CENTRAL EXTENSIONS OF SOME LIE ALGEBRAS

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Abstract. We consider three Lie algebras: Der \( C((t)) \), the Lie algebra of all derivations on the algebra \( C((t)) \) of formal Laurent series; the Lie algebra of all differential operators on \( C((t)) \); and the Lie algebra of all differential operators on \( C((t)) \otimes C^n \). We prove that each of these Lie algebras has an essentially unique nontrivial central extension.

The Lie algebra of all derivations on the Laurent polynomial algebra \( C[t, t^{-1}] \) can also be characterized as the Lie algebra of vector fields on the circle. The analogous object over a field \( F \) of characteristic \( p > 0 \), \( \text{Der}_F [t] / \langle t^p \rangle \), is called the Witt algebra \([C]\), and this name is sometimes applied to \( \text{Der} \ C[t, t^{-1}] \) as well. It is known \([Bl]\) that \( \text{Der} F[t] / \langle t^p \rangle \) has an essentially unique nontrivial one-dimensional central extension, and also \([GF]\) that \( \text{Der} C[t, t^{-1}] \) has an essentially unique nontrivial one-dimensional central extension. The proofs of these facts are similar. The nontrivial one-dimensional central extension of \( \text{Der} C[t, t^{-1}] \) is called the Virasoro algebra. It is one of the fundamental objects in representation theory as well as in theoretical physics.

For a positive integer \( n \), the Lie algebra of all differential operators on \( C[t, t^{-1}] \otimes C^n \) has a nontrivial one-dimensional central extension, and the extended Lie algebra is related to the representation theory of affine Lie algebras \([KP]\). It is proved in \([L]\) that this extension is essentially unique (also see \([F]\)). When \( n = 1 \), the Lie algebra of all differential operators on the Laurent polynomial ring \( C[t, t^{-1}] \) can also be characterized as the Lie algebra of differential operators on the circle; the corresponding extension is referred to, particularly in the physics literature, as \( W_{1+\infty} \). Some representations of \( W_{1+\infty} \) have been studied recently (see, e.g., \([KR]\), \([FKRW]\)). In \([FKRW]\), it is shown that some representations of \( W_{1+\infty} \) have natural structures of vertex operator algebras (see, e.g., \([Bo]\) and \([FLM]\) for definitions).

Each of these constructions involves the Laurent polynomial algebra \( C[t, t^{-1}] \). This algebra is, of course, contained in \( C((t)) \), the algebra of formal Laurent series. In this paper, we consider the Lie algebras obtained by replacing \( C[t, t^{-1}] \) by \( C((t)) \) in each of these constructions. We show that each of the resulting Lie algebras has an essentially unique nontrivial one-dimensional central extension.

We work over the field of complex numbers, though all results hold over any field of characteristic zero.

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1. Some basic definitions

Let \( L \) and \( \hat{L} \) be two Lie algebras over \( \mathbb{C} \). The Lie algebra \( \hat{L} \) is said to be a one-dimensional central extension of \( L \) if there is a Lie algebra exact sequence
\[ 0 \rightarrow \mathbb{C}c \rightarrow \hat{L} \rightarrow L \rightarrow 0, \]
where \( \mathbb{C}c \) is the one-dimensional trivial Lie algebra and the image of \( \mathbb{C}c \) is contained in the center of \( \hat{L} \). It is well-known that \( \hat{L} \) is a one-dimensional central extension of \( L \) if and only if \( \hat{L} \) is the direct sum of \( L \) and \( \mathbb{C}c \) as vector spaces and the Lie bracket \([\cdot, \cdot] \) in \( \hat{L} \) is given by
\[ [x, y]_1 = [x, y] + \varphi(x, y)c, \]
\[ [x, c]_1 = 0 \]
for all \( x, y \in L \), where \([\cdot, \cdot] \) is the Lie bracket in \( L \) and \( \varphi : L \times L \rightarrow \mathbb{C} \) is a bilinear form on \( L \) satisfying the following conditions:
\begin{align*}
(1) & \quad \varphi(x, y) = -\varphi(y, x), \\
(2) & \quad \varphi([x, y], z) + \varphi([y, z], x) + \varphi([z, x], y) = 0
\end{align*}
for all \( x, y, z \in L \). The bilinear form \( \varphi \) is called a 2-cocycle on \( L \). A central extension is trivial if \( \hat{L} \) is the direct sum of a subalgebra \( M \) and \( \mathbb{C}c \) as Lie algebras, where the subalgebra \( M \) is isomorphic to \( L \). A 2-cocycle \( \varphi \) corresponding to a trivial central extension is called a 2-coboundary, or a trivial 2-cocycle, and is given by a linear function \( f \) from \( L \) to \( \mathbb{C} \):
\[ \varphi(x, y) = f([x, y]) \]
for all \( x, y \in L \). The 2-coboundary defined by \( f \) is denoted by \( \alpha_f \). The set of all 2-cocycles on \( L \) is a vector space, denoted by \( Z^2(L, \mathbb{C}) \). The set of all 2-coboundaries is a subspace of \( Z^2(L, \mathbb{C}) \), denoted by \( B^2(L, \mathbb{C}) \). The quotient space \( Z^2(L, \mathbb{C})/B^2(L, \mathbb{C}) \) is called the 2nd cohomology group of \( L \) with coefficients in \( \mathbb{C} \), and denoted by \( H^2(L, \mathbb{C}) \). If \( \dim H^2(L, \mathbb{C}) = 1 \), we say that \( L \) has an essentially unique nontrivial one-dimensional central extension. We say that 2-cocycles \( \varphi, \psi \) are equivalent if \( \varphi - \psi \) is a 2-coboundary.

The following two lemmas will be used in the proofs of our main results.

**Lemma 1.** Let \( L \) be a Lie algebra and \( S \) a subset of \( L \) such that \( S \) spans \( L \) and for each \( x \in S \), \( x = [y_x, z_x] \) for some \( y_x, z_x \in L \). If a 2-cocycle \( \varphi \) satisfies \( \varphi(y_x, z_x) = 0 \) for all \( x \in S \), then either \( \varphi = 0 \) or \( \varphi \) is nontrivial.

**Proof.** Suppose that \( \varphi \) is trivial, so that \( \varphi = \alpha_f \) for some linear function \( f \). Then for each \( x \in S \),
\[ f(x) = f([y_x, z_x]) = \varphi(y_x, z_x) = 0. \]
Thus \( f = 0 \) since \( S \) spans \( L \). This implies that \( \varphi = \alpha_f = 0 \). \( \square \)

**Lemma 2.** Let \( L \) be a Lie algebra and \( \varphi \) a 2-cocycle on \( L \). Suppose there are linear endomorphisms \( E \) and \( F \) of \( L \) such that
\[ \varphi(Ex, y) = \varphi(x, Fy) \]
for all \( x, y \in L \), \( E \) is surjective and \( F \) is locally nilpotent (i.e., for any \( y \in L \), there is a positive integer \( n \) such that \( F^n y = 0 \)). Then the 2-cocycle \( \varphi \) is 0.

**Proof.** For \( x, y \in L \), let \( n \) be a positive integer such that \( F^n y = 0 \). Since \( E \) is surjective, we have \( x' \in L \) such that \( x = E^n x' \). Thus,
\[ \varphi(x, y) = \varphi(E^n x', y) = \varphi(x', F^n y) = 0. \]  \( \square \)
Let \(A\) be a (not necessarily associative) algebra. A linear map \(\delta: A \to A\) is called a derivation, if \(\delta(ab) = \delta(a)b + a\delta(b)\) for all \(a, b \in A\).

Now consider the algebra of all formal Laurent series

\[
\mathbb{C}((t)) = \left\{ \sum_{i \in \mathbb{Z}, i \geq n} a_i t^i \middle| a_i \in \mathbb{C}, n \in \mathbb{Z} \right\}.
\]

It is known that the set of all derivations on \(\mathbb{C}((t))\) is

\[
\mathcal{A} = \left\{ f(t) \frac{d}{dt} f(t) \in \mathbb{C}((t)) \right\},
\]

where \(\frac{d}{dt}\) is the formal derivation defined by \(\frac{d}{dt}: \mathbb{C}((t)) \to \mathbb{C}((t)), \sum a_i t^i \mapsto \sum i a_i t^{i-1}\). For convenience, we denote \(\frac{d}{dt}\) by \(D\) and denote \(\frac{d}{dt} f(t)\) by \(f'(t)\). For \(f(t) = \sum a_i t^i \in \mathbb{C}((t))\), define \(\text{Res } f(t) = a_{-1}\). If \(\text{Res } f(t) = 0\), we can define the formal integral of \(f(t)\) as

\[
\sum_{i \neq -1} \frac{a_i}{i + 1} t^{i+1},
\]

denoted by \(\int f(t)\). Then \(\mathcal{A}\) is a Lie algebra under the bracket operation:

\[
[f(t)D, g(t)D] = (f(t)D) \circ (g(t)D) - (g(t)D) \circ (f(t)D)
= f(t)(g(t)'D - g(t)f'(t)D)
\]

for \(f(t), g(t) \in \mathbb{C}((t))\), where the \(\circ\) is the composition of operators.

More generally, consider the space of all differential operators on the algebra of formal Laurent series \(\mathbb{C}((t))\):

\[
\mathcal{B} = \text{span} \left\{ f(t)D^l \middle| l \in \mathbb{N}, f(t) \in \mathbb{C}((t)) \right\}.
\]

\(\mathcal{B}\) is a Lie algebra under the bracket operation:

\[
[f(t)D^l, g(t)D^k] = (f(t)D^l) \circ (g(t)D^k) - (g(t)D^k) \circ (f(t)D^l)
= \sum_{i=0}^l \binom{l}{i} f(t) (D^{l-i}g(t)) D^{k+i} - \sum_{j=0}^k \binom{k}{j} g(t) (D^{k-j} f(t)) D^{l+j}.
\]

Furthermore, we may consider \(\mathcal{C} = \mathcal{B} \otimes \text{gl}_n(\mathbb{C})\), the space of differential operators on \(\mathbb{C}((t)) \otimes \mathbb{C}^n\). Note that \(\mathcal{C} \subset \text{End } (\mathbb{C}((t)) \otimes \mathbb{C}^n)\). Define the bracket operation on \(\mathcal{C}\) by linearity and the commutator

\[
[f(t)D^l \otimes A, g(t)D^k \otimes B] = (f(t)D^l \otimes A) \circ (g(t)D^k \otimes B) - (g(t)D^k \otimes B) \circ (f(t)D^l \otimes A)
= \sum_{i=0}^l \binom{l}{i} f(t) (D^{l-i}g(t)) D^{k+i} \otimes AB - \sum_{j=0}^k \binom{k}{j} g(t) (D^{k-j} f(t)) D^{l+j} \otimes BA.
\]

Hence \(\mathcal{C}\) is a Lie algebra.
2. Main results and proofs

In this section, we will give our main results and their proofs. In each case we exhibit (using Lemma 1) a nontrivial 2-cocycle on the Lie algebra under consideration. The 2-cocycle is analogous to the standard nontrivial 2-cocycle on the Lie algebra obtained from Laurent polynomial algebra. Then for any given 2-cocycle on the Lie algebra, we reduce the 2-cocycle to a 2-cocycle which is equivalent to the original one and takes value 0 whenever the standard 2-cocycle takes value 0. We use Lemma 2 to show that the reduced 2-cocycle is a multiple of the standard one.

**Theorem 1.** \( \dim H^2(A, \mathbb{C}) = 1. \)

**Proof.** Let \( \beta \) be a 2-cocycle on \( A \). Define a linear function \( f_\beta : A \to \mathbb{C} \) by

\[
f_\beta(g(t)D) = \beta\left(D, \int g(t)D\right) \quad \text{for} \quad g(t) \in \mathbb{C}((t)), \quad \text{Res } g(t) = 0
\]

and

\[
f_\beta(t^{-1}D) = \frac{1}{2} \beta(2(t^{-1}D,tD)).
\]

Then \( \beta_1 = \beta - \alpha f_\beta \) is a 2-cocycle on \( A \) which is equivalent to \( \beta \).

For \( f(t) = \sum_{i \neq 0} a_i t^i \in \mathbb{C}((t)) \),

\[
\beta_1(D, f(t)D) = \beta(D, f(t)D) - f_\beta([D, f(t)D]) = \beta(D, f(t)D) - f_\beta(f'(t)D) = 0,
\]

and

\[
\beta_1(t^{-1}D, tD) = \beta(t^{-1}D, tD) - f_\beta([t^{-1}D, tD]) = \beta(t^{-1}D, tD) - f_\beta(2t^{-1}D) = 0.
\]

**Lemma 3.** \( \beta_1(D, A) = 0 \) and \( \beta_1(tD, A) = 0 \).

**Proof of Lemma 3.** From (3) and \( \beta_1(D, D) = -\beta_1(D, D) \), we have \( \beta_1(D, A) = 0 \). For \( f(t) \in \mathbb{C}((t)) \) and \( \text{Res } f(t) = 0 \),

\[
\beta_1(tD, f(t)D) = \beta_1\left(tD, \left[D, \int f(t)D\right]\right) = \beta_1\left([tD, D], \int f(t)D\right) + \beta_1\left(D, \left[tD, \int f(t)D\right]\right) = 0.
\]

\( \beta_1(tD, A) = 0 \) follows from this and (4).

**Lemma 4.** \( \beta_1(t^2D, A) = 0 \).
**Proof of Lemma 4.** For \( f(t) \in \mathbb{C}((t)) \) and \( \text{Res } f(t) = 0 \), we have

\[
\beta_1(t^2D, f(t)D) \\
= \beta_1\left(t^2D, \left[ D, \int f(t)D \right] \right) \\
= \beta_1\left([t^2D, D], \int f(t)D \right) + \beta_1\left(D, \left[ t^2D, \int f(t)D \right] \right) = 0.
\]

Also

\[
\beta_1(t^2D, t^{-1}D) \\
= \beta_1\left(t^2D, -\frac{1}{2} [tD, t^{-1}D] \right) \\
= -\frac{1}{2} \beta_1([t^2D, tD], t^{-1}D) - \frac{1}{2} \beta_1(tD, [t^2D, t^{-1}D]) \\
= -\frac{1}{2} \beta_1(-t^2D, t^{-1}D) = \frac{1}{2} \beta_1(t^2D, t^{-1}D).
\]

This implies that \( \beta_1(t^2D, t^{-1}D) = 0 \).

**Lemma 5.** If \( f(t) \in \mathbb{C}((t)) \) and \( \text{Res } f(t) = 0 \), then \( \beta_1(t^3D, f(t)D) = 0 \).

**Proof of Lemma 5.** We have

\[
\beta_1(t^3D, f(t)D) \\
= \beta_1\left(t^3D, \left[ D, \int f(t)D \right] \right) \\
= \beta_1\left([t^3D, D], \int f(t)D \right) + \beta_1\left(D, \left[ t^3D, \int f(t)D \right] \right) \\
= \beta_1\left(-3t^2D, \int f(t)D \right) = 0.
\]

Define \( \alpha : \mathcal{A} \times \mathcal{A} \to \mathbb{C} \) by

\[
\alpha\left(\sum_i a_i t^{i+1}D, \sum_j b_j t^{j+1}D \right) = \sum_i a_i b_{-i}(i^3 - i)
\]

for \( \sum_i a_i t^{i+1}D, \sum_j b_j t^{j+1}D \in \mathcal{A} \). Note that the sum on the right-hand side is finite and \( \alpha \) is a 2-cocycle on \( \mathcal{A} \). Let \( S \) be the subset of \( \mathcal{A} \) given by

\[
S = \{t^{-1}D\} \cup \left\{ f(t)D \in \mathcal{A} \mid \text{Res } f(t) = 0 \right\}.
\]

Now for \( f(t) \in \mathbb{C}((t)) \), \( \text{Res } f(t) = 0 \), we have

\[
t^{-1}D = \left[ \frac{1}{2} t^{-1}D, tD \right], \\
f(t)D = \left[ D, \int f(t)D \right], \\
\alpha(t^{-1}D, tD) = 0,
\]

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and

\[ \alpha \left( D, \int f(t)D \right) = 0. \]

Since \( \alpha \) is nonzero (in fact, \( \alpha(t^3D, t^{-1}D) = 6 \)), Lemma 1 shows that \( \alpha \) is nontrivial. Also \( \alpha = \alpha_1 \). Applying Lemma 3, Lemma 4 and Lemma 5 to \( \alpha \), we have

\[ \alpha(D, A) = \alpha(tD, A) = \alpha(t^2D, A) = 0, \]

and

\[ \alpha(t^3D, f(t)D) = 0 \quad \text{for} \quad f(t) \in \mathbb{C}((t)), \quad \text{Res} f(t) = 0. \]

Suppose that \( \beta_1(t^3D, t^{-1}D) = 6r \) for some \( r \in \mathbb{C} \). Define \( \beta_2 = \beta_1 - r\alpha \); then we have

\[ \beta_2(D, A) = \beta_2(tD, A) = \beta_2(t^2D, A) = 0 \]

and

\[ \beta_2(t^3D, A) = 0. \]

We now show that \( \beta_2 = 0 \), completing the proof of Theorem 1.

Let \( \text{ad}: \mathcal{A} \to \mathcal{A}, \text{ad}(a)b = [a, b] \), be the adjoint operator; then

\[ \beta_2(\text{ad}(f(t)D), g(t)D) = \beta_2([D, f(t)D], g(t)D) = \beta_2([D, g(t)D], f(t)D) + \beta_2(D, [f(t)D, g(t)D]) = -\beta_2(f(t)D, \text{ad}(g(t)D)). \]

Similarly,

\[ \beta_2(\text{ad}(tD)(f(t)D), g(t)D) = -\beta_2(f(t)D, \text{ad}(tD)(g(t)D)), \]

\[ \beta_2(\text{ad}(t^3D)(f(t)D), g(t)D) = -\beta_2(f(t)D, \text{ad}(t^3D)(g(t)D)). \]

Now we want to use formulas (7), (8) and (9) to construct two linear endomorphisms \( E \) and \( F \) on \( \mathcal{A} \) so that we can use Lemma 2. Set

\[ E = (\text{ad}(D))^2\text{ad}(t^3D) - (\text{ad}(tD))^3 - 3(\text{ad}(tD))^2 + 4\text{ad}(tD), \]

\[ F = -\text{ad}(t^3D)(\text{ad}(D))^2 + (\text{ad}(tD))^3 - 3(\text{ad}(tD))^2 - 4\text{ad}(tD). \]

Then, using (7), (8) and (9), we have

\[ \beta_2(E(f(t)D), g(t)D) = \beta_2(f(t)D, F(g(t)D)). \]
For $f(t) = \sum_i a_i t^{i+1}$, $g(t) = \sum_j b_j t^{j+1} \in \mathbb{C}((t))$, we have

$$(\mathrm{ad}D)^2 \mathrm{ad}(t^3 D)(f(t)D)$$

$$= [D, [D, [t^3 D, f(t)D]]]$$

$$= [D, \sum_i (i-2)a_i t^{i+3}D]$$

$$= \sum_i (i-2)(i+3)a_i t^{i+2}D$$

$$= \sum_i (i^3 + 3i^2 - 4i - 12)a_i t^{i+1}D.$$

Also

$$(\mathrm{ad}t D)^k(f(t)D) = \sum_i t^k a_i t^{i+1}D$$

for all $k \in \mathbb{N}$. This implies that $E(f(t)D) = -12 f(t)D$ for all $f(t) \in \mathbb{C}((t))$. Thus $E$ is invertible. Similarly, for $g(t) = \sum_j b_j t^{j+1} \in \mathbb{C}((t))$, we have

$$-\mathrm{ad}(t^3 D)(\mathrm{ad}D)^2(g(t)D) = \sum_j (-j^3 + 3j^2 + 4j)b_j t^{j+1}D.$$

Thus $F(g(t)D) = 0$ for all $g(t) \in \mathbb{C}((t))$. From Lemma 2, we have $\beta_2 = 0$ or $\beta_1 = r\alpha$. \hfill $\square$

**Theorem 2.** $\dim H^2(\mathcal{B}, \mathbb{C}) = 1$.

**Proof.** For any 2-cocycle $\psi$ on $\mathcal{B}$, define

$$f_\psi(g(t)D^l) = \frac{-1}{i+1} \psi(t, g(t)D^{i+1})$$

for $g(t) \in \mathbb{C}((t))$. Then $\psi_1 = \psi - \alpha f_\psi$ is a 2-cocycle and equivalent to $\psi$. We have

$$\psi_1(t, g(t)D^{l+1}) = 0$$

for all $g(t) \in \mathbb{C}((t))$ and $l \in \mathbb{N}$.

**Lemma 6.** If $g(t) \in \mathbb{C}((t))$ and $\mathrm{Res } g(t) = 0$, then $\psi_1(t, g(t)) = 0$.

**Proof of Lemma 6.** For $f(t) \in \mathbb{C}((t))$ and $\mathrm{Res } f(t) = 0$, we have

$$\psi_1(t, f(t))$$

$$= \psi_1([tD, t], f(t))$$

$$= \psi_1(tD, [t, f(t)]) + \psi_1([tD, f(t)], t)$$

$$= -\psi_1(t, tf'(t)).$$

Therefore, $\psi_1(t, f(t) + tf'(t)) = 0$. Note that every element $g(t) \in \mathbb{C}((t))$ with $\mathrm{Res } g(t) = 0$ can be written in the form $f(t) + tf'(t)$ for some $f(t) \in \mathbb{C}((t))$ with $\mathrm{Res } f(t) = 0$. \hfill $\square$
Define \( \varphi : \mathcal{B} \times \mathcal{B} \to \mathbb{C}, \)
\[
\varphi \left( \sum_m a_m t^{l+m} D^l, \sum_n b_n t^{k+n} D^k \right) = \sum_m a_m b_{-m} (-1)^l l! k! \left( \frac{m + l}{l + k + 1} \right).
\]
Then \( \varphi \) is a 2-cocycle on \( \mathcal{B}. \) Let \( S \) be the subset of \( \mathcal{B} \) given by
\[
S = \left\{ f(t) D^l \mid l \in \mathbb{N}, f(t) \in \mathbb{C}((t)) \right\}.
\]
For any \( l \in \mathbb{N} \) and \( f(t) \in \mathbb{C}((t)), \)
\[
f(t) D^l = - \frac{1}{l + 1} \left[ t, f(t) D^{l+1} \right]
\]
and
\[
\varphi \left( t, f(t) D^{l+1} \right) = 0.
\]
From Lemma 1 and the fact that \( \varphi(t, t^{-1}) = 1, \) we have that \( \varphi \) is nontrivial. If \( \psi_1(t, t^{-1}) = s, \) we define \( \psi_2 = \psi_1 - s \varphi. \) Then using Lemma 6, we have \( \psi_2(t, \mathcal{B}) = 0. \)

Note that
\[
\psi_2 \left( \text{ad}(t(t)) D^l, g(t) D^k \right) = \psi_2 \left( [t, f(t) D^l], g(t) D^k \right) = \psi_2 \left( [t, g(t) D^k], f(t) D^l \right) + \psi_2 \left( f(t) D^l, [g(t) D^k] \right)
\]
(11)
\[
= - \psi_2 \left( f(t) D^l, \text{ad}(g(t) D^k) \right).
\]
For \( f(t) = \sum_m a_m t^{l+m} \in \mathbb{C}((t)), \)
\[
\left[ t, f(t) D^l \right] = - l f(t) D^{l-1}.
\]
Therefore the operator \( \text{ad} t \) is surjective and locally nilpotent. Let \( E = \text{ad} t \) and \( F = - \text{ad} t. \) From equation (11) and Lemma 2, we have \( \psi_2 = 0. \) This gives \( \psi_1 = s \varphi. \)

**Remark.** This method gives a simplified proof of Theorem 2.1 of [L].

Consider the Lie algebra \( \mathcal{C} = \mathcal{B} \otimes gl_n(\mathbb{C}). \) Define a bilinear map \( \phi : \mathcal{C} \times \mathcal{C} \to \mathbb{C} \) by
\[
\phi \left( \sum_m a_m t^{l+m} D^l \otimes A, \sum_n b_n t^{k+n} D^k \otimes B \right) = \sum_m a_m b_{-m} (-1)^l l! k! \left( \frac{m + l}{l + k + 1} \right) \text{tr}(AB)
\]
for \( \sum_m a_m t^{l+m}, \sum_n b_n t^{k+n} \in \mathbb{C}((t)), l, k \in \mathbb{N} \) and \( A, B \in gl_n(\mathbb{C}). \) Then \( \phi \) is a 2-cocycle on \( \mathcal{C}. \) Using a method similar to the proof of Theorem 2.2 of [L], we have

**Corollary.** \( \text{dim} H^2(\mathcal{C}, \mathbb{C}) = 1. \)

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REFERENCES


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