dimensional modules generated by $X^{n-i}Y^i$, $i = 0, \dots, n$, so $GL(R_n)^T$ is a group of diagonal matrices by Schur's lemma. Then $\Psi_E|_{R_2^T}$ has the following form:

$$\Psi_E(\lambda XY) = \text{diag}(q_0(\lambda), q_1(\lambda), \dots, q_n(\lambda)) \quad \text{for } \lambda XY \in R_2^T = \{\lambda XY \mid \lambda \in \mathbb{C}\},$$

with respect to the basis $\{X^{n-i}Y^i\}_{i=0, \dots, n}$ of R_n , where $\{q_i(\lambda)\}$ are rational functions in λ with $q_i(\lambda) = q_{n-i}(-\lambda)$ for $i = 0, 1, \dots, n$. The relations among $q_i(\lambda)$'s come from the compatibility with the W -action. Composing the isomorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$, we can normalize $\Psi_E|_{R_2^T}$ such that $\Psi_E(0) = I$, and expand it into a formal power series (cf. [3]). Since SL_2 -orbit of R_2^T is dense in R_2 , the inclusion $i : R_2^T \rightarrow R_2$ induces an injection

$$i^* : \text{Mor}(R_2, \text{End } R_n)^{SL_2} \rightarrow \text{Mor}(R_2^T, (\text{End } R_n)^T)^W.$$

Denote by M_k^n (resp. N_k^n) the linear space of the homogeneous elements of degree k in $i^* \text{Mor}(R_2, \text{End } R_n)^{SL_2}$ (resp. $\text{Mor}(R_2^T, (\text{End } R_n)^T)^W$) and set $M^n = \prod_{k \geq 1} M_k^n$ (resp. $N^n = \prod_{k \geq 1} N_k^n$). It is known (cf. [5]) that an arbitrary element in $\text{Mor}(R_2^T, (\text{End } R_n)^T)^W$ is of the following form:

$$\rho(\lambda XY) = \text{diag}(f_0(\lambda), \dots, f_n(\lambda))$$

where $f_i(\lambda)$'s are polynomials in λ , such that $f_i(-\lambda) = f_{n-i}(\lambda)$ for $i = 0, \dots, n$. Then $\Psi_E|_{R_2^T}$ belongs to $I + N^n$. If $\Psi_E|_{R_2^T}$ belongs to $I + M^n$, it turns out that Ψ_E can be extended to an isomorphism over $\Delta^{-1}(\mathbb{C} - S)$ and E is isomorphic to a trivial bundle. By Schwarz [4], [7] (cf. [5]), it is proved that the correspondence $E \mapsto \Psi_E|_{R_2^T}$ induces a bijection

$$\text{VEC}(R_2, R_n) \longrightarrow (I + N^n)/(I + M^n).$$

We will introduce some notation to explain our representative. Let φ be the element in $\text{Mor}(R_2, \text{Hom}(R_{m-2}, R_m))^{SL_2}$ such that $\varphi(u) : R_{m-2} \rightarrow R_m$ is multiplication by $u \in R_2$. Since $\varphi(u)$ is an injection unless $u = 0$, its dual $\varphi(u)^* : R_m^* \rightarrow R_{m-2}^*$ is a surjection unless $u = 0$. Since R_m is self-dual for any m , $\varphi(u)^*$ defines an element ψ in $\text{Mor}(R_2, \text{Hom}(R_m, R_{m-2}))^{SL_2}$. The composition $\psi \circ \varphi$ is an automorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$. Let σ be the inverse of $\psi \circ \varphi$ over $\Delta^{-1}(\mathbb{C} - \{0\})$; then

$$\sigma \circ \psi \circ \varphi = I_{R_{m-2}} = \psi \circ \varphi \circ \sigma$$

over $\Delta^{-1}(\mathbb{C} - \{0\})$. Let ϕ be $\varphi \circ \sigma$; then $\psi \circ \phi = I_{R_{m-2}}$ over $\Delta^{-1}(\mathbb{C} - \{0\})$. The elements ψ and ϕ actually depend on m , but to simplify the notation, we did not give ψ and ϕ any suffixes. We write $\psi^i(u) : R_m \rightarrow R_{m-2i}$ and $\phi^i(u) : R_{m-2i} \rightarrow R_m$ as the i -fold compositions of the ψ 's and ϕ 's respectively. We remark that ψ and ϕ are represented by the following matrices respectively on R_2^T :

$$\psi = \begin{pmatrix} 0 & * & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & & * & 0 \end{pmatrix}, \quad \phi = \begin{pmatrix} 0 & \dots & 0 \\ * & & \\ & \ddots & \\ & & * \\ 0 & \dots & 0 \end{pmatrix}.$$

Then, for $\tau_{n-2i} \in \text{Mor}(R_2, \text{End } R_{n-2i})^{SL_2}$ with $\tau_{n-2i}(0) = 0$, $i = 1, \dots, [\frac{n}{2}]$, we have

$$(\phi^i \tau_{n-2i} \psi^i)(\lambda XY) = \text{diag}(0, \dots, 0, q_0^i(\lambda), \dots, q_i^i(\lambda), 0, \dots, 0)$$

and $\text{diag}(q_0^i(\lambda), \dots, q_i^i(\lambda)) = \tau_{n-2i}(\lambda XY)$ has polynomials as diagonal entries. Hence the restriction of $I + \phi^i \tau_{n-2i} \psi^i$ to R_2^T belongs to $I + N^n$. We remark that $I + N^n$ is commutative with respect to the composition.

Proposition 1.1. $(I + N^n)/(I + M^n)$ is multiplicatively generated by the restriction of the following morphisms to R_2^T :

$$\{(I + \phi^i \tau_{n-2i} \psi^i)|_{\tau_{n-2i}} \in \text{Mor}(R_2, \text{End } R_{n-2i})^{SL_2} \text{ with } \tau_{n-2i}(0) = 0, \quad i = 1, \dots, [\frac{n}{2}]\}.$$

To prove Proposition 1.1 we use the fact that $\text{Mor}(R_2, \text{End } R_m)^{SL_2}$ is an algebra over $\mathcal{O}(R_2)^{SL_2} = \mathbb{C}[\Delta]$ with one generator, for every m . The following Lemma is in Masuda-Nagase [5].

Lemma 1.2. $\text{Mor}(R_2, \text{End } R_m)^{SL_2}$ is isomorphic to

$$\mathbb{C}[\Delta][\gamma_m] / \prod_{i=0}^m (\gamma_m - (m - 2i)\sqrt{\Delta}),$$

as an $\mathcal{O}(R_2)^{SL_2} = \mathbb{C}[\Delta]$ -algebra where γ_m is a homogeneous element of degree one such that

$$\gamma_m(\lambda XY) = \text{diag}(m\lambda, (m - 2)\lambda, (m - 4)\lambda, \dots, -m\lambda)$$

for $\lambda XY \in R_2^T$ with respect to the basis $\{X^{m-i}Y^i\}_{i=0,1,\dots,m}$ of R_m .

Proof of Proposition 1.1. Take an arbitrary element $\rho \in I + N^n$. Then ρ is of the following form:

$$\rho(\lambda XY) = \text{diag}(f_0(\lambda), f_1(\lambda), \dots, f_n(\lambda)) \quad \text{for } \lambda XY \in R_2^T$$

where $f_i(\lambda)$ are power series such that $f_i(-\lambda) = f_{n-i}(\lambda)$ for all $i = 0, \dots, n$. So ρ has $1 + \lfloor \frac{n}{2} \rfloor$ independent power series in its entries. On the other hand for each natural number k , $\{\phi^i \gamma_{n-2i}^k \psi^i, i = 0, \dots, \lfloor \frac{n}{2} \rfloor\}$ are linearly independent on R_2^T . Here γ_m is the degree 1 generator in $\text{Mor}(R_2, \text{End } R_m)^{SL_2}$. Since the restriction to R_2^T is injective, although our discussion is on R_2^T , we may use the same notation. Now we can choose complex numbers $\alpha_0, \alpha_1, \dots, \alpha_{\lfloor \frac{n}{2} \rfloor}$ such that $\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} (\phi^i \alpha_i \gamma_{n-2i} \psi^i)$ is the degree 1 term of ρ . Then the degree 1 term vanishes in $\rho \prod_{i=0}^{\lfloor \frac{n}{2} \rfloor} (I - \phi^i \alpha_i \gamma_{n-2i} \psi^i)$. Repeating the same method for $k = 2, \dots, n - 2$, we have

$$\rho \prod_{i=0}^{\lfloor \frac{n}{2} \rfloor} (I + \phi^i g_{n-2i} \psi^i) = I + \lambda^{n-1} \eta,$$

where $g_{n-2i}, i = 0, 1, \dots, \lfloor \frac{n}{2} \rfloor$, are polynomials of γ_{n-2i} with no constant term, and η is a diagonal matrix with formal power series of λ as its entries. Since $I + g_m$ has its inverse in $I + M^m$, let $I + \hat{\tau}_m$ be the inverse of $I + g_m$, where $\hat{\tau}_m(0) = 0$. Then,

$$\rho = \left(\prod_{i=0}^{\lfloor \frac{n}{2} \rfloor} (I + \phi^i \hat{\tau}_{n-2i} \psi^i) \right) (I + \lambda^{n-1} \eta).$$

Since $N_k^n = M_k^n$ for $k \geq n - 1$ (cf. [5]),

$$\rho = \prod_{i=1}^{\lfloor \frac{n}{2} \rfloor} (I + \phi^i \tau_{n-2i} \psi^i) \quad \text{in } (I + N^n) / (I + M^n),$$

where τ_m is $\hat{\tau}_m$ with the terms of degree greater than $n - 2$ discarded. □

2. THE CONSTRUCTION OF THE ALGEBRAIC SL_2 -VECTOR BUNDLES

In this section we construct an SL_2 -vector bundle with the transition function mentioned in Proposition 1.1.

Let τ_m be an element in $\text{Mor}(R_2, \text{End } R_m)^{SL_2}$ such that $\tau_m(0) = 0$, and let

$$L : R_2 \times (R_n \oplus R_{n-2} \oplus \dots \oplus R_{n-2\lfloor \frac{n}{2} \rfloor}) \longrightarrow R_2 \times (R_{n-2} \oplus \dots \oplus R_{n-2\lfloor \frac{n}{2} \rfloor})$$

be an SL_2 -bundle morphism over R_2 given by

$$L = \begin{pmatrix} \psi & -(I + \tau_{n-2}) & & & & \\ & \psi & -(I + \tau_{n-4}) & & & \\ & & \ddots & \ddots & & \\ & & & \psi & -(I + \tau_{n-2[\frac{n}{2}]}) & \end{pmatrix}.$$

Here the ψ 's are elements in $\text{Mor}(R_2, \text{Hom}(R_m, R_{m-2}))^{SL_2}$ defined in §1. They are surjections at non-zero u in R_2 , and $(I + \tau_j)(0) = I$ for any j , so L is a surjection over R_2 . Its kernel

$$\ker L = \{(u, (z_n, z_{n-2}, \dots, z_{n-2[\frac{n}{2}]}) \mid \psi(u)z_{n-2i} = (I + \tau_{n-2i-2})(u)z_{n-2i-2}, \\ u \in R_2, z_{n-2i} \in R_{n-2i}, \quad i = 1, \dots, [\frac{n}{2}]\}$$

is an algebraic SL_2 -vector bundle over R_2 with 0-fiber R_n . In fact, since L has a splitting (cf. [1]), $\ker L$ becomes trivial when the trivial bundle with fiber $R_{n-2} \oplus \dots \oplus R_{n-2[\frac{n}{2}]}$ is added.

We will show that $\ker L$ is trivial when it is restricted to Zariski open subsets $\Delta^{-1}(\mathbb{C} - \{0\})$ and $\Delta^{-1}(\mathbb{C} - S)$ in R_2 , where

$$S = \{\Delta(u) \in \mathbb{C} \mid \prod_{i=1}^{[\frac{n}{2}]} \det(I + \tau_{n-2i})(u) = 0\}$$

is a finite subset in $\mathbb{C} - \{0\}$. Trivializations of $\ker L$ on each open subset are given in the following.

Over $\Delta^{-1}(\mathbb{C} - S)$, define

$$(2.1) \quad t_S : \ker L|_{\Delta^{-1}(\mathbb{C} - S)} \longrightarrow \Delta^{-1}(\mathbb{C} - S) \times R_n$$

by

$$t_S(u, (z_n, z_{n-2}, \dots, z_{n-2[\frac{n}{2}]})) = (u, z_n).$$

Proposition 2.1. t_S is an SL_2 -isomorphism over $\Delta^{-1}(\mathbb{C} - S)$.

Proof. Since $\psi(u)z_{n-2j} = (I + \tau_{n-2j-2})(u)z_{n-2j-2}$ for $j = 0, 1, \dots, [\frac{n}{2}] - 1$ and $(I + \tau_{n-2j-2})(u)$ are isomorphisms over $\Delta^{-1}(\mathbb{C} - S)$, t_S is an injection, so t_S is an SL_2 -isomorphism over $\Delta^{-1}(\mathbb{C} - S)$. \square

Define

$$(2.2) \quad t_0 : \ker L|_{\Delta^{-1}(\mathbb{C} - \{0\})} \longrightarrow \Delta^{-1}(\mathbb{C} - \{0\}) \times R_n$$

by

$$t_0(u, (z_n, z_{n-2}, \dots, z_{n-2[\frac{n}{2}]})) = (u, \sum_{i=0}^{[\frac{n}{2}]} \phi(u)^i (I - \phi(u)\psi(u))z_{n-2i})$$

where $\phi(u)\psi(u)$ is understood to be zero for $i = [\frac{n}{2}]$.

Proposition 2.2. t_0 is an SL_2 -isomorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$. Its inverse is given by the following formula. For $(u, w) \in \Delta^{-1}(\mathbb{C} - \{0\}) \times R_n$, if we write

$$t_0^{-1}(u, w) = (u, (z'_n, z'_{n-2}, \dots, z'_{n-2[\frac{n}{2}]}))$$

where $z'_m \in R_m$, then

$$z'_{n-2j} = \left(\prod_{i=1}^{\lfloor \frac{n}{2} \rfloor - j} (I + \phi^i \tau_{n-2j-2i} \psi^i) \right) \psi^j(u) w \quad \text{for } j = 0, \dots, \lfloor \frac{n}{2} \rfloor - 1,$$

$$z'_{n-2\lfloor \frac{n}{2} \rfloor} = \psi(u)^{\lfloor \frac{n}{2} \rfloor} w.$$

Proof. t_0^{-1} is well defined. Indeed, by using the fact that $\psi\phi = I$ over $\Delta^{-1}(\mathbb{C} - \{0\})$, we have

$$\psi(I + \phi^i \tau_{n-2j-2i} \psi^i) = (I + \phi^{i-1} \tau_{n-2(j+1)-2(i-1)} \psi^{(i-1)}) \psi.$$

This relation implies $\psi z'_{n-2j} = (I + \tau_{n-2(j+1)}) z'_{n-2(j+1)}$. The proof of $t_0 \circ t_0^{-1} = I$ follows easily below:

Since $\psi\phi = I$ over $\Delta^{-1}(\mathbb{C} - \{0\})$, $(I - \phi\psi)(I + \phi^k \tau_{n-2i-2k} \psi^k) = (I - \phi\psi)$; so we have

$$\begin{aligned} t_0(t_0^{-1}(u, w)) &= (u, \left(\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \phi^i (I - \phi\psi) \left(\prod_{k=1}^{\lfloor \frac{n}{2} \rfloor - i} (I + \phi^k \tau_{n-2i-2k} \psi^k) \right) \psi^i \right) (u) w)) \\ &= (u, \left(\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \phi^i (I - \phi\psi) \psi^i \right) (u) w) \\ &= (u, w). \end{aligned}$$

Hence t_0 is a surjection and an isomorphism over $\Delta^{-1}(\mathbb{C} - \{0\})$. □

The transition function of $\ker L$,

$$\Psi_{\ker L} : \Delta^{-1}(\mathbb{C} - (\{0\} \cup S)) \longrightarrow GL(R_n),$$

is, in the above notation,

$$\begin{aligned} \Psi_{\ker L}(u)(w) &= t_S \circ t_0^{-1}(u, w) \\ &= (u, \prod_{i=1}^{\lfloor \frac{n}{2} \rfloor} (I + \phi^i \tau_{n-2i} \psi^i) (u) w). \end{aligned}$$

That is,

$$\Psi_{\ker L}(u) = \prod_{i=1}^{\lfloor \frac{n}{2} \rfloor} (I + \phi^i \tau_{n-2i} \psi^i) (u).$$

Proof of the Theorem. Let $E(\tau_{n-2}, \dots, \tau_{n-2\lfloor \frac{n}{2} \rfloor})$ be the SL_2 -vector bundle $\ker L$ which is constructed above. The transition function of $E(\tau_{n-2}, \dots, \tau_{n-2\lfloor \frac{n}{2} \rfloor})$ was $\prod_{i=1}^{\lfloor \frac{n}{2} \rfloor} (I + \phi^i \tau_{n-2i} \psi^i)$. It follows from Proposition 1.1 that the inclusion

$$\text{VEC}(R_2, R_n : R_{n-2} \oplus \dots \oplus R_{n-2\lfloor \frac{n}{2} \rfloor}) \hookrightarrow \text{VEC}(R_2, R_n)$$

is surjective. Thus the theorem has been proved. □

The decomposition of the transition function is not unique. Using the results of Schwarz mentioned in §1, one can show that

Corollary 2.3.

$$E(\tau_{n-2}, \dots, \tau_{n-2[\frac{n}{2}]}) \cong E(\tau'_{n-2}, \dots, \tau'_{n-2[\frac{n}{2}]})$$

if and only if

$$\prod_{i=1}^{[\frac{n}{2}]} (I + \phi^i \tau_{n-2i} \psi^i) \equiv \prod_{i=1}^{[\frac{n}{2}]} (I + \phi^i \tau'_{n-2i} \psi^i) \quad \text{mod } j^* \text{Mor}(R_2, \text{End } R_n)^{SL_2}$$

where j is the inclusion $\Delta^{-1}(\mathbb{C} - \{0\}) \hookrightarrow R_2$.

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