

ON CHOGOSHVILI'S CONJECTURE

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ABSTRACT. There exists a two-dimensional compact subset of \mathbb{R}^4 having unstable intersection with every affine 2-plane.

1. INTRODUCTION

Let N and X be subsets in the euclidean space \mathbb{R}^n ; we say that X intersects N unstably if X can be removed from N by an arbitrarily small move of X in \mathbb{R}^n , i.e. for every $\epsilon > 0$ there exists a continuous map $f : X \rightarrow \mathbb{R}^n$, ϵ -closed to the identity and with $f(X) \cap N = \emptyset$. In the late '20s P.S. Alexandroff proved [1] that a compact subset $X \subset \mathbb{R}^n$ is at most k -dimensional if and only if X has an unstable intersection with every $(n - k - 1)$ -dimensional piece-wise linear polyhedron in \mathbb{R}^n . In the late '30s G. Chogoshvili made an attempt to generalize Alexandroff's theorem [2]. He published the theorem which is known now as Chogoshvili's Conjecture.

Conjecture. *A compact subset $X \subset \mathbb{R}^n$ is at most k -dimensional if and only if X has unstable intersection with every $(n - k - 1)$ -dimensional plane.*

The original formulation of the Conjecture was for an arbitrary subset of \mathbb{R}^n but in the non-compact case it was disproved by K.A. Sitnikov in the '50s [3].

We note that in one direction the Conjecture is obvious. Namely, if a compact space X is at most k -dimensional, then according to the general position property the intersection of X with a given $(n - k - 1)$ -plane α is empty after a small perturbation. The way to see this is as follows. Consider the projection $\pi : \mathbb{R}^n \rightarrow \beta$ onto the orthogonal plane. Let $O_z(\epsilon)$ denote an open ϵ -ball in β centered at $z = \pi(\alpha)$. We define a partial map $\phi : A \rightarrow \beta - O_z(\epsilon)$ as the restriction $\pi|_A$ where $A = X \cap \pi^{-1}(\beta - O_z(\epsilon))$. Since X is at most k -dimensional and $\beta - O_z(\epsilon)$ is homotopy equivalent to the k -dimensional sphere S^k , there is an extension $\bar{\phi} : X \rightarrow \beta - O_z(\epsilon)$ of ϕ over X . Now we can define an ϵ -move $f(x, y) = (\bar{\phi}(x), y)$ for every $(x, y) \in X \subset \beta \times \alpha$.

Thus, the Conjecture can be formulated as follows.

If a compactum $X \subset \mathbb{R}^n$ has unstable intersection with every $(n - k)$ -dimensional linear plane, then $\dim X < k$.

In other words this means that every k -dimensional compactum $X \subset \mathbb{R}^n$ has a stable intersection with some $(n - k)$ -dimensional plane. This means precisely that the orthogonal projection of X onto some k -dimensional plane has a stable

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value (or essential in other terminology). It was proved in the '30s by Nobeling [4] (see also [5]) that for every k -dimensional compactum $X \subset \mathbb{R}^n$ there is a k -plane such that the image of X under the orthogonal projection onto that plane has a non-empty interior. It was a significant result supporting the Conjecture.

We note that the Conjecture is still open for the case when X is a manifold. For the case of a 2-dimensional manifold the Conjecture was recently proved by M. Levin [6]. Also I should mention that O.D. Kiguradze has another positive result toward the Conjecture [7], [10]. He considered compacta X with certain restrictions on the imbedding of X in \mathbb{R}^n . The next contribution to the problem was an example by Y. Sternfeld [8]. He constructed an embedding of a 2-dimensional Bing's hereditarily indecomposable compactum X in \mathbb{R}^{21} such that X intersects unstably every 19-dimensional plane which is parallel to some coordinate plane. After that example there was a strong feeling that Chogoshvili's Conjecture is not valid even for compact spaces and the technique of functional analysis used in the construction of Sternfeld's example would allow one to disprove the Conjecture. Nevertheless the recent work of F. Ancel and T. Dobrowolski [9] shows that Sternfeld's technique does not help much in the general situation.

In this paper we construct a 2-dimensional compactum in \mathbb{R}^4 which has unstable intersection with every 2-plane.

2. CONSTRUCTION OF THE COUNTEREXAMPLE

Let X be a compact subset of \mathbb{R}^n and let $f : X \rightarrow \mathbb{R}^n$ be a continuous map. Then the norm of f is the number $\|f\| = \max\{\text{dist}(x, f(x)) \mid x \in X\}$. We recall that the Kolmogorov diameter of a compactum $X \subset \mathbb{R}^n$ of dimension r is the number $K_r(X) = \min\{\|\pi_\alpha\| \mid \pi_\alpha : X \rightarrow \alpha\}$ where α runs over all affine planes of the dimension r and π_α is the orthogonal projection.

Let $c \in Z_1(X)$ be a one-dimensional singular cycle in X . We define

$$K_2(c) = \inf\{\max_i\{K_2(\text{supp}(c_i))\} \mid c = \sum c_i, c_i \in Z_1(X)\}.$$

If $a \in H_1(X)$ is an element of the 1-dimensional singular homology group of $X \subset \mathbb{R}^n$, then we define the Kolmogorov diameter $K_2(a)$ of the element a as $\inf\{K_2(b) \mid b \in a\}$, where b runs over all cycles of the class a . We recall that the support $\text{supp}(b)$ of a singular chain b is the union of images of singular simplices appearing in $b = \sum \lambda_\phi \sigma_\phi$ with non-zero coefficients λ_ϕ .

Remark 1. If $K_2(a) > 0$ for $a \in H_1(X)$, then $a \neq 0$.

Remark 2. $K_2(c_1 + c_2) \leq \max\{K_2(c_1), K_2(c_2)\}$ for every $c_1, c_2 \in Z_1(X)$.

Proposition 1. *Let $X \subset \mathbb{R}^n$ be a piece-wise smoothly imbedded compact polyhedron. Then for every $a \in H_1(X)$ with $K_2(a) > 0$ there exists a regular neighborhood $N_\epsilon X = \{x \mid \text{dist}(x, X) \leq \epsilon\} = N$ of X with $K_2(i_*(a)) > 0$, where $i_* : H_1(X) \rightarrow H_1(N)$ is the inclusion homomorphism.*

Proof. For small enough ϵ there is a regular neighborhood $N_\epsilon X$ with retraction $r : N_\epsilon X \rightarrow X$ having the norm $\|r\| < \epsilon$. Then $r_* i_*(a) = a$. If $i_*(a)$ has a representing cycle b with $K_2(b) < K_2(a)/2$, then $K_2(r(b)) = K_2(r(b)) < K_2(a)/2 + \epsilon < K_2(a)$ for sufficiently small ϵ . Since $r(b) \in a$, this is a contradiction.

Proposition 2. *There exists a smooth map $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that the intersection of the graph $\Gamma_f \subset \mathbb{R}^2 \times \mathbb{R}^2 = \mathbb{R}^4$ with every 2-dimensional plane in \mathbb{R}^4 is zero-dimensional.*

Proof. Define $f(x, y) = (\sin(x + y^2), \sin(x^2 + y^2))$. An arbitrary plane α is given by a linear system

$$\begin{aligned} a_1x + b_1y + c_1z + d_1t + e_1 &= 0, \\ a_2x + b_2y + c_2z + d_2t + e_2 &= 0 \end{aligned}$$

with non-proportional (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2) . Then $\Gamma_f \cap \alpha$ is homeomorphic to the solution of the system

$$\begin{aligned} a_1x + b_1y + e_1 + c_1 \sin(x + y^2) + d_1 \sin(x^2 + y^2) &= 0, \\ a_2x + b_2y + e_2 + c_2 \sin(x + y^2) + d_2 \sin(x^2 + y^2) &= 0. \end{aligned}$$

The system implies the following:

$$\begin{aligned} l_1(x, y) + k \sin(x + y^2) &= 0, \\ l_2(x, y) + k \sin(x^2 + y^2) &= 0, \end{aligned}$$

where $l_i(x, y)$ is a linear function. If $k \neq 0$, then the new system is equivalent to the old one. Assume that the intersection of these two curves in \mathbb{R}^2 is one-dimensional. Then the intersection will be more than 0-dimensional for corresponding complex analytic sets and hence these two sets coincide since they are irreducible [11]. Since $\sin t$ is a bounded function, the curve is quasi-isometric to the lines $l_i(x, y) = 0$. Therefore these two lines $l_1(x, y) = 0$ and $l_2(x, y) = 0$ are parallel. Then it follows that our curve lies in the set $\sin(x^2 + y^2) + k_2 \sin(x + y^2) + k_3 = 0$. That set does not contain a curve quasi-isometric to a line. Contradiction.

If $k = 0$, then it follows that our original system is equivalent to the system

$$\begin{aligned} l_1(x, y) &= 0, \\ a_2x + b_2y + c_2 \sin x + y^2 + d_2 \sin x^2 + y^2 + e_2 &= 0 \end{aligned}$$

with $(c_2, d_2) \neq (0, 0)$ which has a discrete set of solutions.

Remark. Proposition 2 is correct for a multiple kf of f with $k \neq 0$.

Proposition 3. *Let $f' = kf$ be a non-zero multiple of the \mathbb{R}^2 -valued function f of Proposition 2 restricted to a compact subset $Y \subset \mathbb{R}^2$. Then for every positive σ there exists $\delta > 0$ such that for every continuum $C \subset Y$ of diameter $\text{diam}C \geq \sigma$, the Kolmogorov diameter $K_2(C^\Gamma)$ is greater than δ , where $C^\Gamma = (C \times \mathbb{R}^2) \cap \Gamma_{f'} \subset \mathbb{R}^2 \times \mathbb{R}^2 = \mathbb{R}^4$.*

Proof. Assume the contrary, i.e. there is a sequence of continua $\{C_i\}$ in Y with $\text{diam}C_i \geq \sigma$ and with $K_2(C_i^\Gamma) < 1/i$. It means that for every i there is a 2-plane α_i such that the orthogonal projection $\pi_{\alpha_i} : C_i^\Gamma \rightarrow \alpha_i$ is a $1/i$ -move. Since Y is compact and the set of planes intersecting $\Gamma_{f'}$ is compact, we may assume that there are limits $C = \lim C_i$ and $\alpha = \lim \alpha_i$. Then from one side C is a continuum of diameter $\geq \sigma$ and hence $\text{dim}C^\Gamma > 0$. On the other hand $C^\Gamma \subset \alpha \cap \Gamma_{f'}$, contradicting Proposition 2.

Let $X \subset \mathbb{R}^4$ be a submanifold (with boundary). We call a plane α transverse to X if the intersection $\alpha \cap X$ is a manifold smoothly imbedded in α . Note that by the Sard theorem every plane α can be translated by an arbitrarily small translation T to a transversal plane α' .

By the distance, $\text{dist}(A, Y)$, between two closed subsets A and Y in some metric space we mean the infimum of distances between their points.

Proposition 4. *Let $X \subset B \subset \mathbb{R}^4$ be a smooth manifold lying in the interior of some standard ball and let α be a transversal plane with $\alpha \cap (X \cup (\mathbb{R}^4 - \text{Int}B)) = A$. Then for every $\sigma > 0$ and for every $\epsilon > 0$ there is a $\delta > 0$ and there is an ϵ -homeomorphism $h : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ such that*

- 1) $h|_{X \cup (\mathbb{R}^4 - \text{Int}B)} = \text{id}$,
 2) for every continuum C in α of diameter $\text{diam}C \geq \sigma$ with $\text{dist}(C, A) > \sigma/2$ the Kolmogorov diameter $K_2(h(C))$ is greater than δ .

Proof. Let $d : \alpha \rightarrow \mathbb{R}$ be the distance function $d(z, X \cup (\mathbb{R}^4 - \text{Int}B))$. Let α^\perp be the orthogonal plane; then $\mathbb{R}^4 = \alpha \times \alpha^\perp$. We define a compactum $Y = \alpha \cap B - O_{\sigma/2}(A)$ to be the complement in $\alpha \cap B$ to the open $\sigma/2$ -neighborhood of A . We consider a function $\phi : \alpha \rightarrow \mathbb{R}$ with the following properties: 1) $0 \leq \phi \leq \epsilon/2$, 2) $\phi^{-1}(0) = A$, 3) $\phi^{-1}(\epsilon/2) = Y$ and 4) $\phi \leq d$. Then we define a map h on α , $h : \alpha \rightarrow \alpha \times \alpha^\perp$ by the formula $(\text{id}_\alpha, \phi f)$ where f is the map of Proposition 2. The map h can be extended over \mathbb{R}^4 to an ϵ -homeomorphism. Here is one of the ways how to see that. We may assume that ϵ is so small that the ϵ -tube around α intersects X regularly. Then we have the following extension problem: to extend a fiber-wise map of $\alpha \times 0 \cup \alpha \times \partial D^2 \rightarrow \alpha \times D^2$ which is the identity on the boundary $\alpha \times \partial D^2$ to a homeomorphism $h : \alpha \times D^2 \rightarrow \alpha \times D^2$. Hence the extension problem is reduced to the problem of finding a homeomorphism h_x of the disk D^2 fixed on the boundary ∂D^2 and sending the center 0 to a given point $x \in \text{Int}D^2$ in such a way that h_x continuously depends on x and if $x = 0$ then $h_x = \text{id}_{D^2}$. There is no need to say that such a family h_x exists.

Now we take δ as in Proposition 3 for $f' = (\epsilon/2)f$ on the compactum Y and for a given σ to obtain δ . Then by Proposition 3 for every continuum of $\text{diam}C \geq \sigma$ with $\text{dist}(C, A) \geq \sigma/2$ we have $K_2(C^\Gamma) > \delta$. Note that $C^\Gamma = h(C)$.

Proposition 5. *Let $X \subset \text{Int}B \subset \mathbb{R}^4$ be a smooth compact manifold lying in the interior of a ball B and let α be a transversal plane. Suppose that $K_2(a) = r > 0$ and $K_2(i_*(a)) = 0$ for some element $a \in H_1(X)$, where $i_* : H_1(X) \rightarrow H_1(X \cup \alpha)$ is the inclusion homomorphism. Then there exists $\sigma > 0$ such that for every cycle $b \in i_*(a)$ with $K_2(b) < r/2$ and for every representation $b = c_1 + \dots + c_l$ as a sum of cycles with connected supports, $\text{supp}(c_i)$, the diameter of some component of the intersection $\text{supp}(c_i) \cap (\alpha - O_{\sigma/2}(A))$ is greater than σ for some i , where $A = \alpha \cap (X \cup (\mathbb{R}^4 - \text{Int}B))$.*

Proof. First we note that the support of a connected cycle is a Peano continuum. Assume the contrary: for every $\sigma > 0$ there exists a cycle $b \in i_*(a)$ with $K_2(b) < r/2$ such that $\text{diam}C < \sigma$ for every component of $\text{supp}(c_i) \cap (\alpha - O_{\sigma/2}(A))$. We may assume that $O_{\sigma/2}(A)$ is a regular neighborhood. If $\text{supp}(c_i) \subset \alpha$ then the cycle c_i is homologous to zero in $X \cup \alpha$. Delete all such c_i and consider the reduced sum $b' = c'_1 + \dots + c'_k$. There are two possibilities: $k > 0$ and $k \leq 0$. In the first case b is homologous to a cycle $d \in Z_1(X)$ obtained from b' by the deformation retraction $w : O_{3\sigma/2} \rightarrow A$. In the second case b is homologous to zero in $X \cup \alpha$. In both cases every cycle $a' \in a$ is homologous in $X \cup \alpha$ to a cycle $d \in Z_1(X)$ with $K_2(d) \leq r/2 + 2\sigma$. Let z be a 2-dimensional chain with $\partial z = a' - d$. We may assume that $z = z' + z_\alpha$ where z_α has $\alpha \cap \text{supp}(z)$ as the support, $\text{supp}(\partial z_\alpha) \subset X$ and $\text{supp}(z') \subset X$. Then $\partial z' = \partial z - \partial z_\alpha = a' - d - \partial z_\alpha$, i.e. a' is homologous to $d + \partial z_\alpha$ in X . Note that $K_2(a) \leq K_2(d + \partial z_\alpha) \leq \max\{K_2(d), K_2(\partial z_\alpha)\} = K_2(d) \leq r/2 + 2\sigma$. Since σ can be arbitrarily small, we have a contradiction.

Lemma 1. *Let $X \subset \mathbb{R}^4$ be a compact smooth manifold, $a \in H_1(X)$ with $K_2(a) > 0$. Then for every transversal 2-plane α and for any $\epsilon > 0$ there is an ϵ -homeomorphism $h : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ fixing X , such that $K_2(i_*(a)) > 0$ where $i_* : H_1(X) \rightarrow H_1(X \cup h(\alpha))$ is the inclusion homomorphism.*

Proof. Let j_* be the homomorphism induced by inclusion $j : X \rightarrow X \cup \alpha$. If $K_2(j_*(a)) \neq 0$, we take $h = id$. Assume that $K_2(j_*(a)) = 0$. Then Proposition 5 defines the number σ . Then we apply Proposition 4 for σ and $\epsilon < K_2(a)/8$ to obtain a homeomorphism $h : \mathbb{R}^4 \rightarrow \mathbb{R}^4$. Now we claim that $K_2(i_*(a)) > 0$.

Assume the contrary, then there are $b' \in i_*(a)$ with arbitrarily small Kolmogorov diameters, in particular $K_2(b') < K_2(a)/4$. Hence $K_2(h^{-1}(b')) < K_2(a)/4 + 2\epsilon < K_2(a)/2$. Since $h^{-1}(b') \in j_*(a)$, we can apply Proposition 5 to $b = h^{-1}(b')$. If $c_1 + \dots + c_l = b'$ is a partition with $K_2(\text{supp}(c_i)) < \delta$, then $b = h^{-1}(c_1) + \dots + h^{-1}(c_l)$ with $K_2(\text{supp}(h^{-1}(c_i))) < \delta + \epsilon$. By Proposition 5 there is an i and there is a component C of $\text{supp}(h^{-1}(c_i)) \cap (\alpha - O_{\sigma/2}(A))$ with $\text{diam}C > \sigma$. Then by Proposition 4, $K_2(h(C)) > \delta$. Therefore, $\delta > K_2(\text{supp}(c_i)) = K_2(h(\text{supp}(h^{-1}(c_i)))) \geq K_2(h(C)) > \delta$. This is a contradiction.

We denote by $B(n) = \{x \in \mathbb{R}^4 \mid \|x\| \leq n\}$ the ball of radius n centered at the origin.

Lemma 2. *Assume that X is a compact smooth submanifold, $X \subset \text{Int}B(1)$ and $a \in H_1(X)$ with $K_2(a) > 0$. Then for every sequence of 2-planes $\{\alpha_k\}$ and for every sequence of positive numbers $\{\epsilon_k\}$, $\epsilon_k \rightarrow 0$, there exists an open subset $W \subset \mathbb{R}^4$ such that*

- 1) $X \subset W$ and the image $i_*(a)$ is non-zero in $H_1(W)$,
- 2) for every k there is an ϵ_k -homeomorphism $h_k : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ with $h_k(\alpha_k \cap B(3)) \subset W$.

Proof. We construct $W = X_0 \cup X_1 \cup \dots \cup X_n \cup \dots$ by induction on n . First define $X_0 = X$. Then consider a regular neighborhood $N_0 \subset \text{Int}B(4)$ of X_0 with a positive Kolmogorov diameter of the image of a under the inclusion homomorphism, i.e. $K_2((i_0)_*(a)) > 0$ (see Proposition 1). We replace α_1 if it's necessary with a parallel $\epsilon_1/2$ -close transverse to N_0 plane α'_1 . Let T_1 be an $\epsilon_1/2$ -translation with $T_1(\alpha_1) = \alpha'_1$. We apply Lemma 1 with $\epsilon = \epsilon_1/2$ to obtain a homeomorphism $h'_1 : \mathbb{R}^4 \rightarrow \mathbb{R}^4$. The composition $h_1 = h'_1 \circ T_1 : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is an ϵ_1 -move. We define $X_1 = (N_0 \cup h_1(\alpha_1)) \cap B(4)$. Let N_1 be a smooth regular neighborhood of X_1 in $B(5)$. Then $K_2((i_1)_*(a)) > 0$, where $i_1 : X \hookrightarrow N_1$ and we can apply Lemma 1 to α_2 (or its translation α'_2) with $\epsilon_2/2$ and so on. As the result we will get sequences of $X_k \subset B(k+3)$, $N_k \subset B(k+4)$ and $i_k : X \hookrightarrow N_k$ such that $X_k \subset \text{Int}N_k \subset N_k \subset X_{k+1}$, $X_k = (N_{k-1} \cup h_k(\alpha_k)) \cap B(k+3)$ and $K_2((i_k)_*(a)) > 0$. Hence $W = \bigcup_k X_k = \bigcup_k \text{Int}N_k$ is an open set and $i_*(a)$ is non-zero in W . Note that $h_k(\alpha_k \cap B(3)) \subset h_k(\alpha_k) \cap h_k(B(3)) \subset h_k(\alpha_k) \cup B(4) \subset h_k(\alpha_k) \cap B(k+3) \subset X_k \subset W$.

Theorem. *There exists a 2-dimensional compactum $Z \subset \mathbb{R}^4$ which intersects unstably every 2-dimensional affine plane.*

Proof. Let Π denote the space of planes $\alpha \subset \mathbb{R}^4$ having non-empty intersection with $B(1)$. Let the distance between two planes α and β be the Hausdorff distance between the compacta $\alpha \cap B(1)$ and $\beta \cap B(1)$. Let $\{\alpha_i\}$ be a countable dense set in Π . Let $X \subset \text{Int}B(1)$ be a topological circle with $K_2(a) > 0$, where a is a generator of $H_1(X)$. Define $\epsilon_i = 1/i$; then Lemma 2 gives us an open set $W \subset \mathbb{R}^4$. By Alexander Duality we have $H^2(B(1) - W, \partial B(1) - W) = H_1(W \cap \text{Int}B(1)) \neq 0$. We define $Z = B(1) - W$; then $\dim Z \geq 2$. The compact Z cannot have $\dim Z = 3$, since every compactum X of codimension one satisfies the Chogoshvili Conjecture, i.e. X intersects some affine line stably. Below we show that our compactum Z intersects unstably every 2-dimensional affine plane (and hence every affine line).

Let α be a given 2-plane with $\alpha \cap Z \neq \emptyset$ and let ϵ be a given positive number. Then $\alpha \in \Pi$. There exists i such that $\text{dist}(\alpha, \alpha_i) < \epsilon_i < \epsilon/2$. Then there exists an $\epsilon/2$ -homeomorphism $h'_i : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ with $h'_i(\alpha) = \alpha_i$. By Lemma 2 there exists an $\epsilon/2$ -homeomorphism $h_i : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ with $h_i(\alpha_i \cap B(3)) \subset W$. Consider $g = h_i \circ h'_i$. Then $g(\alpha \cap B(2)) \subset W$, therefore $(g^{-1}(\mathbb{R}^4 - W)) \cap (\alpha \cap B(2)) = \emptyset$. Hence $g^{-1}(Z) \cap (\alpha \cap B(2)) = \emptyset$ and hence $g^{-1}(Z) \cap \alpha = \emptyset$. Note that $f = g^{-1}|_Y : Y \rightarrow \mathbb{R}^4$ is an ϵ -move removing Z from α .

Remark. The method of the proof of the Theorem actually allows one to prove more. Namely, for every natural number k there is a compactum $Z_k \subset \mathbb{R}^{2k}$ of the dimension $\dim Z_k = 2k - 2$ such that Z_k has unstable intersection with every k -dimensional affine plane.

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