

## PELCZYŃSKI'S PROPERTY (V\*) FOR SYMMETRIC OPERATOR SPACES

NARCISSE RANDRIANANTOANINA

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ABSTRACT. We show that if a rearrangement invariant Banach function space  $E$  on the positive semi-axis contains no subspace isomorphic to  $c_0$  then the corresponding space  $E(\mathcal{M})$  of  $\tau$ -measurable operators, affiliated with an arbitrary semifinite von-Neumann algebra  $\mathcal{M}$  equipped with a distinguished faithful, normal and semifinite trace  $\tau$ , has Pelczyński's property (V\*).

### 1. INTRODUCTION

Let  $X$  be a Banach space. We say that  $X$  has Pelczyński's property (V\*) if any subset  $K \subset X$  such that  $\lim_{n \rightarrow \infty} \sup_{x \in K} x_n^*(x) = 0$  for every weakly unconditionally Cauchy series (w.u.c.)  $\sum_{n=1}^{\infty} x_n^*$  in  $X^*$ , is relatively weakly compact. This property was introduced by Pelczyński in [9]. The most notable examples of Banach spaces that have property (V\*) are  $L^1$ -spaces and it was generalized by Saab and Saab in [13] and independently by Emmanuele in [4] that Banach lattices that do not contain any copy of  $c_0$  have property (V\*). On the other hand, one can deduce from a result of Pfitzner in [10] (see also [11]) that non-commutative  $L^1$ -spaces have property (V\*).

It is the intention of the present note to give a common generalization of these results in the setting of symmetric spaces of measurable operators (see definition below). Let  $\mathcal{M}$  be a semifinite von-Neumann algebra equipped with a distinguished faithful normal semifinite trace  $\tau$ . Our principal result shows that if  $E$  is a rearrangement invariant function space on  $(0, \infty)$  that does not contain any copy of  $c_0$  then the corresponding symmetric space of  $\tau$ -measurable operators  $E(\mathcal{M})$  has property (V\*). This was motivated by [3] where it was shown that such a space is weakly sequentially complete (a property weaker than property (V\*)). Non-commutative symmetric spaces associated with semifinite von-Neumann algebras have been considered by several authors. Some Banach space properties of these spaces can be found in [1], [3], [16].

We refer to [15] and [8] for general information concerning von-Neumann algebras as well as basic notions of non-commutative integration, to [2] and [7] for Banach space theory.

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## 2. PRELIMINARIES

Let  $\mathcal{M}$  be a semifinite von-Neumann algebra acting on a Hilbert space  $H$ . Let  $\tau$  be a faithful semifinite normal trace on  $\mathcal{M}$ . A densely-defined closed operator  $x$ , affiliated with  $\mathcal{M}$ , is called  $\tau$ -measurable (in the sense of [8]) if for each  $\varepsilon > 0$ , there exists a projection  $e \in \mathcal{M}$  such that  $xe \in \mathcal{M}$  and  $\tau(1 - e) < \varepsilon$ . The set  $\widetilde{\mathcal{M}}$  of all  $\tau$ -measurable operators is a  $*$ -algebra with respect to the strong sum, the strong product, and the adjoint operation [8]. The rearrangement of an operator  $x \in \widetilde{\mathcal{M}}$  is the function  $\mu(x)$  defined by

$$\mu_t(x) = \inf \{ \|xe\| : e \text{ projection} \in \mathcal{M}, \tau(1 - e) \leq t \}, \quad t \geq 0.$$

The function  $t \rightarrow \mu_t(x)$  from  $(0, \infty)$  to  $[0, \infty)$  is non-increasing, continuous from the right and  $\lim_{t \rightarrow 0^+} \mu_t(x) = \|x\| \forall x \in \mathcal{M}$ . For complete detailed study of  $\mu(\cdot)$ , we refer to [5].

Unless stated otherwise,  $E$  will always denote an order continuous rearrangement invariant Banach function space on  $(0, \infty)$  (in the sense of [7]). We define the symmetric operator space  $E(\mathcal{M})$  of measurable operators by setting

$$E(\mathcal{M}) = \{x \in \widetilde{\mathcal{M}} : \mu(x) \in E\}$$

and

$$\|x\|_{E(\mathcal{M})} = \|\mu(x)\|_E, \quad x \in E(\mathcal{M}).$$

It is well known that  $E(\mathcal{M})$  is a Banach space and if  $E = L^p(0, \infty)$  ( $1 \leq p < \infty$ ), then  $E(\mathcal{M})$  coincides with the usual non-commutative  $L^p$ -space associated with the semifinite von Neumann algebra  $\mathcal{M}$ . The space  $E(\mathcal{M})$  is often referred to as the non-commutative version of the function space  $E$ .

We will now recall some facts about property  $(V^*)$ .

**Definition 1.** Let  $X$  be a Banach space. A series  $\sum_{n=1}^{\infty} x_n$  is said to be weakly unconditionally Cauchy (w.u.c.) if for every  $x^* \in X^*$ , the series  $\sum_{n=1}^{\infty} |x^*(x_n)|$  is convergent.

There are many criteria for a series to be a w.u.c. series (see [2]).

The following property was introduced by Pełczyński in [9]:

**Definition 2.** A Banach space  $X$  is said to have property  $(V^*)$  if a bounded subset  $K$  of  $X$  is relatively weakly compact whenever  $\lim_{n \rightarrow \infty} \sup_{x \in K} |x_n^*(x)| = 0$  for every w.u.c. series  $\sum_{n=1}^{\infty} x_n^*$  in  $X^*$ .

Perhaps the main importance of property  $(V^*)$  is that it provides good information about existence of a complemented copy of  $\ell^1$ . In fact we have the following characterization: A Banach space  $X$  has property  $(V^*)$  if and only if it is weakly sequentially complete and every sequence that is equivalent to  $\ell^1$  has a subsequence that generates a complemented copy of  $\ell^1$ . For more information and examples of spaces with property  $(V^*)$ , we refer to [4], [6], [12] and [14].

## 3. MAIN THEOREM

**Theorem 1.** Let  $E$  be a symmetric function space on  $(0, \infty)$  and  $(\mathcal{M}, \tau)$  be a semifinite von-Neumann algebra. Then  $E(\mathcal{M})$  has property  $(V^*)$  if and only if  $E$  does not contain any copy of  $c_0$ .

*Proof.* We shall assume without loss of generality that  $E$  is a separable function space. In this case,  $E(\mathcal{M})^*$  is completely described in [3] as follows:

$$E(\mathcal{M})^* = \{x \in \widetilde{\mathcal{M}} : \|x\|_{E(\mathcal{M})^*} = \sup\{\tau(|xy|); y \in E(\mathcal{M}), \|y\|_{E(\mathcal{M})} \leq 1\} < \infty\},$$

and the action of  $x \in E(\mathcal{M})^*$  on  $y \in E(\mathcal{M})$  is  $\tau(xy^*)$ .

Recall also that  $L^1(\mathcal{M}, \tau) \cap \mathcal{M} \subset E(\mathcal{M}) \subset L^1(\mathcal{M}, \tau) + \mathcal{M}$ .

Assume first that  $\tau$  is a finite trace. For simplicity, we assume without loss of generality that  $\tau(1) = 1$ . Let  $K \subset E(\mathcal{M})$  be a (V\*)-set, i.e.,  $\lim_{n \rightarrow \infty} \sup_{a \in K} x_n^*(a) = 0$  for every w.u.c. series  $\sum_{n=1}^\infty x_n^*$  in  $E(\mathcal{M})^*$ . Our goal is to show that  $K$  is relatively weakly compact in  $E(\mathcal{M})$ .

Since  $\tau(1) = 1$ , we have  $\mathcal{M} \subset L^1(\mathcal{M}, \tau)$  and therefore  $\mathcal{M} \subset E(\mathcal{M}) \subseteq L^1(\mathcal{M}, \tau)$  with the inclusions being continuous.

The fact that  $L^1(\mathcal{M}, \tau)$  has property (V\*) (see [10]) implies that  $K$  is relatively weakly compact in  $L^1(\mathcal{M}, \tau)$ . Hence, there exists a sequence  $(a_n)_n$  in  $K$  that converges weakly to  $a$  in  $L^1(\mathcal{M}, \tau)$ . We claim that  $(a_n)_n$  is weakly Cauchy in  $E(\mathcal{M}, \tau)$ .

Let  $h \in E(\mathcal{M}, \tau)^*$ . Since  $h$  is a measurable operator,  $\forall \varepsilon > 0$ , there exists a projection  $e \in \mathcal{M}$  with  $\tau(1 - e) < \varepsilon$  and  $he \in \mathcal{M}$ .

For each  $n \in \mathbb{N}$ , let  $\varepsilon = \frac{1}{2^n}$  and choose  $e_n$  a projection in  $\mathcal{M}$  with  $\tau(1 - e_n) < \frac{1}{2^n}$  and  $he_n \in \mathcal{M}$ . We define the following sequence of projections:

$$\begin{aligned} f_1 &= e_1, \\ f_2 &= e_2 \wedge (1 - f_1), \\ &\vdots \\ f_n &= e_n \wedge (1 - f_1 - f_2 - \dots - f_{n-1}). \end{aligned}$$

Clearly the family  $(f_n)_{n \geq 1}$  consists of pairwise disjoint projections that satisfy  $1 - f_1 - f_2 - \dots - f_n \leq 1 - e_n \quad \forall n \in \mathbb{N}$ . Moreover since  $he_n \in \mathcal{M}$ , the operator  $hf_n \in \mathcal{M}$ .

**Lemma 2.** *The series  $\sum_{n=1}^\infty hf_n$  is a w.u.c. series in  $E(\mathcal{M}, \tau)^*$  and  $h = \text{weak}^* \sum_{n=1}^\infty hf_n$ .*

To prove the lemma, let  $\sigma$  be a finite subset of  $\mathbb{N}$ . We have

$$\left\| \sum_{n \in \sigma} he_n \right\| = \left\| h \left( \sum_{n \in \sigma} e_n \right) \right\| \leq \|h\|_{E(\mathcal{M}, \tau)^*}.$$

So  $\sum_{n=1}^\infty hf_n$  is a w.u.c. series. Moreover if  $a \in E(\mathcal{M}, \tau)$ , the operator  $ha \in L^1(\mathcal{M}, \tau)$  and

$$\begin{aligned} \left| \left\langle \sum_{n=1}^m hf_n - h, a \right\rangle \right| &= \left| \left\langle h \left( 1 - \sum_{n=1}^m f_n \right), a \right\rangle \right| \\ &= \left| \left\langle 1 - \sum_{n=1}^m f_n, ah^* \right\rangle \right| \\ &\leq \tau \left( \left( 1 - \sum_{n=1}^m f_n \right) |ah^*| \right). \end{aligned}$$

Since  $ah^* \in L^1(\mathcal{M}, \tau)$  and the projections  $\theta_m = 1 - \sum_{n=1}^m f_n$  satisfy  $\tau(\theta_m) \leq 1/2^m$   $\forall m \in \mathbb{N}$ , we get that  $\lim_{m \rightarrow \infty} \tau(\theta_m | ah^* |) = 0$  which shows that  $\sum_{n=1}^{\infty} \langle hf_n, a \rangle = \langle h, a \rangle$ . The lemma is proved.

Now since  $K$  is a  $(V^*)$ -subset of  $E(\mathcal{M}, \tau)$ , the series  $\sum_{n=1}^{\infty} hf_n$  converges unconditionally uniformly on  $K$ . If not, one can find  $\delta > 0$ ,  $p_1 < p_2 < \dots < p_n < \dots$  such that for every  $n \geq 1$ ,

$$\sup_{a \in K} \left( \sum_{j=p_n+1}^{p_{n+1}} \langle hf_j, a \rangle \right) > \delta.$$

For each  $n \in \mathbb{N}$ , let  $b_n = \sum_{j=p_n+1}^{p_{n+1}} hf_j$ . The series  $\sum_{n=1}^{\infty} b_n$  is also a w.u.c. series in  $E(\mathcal{M}, \tau)^*$  and  $\sup_{a \in K} \langle b_n, a \rangle > \delta \forall n \in \mathbb{N}$ , thus contradicting the fact that  $K$  is a  $(V^*)$ -subset.

Fix  $\varepsilon > 0$ ; there exists  $m > 0$  such that for all  $n \geq 1$ ,

$$\left| \sum_{j=m+1}^{\infty} \langle hf_j, a_n \rangle \right| < \frac{\varepsilon}{3}.$$

Let  $\mathcal{E} = \sum_{j=1}^m hf_j$ , by the construction of  $(f_j)_j$ , the operator  $\mathcal{E} \in \mathcal{M}$ .

Now since  $(a_n)_n$  is weakly convergent in  $L^1(\mathcal{M}, \tau)$ , the sequence  $(\mathcal{E}(a_n))_{n \geq 1}$  is convergent so there exists  $N \in \mathbb{N}$  such that if  $p, q > N$ ,

$$|\mathcal{E}(a_p) - \mathcal{E}(a_q)| < \frac{\varepsilon}{3}.$$

Hence for  $p, q > N$ ,

$$\begin{aligned} |h(a_p) - h(a_q)| &\leq \left| \sum_{j=1}^{\infty} hf_j(a_p) - hf_j(a_q) \right| \\ &\leq |\mathcal{E}(a_p) - \mathcal{E}(a_q)| + \left| \sum_{j=m+1}^{\infty} hf_j(a_p) \right| + \left| \sum_{j=m+1}^{\infty} hf_j(a_q) \right|. \end{aligned}$$

So  $|h(a_p) - h(a_q)| < \varepsilon$  which shows that  $(a_n)_n$  is weakly Cauchy in  $E(\mathcal{M}, \tau)$  and since  $E(\mathcal{M}, \tau)$  is weakly sequentially complete, the sequence  $(a_n)_n$  is weakly convergent so  $E(\mathcal{M}, \tau)$  has property  $(V^*)$ . This ends the proof for the case where the trace  $\tau$  is finite.

For the general case, let  $\mathcal{M}$  be a semifinite von-Neumann algebra with a distinguished faithful, normal semifinite trace  $\tau$ .

Choose a mutually orthogonal family  $(e_i)_{i \in I}$  of projections in  $\mathcal{M}$  with  $\sum_{i \in I} e_i = 1$  for the strong operator topology and  $\tau(e_i) < \infty$  for all  $i \in I$ .

Let  $K$  be a  $(V^*)$ -subset of  $E(\mathcal{M}, \tau)$  and fix  $(a_n)_{n \geq 1}$  a sequence in  $K$ . Using a similar argument as in [16], one can get an at most countable subset  $(e_k)_{k \in \mathbb{N}}$  of  $(e_i)_{i \in I}$  such that for each  $e_i$  outside of  $(e_k)_{k \in \mathbb{N}}$ ,  $e_i a_n = a_n e_i = 0 \forall n \in \mathbb{N}$ . Let  $e = \sum_{k \in \mathbb{N}} e_k$ . We have  $ea_n = a_n e = a_n \forall n \in \mathbb{N}$ . Replacing  $\mathcal{M}$  by  $e\mathcal{M}e$  and  $\tau$  by its restriction on  $e\mathcal{M}e$ , we may assume that  $e = 1$ .

For  $l \geq 1$  and  $k \geq 1$ , let  $e_{lk} = e_l \vee e_k$ . It is clear that  $\tau(e_{lk}) < \infty$  and the sequence  $(e_l a_n e_k)_{n \geq 1}$  is a  $(V^*)$ -subset of  $E(e_{lk} \mathcal{M} e_{lk}, \tau_{lk})$  where  $\tau_{lk}$  is the restriction of  $\tau$  on  $e_{lk} \mathcal{M} e_{lk}$ . By the previous case, the space  $E(e_{lk} \mathcal{M} e_{lk}, \tau_{lk})$  has property  $(V^*)$  so there exists a subsequence  $(a_{n_j})_{n_j \geq 1}$  of  $(a_n)_{n \geq 1}$  so that  $(e_l a_{n_j} e_k)_j$  converges weakly to  $\varphi_{lk} \in E(e_{lk} \mathcal{M} e_{lk}, \tau_{lk})$ . Using diagonal argument, one can assume that this is true for all  $l \geq 1$  and  $k \geq 1$  (i.e., the subsequence  $(a_{n_j})_j$  is independent of  $l$  and  $k$ ).

Note that for each  $l \geq 1$  and  $k \geq 1$ ,  $\varphi_{lk} \in E(\mathcal{M}, \tau)$ . We claim that the series  $\sum_{l \geq 1} \sum_{k \geq 1} \varphi_{lk}$  is convergent in  $E(\mathcal{M}, \tau)$  and if we let  $a = \sum_{l \geq 1} \sum_{k \geq 1} \varphi_{lk}$  then  $(a_{n_j})_j$  converges weakly to the operator  $a$  in  $E(\mathcal{M}, \tau)$ . To see this, we will show first that for each  $l \geq 1$ , the series  $\sum_{k \geq 1} \varphi_{lk}$  is unconditionally convergent. For that notice that since  $E(\mathcal{M}, \tau)$  does not contain  $c_0$ , it is enough to check that the series  $\sum_{k=1}^{\infty} \varphi_{lk}$  is a w.u.c. series. Using the same argument as in the proof of Lemma 2, one can conclude that for each  $b \in E(\mathcal{M}, \tau)$ , the series  $\sum_{k=1}^{\infty} e_l b e_k$  is a w.u.c. series in  $E(\mathcal{M}, \tau)$ . In fact for each finite subset  $P \subseteq \mathbb{N}$ ,

$$\left\| \sum_{k \in P} e_l b e_k \right\| \leq \|e_l b\|_{E(\mathcal{M}, \tau)}.$$

Now for any finite subset  $P$  of  $\mathbb{N}$ ,

$$\begin{aligned} \left\| \sum_{k \in P} \varphi_{lk} \right\| &\leq \varliminf_{j \rightarrow \infty} \left\| \sum_{k \in P} e_l a_{n_j} e_k \right\| \\ &\leq \overline{\lim}_{j \rightarrow \infty} \|e_l a_{n_j}\| \\ &\leq \overline{\lim}_{j \rightarrow \infty} \|a_{n_j}\| = C. \end{aligned}$$

The constant  $C$  being independent of  $P$  shows that  $\sum_{k=1}^{\infty} \varphi_{lk}$  is a w.u.c. series.

For each  $l \geq 1$ , let  $b_l = \sum_{k=1}^{\infty} \varphi_{lk}$ . The operator  $b_l$  is well defined and the sequence  $(e_l a_{n_j})_j$  converges to  $b_l$  weakly in  $E(\mathcal{M}, \tau)$ . For this recall that the sequence  $(a_{n_j})_j$  is a (V\*)-set and so is the sequence  $(e_l a_{n_j})_j$ . Fix  $y \in E(\mathcal{M}, \tau)^*$ . The series  $\sum_{k=1}^{\infty} y e_k$  is a w.u.c. series in  $E(\mathcal{M}, \tau)^*$  and converges to  $y$  for the weak\*-topology (Lemma 2.).

For  $\varepsilon > 0$ ,  $\exists m \in \mathbb{N}$  so that  $\forall j \geq 1$ ,

$$\left| \sum_{k=m+1}^{\infty} \langle y e_k, e_l a_{n_j} - b_l \rangle \right| < \frac{\varepsilon}{2}.$$

On the other hand,

$$\begin{aligned} \lim_{j \rightarrow \infty} \sum_{k=1}^m \langle y e_k, e_l a_{n_j} - b_l \rangle &= \lim_{j \rightarrow \infty} \sum_{k=1}^m \langle y, e_l a_{n_j} e_k - b_l e_k \rangle \\ &= \lim_{j \rightarrow \infty} \sum_{k=1}^m \langle y, e_l a_{n_j} e_k - \varphi_{lk} \rangle = 0. \end{aligned}$$

Hence  $\exists N \in \mathbb{N}$  so that if  $j \geq N$ ,

$$\left| \sum_{k=1}^m \langle y e_k, e_l a_{n_j} - b_l \rangle \right| < \frac{\varepsilon}{2}$$

and therefore for  $j \geq N$ ,

$$\begin{aligned} |\langle y, e_l a_{n_j} - b_l \rangle| &\leq \left| \sum_{k=1}^m \langle y e_k, e_l a_{n_j} - b_l \rangle \right| + \left| \sum_{k=m+1}^{\infty} \langle y e_k, e_l a_{n_j} - b_l \rangle \right| \\ &\leq \varepsilon \end{aligned}$$

which proves that  $\lim_{j \rightarrow \infty} \langle y, e_l a_{n_j} - b_l \rangle = 0$  so  $(e_l a_{n_j})_j$  converges weakly to  $b_l$  in  $E(\mathcal{M}, \tau)$ .

To complete the proof, we repeat the same argument above for the indices  $l$ . First, one can show as above that the series  $\sum_{l \geq 1} b_l$  is unconditionally convergent

and since  $(e_l a_{n_j})_j$  converges weakly to  $b_l$  for every  $l \geq 1$ , we then deduce that  $(a_{n_j})_j$  converges weakly to  $a = \sum_{l \geq 1} b_l$ . Details are left to the reader.

Since the sequence  $(a_n)_n$  was chosen arbitrary in  $K$ , we conclude that the set  $K$  is relatively weakly compact.  $\square$

*Remark.* The above theorem shows in particular that if  $E$  is a symmetric sequence space that does not contain any copy  $c_0$  then the associated unitary matrix space  $C_E$  has property  $(V^*)$ . As examples, the Lorentz-Schatten classes  $C_{p,1}$  ( $1 \leq p < \infty$ ) have property  $(V^*)$ .

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DEPARTMENT OF MATHEMATICS, THE UNIVERSITY OF TEXAS AT AUSTIN, AUSTIN, TEXAS 78712-1082

*E-mail address:* nrandri@math.utexas.edu

*Current address:* Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056

*E-mail address:* randrin@muohio.edu