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A CHARACTERIZATION OF σ-COMPACTNESS OF A COSMIC SPACE X BY MEANS OF SUBSPACES OF R^X

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Abstract. This work is devoted to the relationship between topological properties of a space X and those of $C_p(X)$ (= the space of continuous real-valued functions on X, with the topology of pointwise convergence). The emphasis is on σ-compactness of X and on location of $C_p(X)$ in R^X. In particular, σ-compact cosmic spaces are characterized in this way.

1. Introduction

In 1974 J.P.R. Christensen [11] proved that a separable metrizable space X is σ-compact if and only if $C_p(X)$ is analytic, that is, $C_p(X)$ is a continuous image of the space of irrational numbers, and posed the following question: Is a completely regular space X σ-compact provided $C_p(X)$ is analytic? In 1980 this question was posed again in [16], Problem 37. In 1985 the second author answered Christensen's question positively [7]. The converse is false in general: the first author constructed a countable Fréchet-Uryshon $\aleph_0$-space (in the sense of E. Michael, see [17]) such that $C_p(X)$ is not analytic (see [8], [9]). Recently O.G. Okunev [18] established that if a Tychonoff space X is σ-bounded, then there exists a $K_{\sigma\delta}$-subspace A of R^X such that $C_p(X) \subset A$ (in this case, we say that $C_p(X)$ *is $K_{\sigma\delta}$-framed in R^X*).

The main goal of this article is to generalize the two first results in the light of Okunev's one. In particular, we obtain that if X is a cosmic space, or a perfectly normal K-analytic space, then X is σ-compact if and only if $C_p(X)$ is K-analytic-framed in R^X (Theorems 2.4 and 2.7). To do this, we blend the suitably adapted technique used by Christensen in the original argument (see the proof that 4) $\Rightarrow$ 5) in Theorem 2.3) with some other technical tools and concepts.

It follows from Theorem 2.4 that if X is a separable metrizable space such that $C_p(X)$ is K-analytic-framed in R^X, then X is σ-compact.

We also give an application of Theorem 2.4 to the problem of embedding a cosmic space in an analytic space (Theorem 2.8, Corollaries 2.9 and 2.10).

By a space we mean a Hausdorff topological space, R^X is the space of real-valued functions on X with the product topology, $C(X)$ is the set of continuous

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real-valued functions on X, and $C_p(X)$ is $C(X)$ with the topology generated by R^X. Our terminology is the same as in [3] and [12].

A mapping φ of a space X into the set $\mathcal{K}(Y)$ of compact subsets of a space Y is called *usco-compact* if $Y = \bigcup\{ \varphi(x) \ : \ x \in X \}$ and if for every $x \in X$ and every open subset $G \supset \varphi(x)$ of Y, there exists an open neighbourhood H of x such that $\varphi(x') \subset G$, for each $x' \in H$. For a subset A of X, we put $\varphi(A) = \bigcup\{ \varphi(x) \ : \ x \in A \}$. Recall that if A is compact, then $\varphi(A)$ is compact.

A space X is called a $K_{\sigma\delta}$-space if it is a $K_{\sigma\delta}$-subset of a space Z, that is, if X is the intersection of a countable family of σ-compact subsets of Z. A space X is *K-analytic* (G. Choquet [10]), if it is a continuous image of a $K_{\sigma\delta}$-space; X is K-analytic if and only if it is an usco-compact image of the space ω^ω of irrational numbers, where ω is the discrete space of integers [14], [13].

A subspace X of Y is *$\mathcal{P}$-framed in Y* (where $\mathcal{P}$ is a topological property) if there exists a subspace Z of Y with the property $\mathcal{P}$ such that $X \subset Z \subset Y$ [4].

A *network* in a topological space X is a collection γ of subsets of X such that, for every $x \in X$ and every open subset G with $x \in G$, there exists $N \in \gamma$ such that $x \in N \subset G$. Spaces with a countable network are often called *cosmic* spaces; these are precisely continuous images of separable metrizable spaces (see [1]).

Recall that a subset Y of a space X is said to be *bounded* (in X) if for every $f \in C_p(X)$, the restriction $f_{|Y}$ is bounded. A space X is called *σ-bounded* if it is a countable union of bounded subspaces. A space X has the property $\mathcal{P}$ projectively if every separable metrizable space Y, which is a continuous image of X, has the property $\mathcal{P}$. Every σ-bounded space is projectively σ-compact, since every σ-bounded paracompact space is σ-compact.

Note that R^X is projectively analytic (it suffices to apply an appropriate factorization theorem for continuous real-valued functions on R^X (see [12])).

2. σ-compactness and K-analytic-frameness

Let X and Y be two sets, A a subset of X, and f a mapping of X onto Y. Then p_A the *projection mapping* of R^X to R^A, that is, $p_A(f) = f_{|A}$, and $\widetilde{f}$ is the *dual mapping* of R^Y into R^X defined by $\widetilde{f}(g) = g \circ f$. We need the following two lemmas, the first of which is obvious:

Lemma 2.1. *Let X be a normal space, A a closed subspace of X, $\mathcal{P}$ a projective property, and let $C_p(X)$ be $\mathcal{P}$-framed in R^X. Then $C_p(A)$ is $\mathcal{P}$-framed in R^A.*

Lemma 2.2. *Let X and Y be topological spaces and f a continuous mapping from X onto Y. Then $\widetilde{f}$ is an embedding of R^Y onto $\widetilde{f}(R^Y)$; $\widetilde{f}(C(Y)) \subset C(X)$; and $\widetilde{f}(R^Y)$ is closed in R^X. Moreover, if $\mathcal{P}$ is a closed-hereditary property and $C_p(X)$ is $\mathcal{P}$-framed in R^X, then $C_p(Y)$ is $\mathcal{P}$-framed in R^Y.*

Proof. Clearly, $\widetilde{f}$ is a homeomorphism of R^Y onto $\widetilde{f}(R^Y)$, $\widetilde{f}(C(Y)) \subset C(X)$, and $\widetilde{f}(R^Y)$ is closed in R^X. Now, let A be a subspace of R^X with the property $\mathcal{P}$ such that $C(X) \subset A$. Note that $\widetilde{f}(C(Y)) \subset A$. Since $\mathcal{P}$ is closed-hereditary, the closed subspace $A \cap \widetilde{f}(R^Y)$ of A has the property $\mathcal{P}$. From $\widetilde{f}(C(Y)) \subset A \cap \widetilde{f}(R^Y) \subset \widetilde{f}(R^Y)$ we deduce that $C_p(Y)$ is $\mathcal{P}$-framed in R^Y.

Theorem 2.3. *Let us consider the following restrictions on a Tychonoff space X:*

1) *X is σ-compact,*
2) *X is σ-bounded,*
3) *$C_p(X)$ is $K_{\sigma\delta}$-framed in R^X,*
4) *$C_p(X)$ is K-analytic-framed in R^X,*
5) *X is projectively σ-compact.*

Then we have: 1) $\Rightarrow$ 2) $\Rightarrow$ 3) $\Rightarrow$ 4) $\Rightarrow$ 5).

Proof. 1) $\Rightarrow$ 2). Evident.

2) $\Rightarrow$ 3). It is a result of O. Okunev [18]. For the sake of completness, we give the proof. Let $\overline{R}$ be the natural two-point compactification of R. Let (X_n) be a countable covering of X such that for every n and every $f \in C(X)$ the restriction of f to X_n is bounded. For each n, $p_{X_n}(C(X)) \subset \bigcup_{k\in\omega}[-k,k]^{X_n}$, and $(\bigcup_{k\in\omega}[-k,k]^{X_n}) \times \overline{R}^{X\setminus X_n}$ is a σ-compact subspace of $\overline{R}^X$. Now one easily verifies that $C(X) \subset \bigcap_n((\bigcup_{k\in\omega}[-k,k]^{X_n}) \times \overline{R}^{X\setminus X_n}) \subset R^X$. Hence, $C_p(X)$ is $K_{\sigma\delta}$-framed in R^X.

3) $\Rightarrow$ 4). Evident.

4) $\Rightarrow$ 5). Let X be a Tychonoff space such that $C_p(X)$ is K-analytic-framed in R^X. Let Y be a separable metrizable space, and let f be a continuous mapping of X onto Y. We must show that Y is σ-compact. Applying Lemma 2.2, we see that $C_p(Y)$ is K-analytic-framed in R^Y. Put $R_+ = \{ x \in R \ : \ x > 0 \}$ and $C^+(Y) = C(Y) \bigcap R_+^Y$. Using the exponential function, we see that $C_p^+(Y)$ is K-analytic-framed in R_+^Y. Let A be a K-analytic subspace of R_+^Y such that $C_p^+(Y) \subset A$, and let φ be an usco-compact mapping from ω^ω into $\mathcal{K}(A)$. For every $\sigma \in \omega^\omega$, put $\lambda(\sigma) = \inf \varphi(\{ \tau \in \omega^\omega \ : \ \tau \leq \sigma \})$, where $\tau \leq \sigma$ means $\tau(n) \leq \sigma(n)$ for all $n \in \omega$, and inf is taken with respect to the usual order on R^Y, that is $f \leq g$ if, for every $y \in Y$, $f(y) \leq g(y)$. Since $\{ \tau \in \omega^\omega \ : \ \tau \leq \sigma \}$ is compact, $\varphi(\{ \tau \in \omega^\omega \ : \ \tau \leq \sigma \})$ is compact. Hence, λ maps ω^ω into R_+^Y. Furthermore, the mapping λ has the properties:

a) $\sigma \leq \tau \Rightarrow \lambda(\tau) \leq \lambda(\sigma)$,

b) for every $f \in C_p^+(Y)$, there exists $\sigma \in \omega^\omega$ such that $\lambda(\sigma) \leq f$.

To see b), fix $f \in C_p^+(Y)$. As $f \in A$ and $\varphi(\omega^\omega) = A$; there is a $\sigma \in \omega^\omega$ such that $f \in \varphi(\sigma)$. Hence, $\lambda(\sigma) \leq f$.

Now, embed Y in a metric compactification $\overline{Y}$ and take a metric d on $\overline{Y}$ which induces the topology of $\overline{Y}$. For every $\sigma \in \omega^\omega$, put

$$\theta(\sigma) = \bigcap_{y\in Y} \left(\overline{Y} \setminus B(y, \lambda(\sigma)(y))\right)$$

where $B(y,\rho)$ is the open ball with center y and with radius ρ. Clearly, the mapping θ goes from ω^ω into $\mathcal{K}(\overline{Y} \setminus Y)$, and, applying a), for every $\sigma \leq \tau$ we obtain that $\theta(\sigma) \leq \theta(\tau)$. Let K be a compact subset of $\overline{Y} \setminus Y$. The restriction of the function $d(K, ...)$ to Y belongs to $C_p^+(Y)$. Hence, applying b), we obtain that $K \subset \theta(\sigma)$ for some σ.

For every non-void compact subset K of ω^ω, put $\sigma(K) = \sup\{ \tau \in \omega^\omega \ : \ \tau \in K \}$ (sup is taken with respect to the usual order on ω^ω). Define $\psi(K) = \theta(\sigma(K))$ for every non-void compact subset K of ω^ω and $\psi(\emptyset) = \emptyset$. We obtain a mapping of

$\mathcal{K}(\omega^\omega)$ into $\mathcal{K}(\overline{Y} \setminus Y)$ with the properties:

a') For every pair of compact subsets K, K' of ω^ω such that $K \subset K'$, we have $\psi(K) \subset \psi(K')$,

b') For each compact subset K' of $\overline{Y} \setminus Y$, there exists a compact subset K of ω^ω such that $K' \subset \psi(K)$.

Hence, the space $\overline{Y} \setminus Y$ is Polish by virtue of a result of Christensen [11] (see also J. Saint-Raymond [21]); therefore Y is σ-compact.

Remarks. It is not possible to replace property 4) by the next more general property:

6) $C_p(X)$ is (projectively analytic)-framed in R^X, since, as we have already mentioned, R^X is projectively analytic for every X.

Note also that in general 5) does not imply 2). Indeed, the one-point lindelöfication of an uncountable discrete space is projectively σ-compact (even projectively countable) while obviously it is not σ-bounded (see [3] and [21] for more on that).

Now we can easily prove the result refered to in the title.

Theorem 2.4. *Let X be a regular cosmic space (in particular, a separable metrizable space). Then the following properties are equivalent:*

1) *X is σ-compact,*
2) *$C_p(X)$ is K-analytic-framed in R^X,*
3) *$C_p(X)$ is analytic-framed in R^X.*

Proof. 1) $\Leftrightarrow$ 2) results immediately from Theorem 2.3 and the following result of O.G. Okunev [20]: If a cosmic space is projectively σ-compact, then it is σ-compact.

3) $\Rightarrow$ 2) is evident.

1) $\Rightarrow$ 3). Let X be a σ-compact cosmic space and let (K_n) be a sequence of compact subsets of X covering X. The topological sum $Y = \sum K_n$ is separable, metrizable and σ-compact, hence $C_p(Y)$ is analytic by Christensen's theorem. Let f be the canonical mapping of Y onto X. In virtue of Lemma 2.2, the space R^X is embedded by the dual mapping $\widetilde{f}$ into R^Y, and the closed subset $\widetilde{f}(R^X) \cap C_p(Y)$ of $C_p(Y)$ is analytic and contains $\widetilde{f}(C_p(X))$.

Remarks. In the preceding proof the analytic space $\widetilde{f}(R^X) \cap C_p(Y)$ is a linear topological space, and its fan-tightness is countable (see [3], chap. 2, Sec. 2). Note that 1) and 2) are not equivalent for Tychonoff spaces in general, since for every pseudocompact space X (which need not be σ-compact) the space $C_p(X)$ is obviously K-analytic-framed in R^X.

Corollary 2.5. *If a separable metrizable space X is such that $C_p(X)$ is K-analytic-framed in R^X, then $C_p(X)$ is analytic.*

Corollary 2.6. *Let $\mathcal{J}$ ($\equiv \omega^\omega$) be the space of irrational numbers, then $C_p(\mathcal{J})$ is not K-analytic-framed in $R^{\mathcal{J}}$.*

Theorem 2.7. *Let X be a perfectly normal K-analytic space. Then the following properties are equivalent:*

1) *X is σ-compact,*
2) *$C_p(X)$ is K-analytic-framed in R^X.*

Proof. 1) $\Rightarrow$ 2) follows from Theorem 2.3.

2) $\Rightarrow$ 1). By contradiction. Suppose that X is not σ-compact. Then, by a result of Jayne and Rogers [15], we have a perfect mapping of a closed subset F of X onto ω^ω. Applying Lemmas 2.1 and 2.2, we obtain that $C_p(\omega^\omega)$ is K-analytic-framed in R^{ω^ω}, which it is not.

By an *analytic uniform space* we mean a uniform space which is analytic as a topological space. We call a regular cosmic space Y *regularly subanalytic*, if there exists a regular analytic space Z containing Y as a subspace. Similarly, a uniform space Y will be called *uniformly subanalytic*, if there exists an analytic uniform space Z containing Y as a uniform subspace. The next application of Theorem 2.4 seems to be of special interest.

Theorem 2.8. *Let X be a regular cosmic space. Then $C_p(X)$, with its natural uniformity, is uniformly subanalytic if (and only if) X is σ-compact.*

Proof. For the "only if" part it suffices to refer to Theorem 2.4 and to note that $C_p(X)$ is not only a topological subspace of R^X but also a uniform subspace of it.

Now, let $C_p(X)$, endowed with the natural uniformity, be a uniform subspace of an analytic uniform space Z. Obviously, we may assume that $C_p(X)$ is dense in Z. Since R^X is a complete uniform space, we can extend the identity mapping of $C_p(X)$ to a (uniformly) continuous mapping f of Z into R^X. Then the image $f(Z)$ is an analytic subspace of R^X containing $C_p(X)$. Thus, $C_p(X)$ is analytic-framed in R^X, and, according to Theorem 2.4, X is σ-compact.

Corollary 2.9. *The uniform space $C_p(\omega^\omega)$ is not uniformly subanalytic.*

Taking into account the remark after the proof of Theorem 2.4, we get the following result:

Corollary 2.10. *A regular cosmic space X is σ-compact if and only if $C_p(X)$ can be represented as a linear topological subspace of an anlytic linear topological space.*

3. $C_pC_p(X)$ and K-analytic-frameness

In this section, all spaces are Tychonoff.

Proposition 3.1. *If a space X is not pseudocompact, then $C_pC_p(X)$ is not K-analytic-framed in $R^{C(X)}$.*

Proof. Indeed, there is an infinite discrete family of non-empty open sets in X. Therefore, the discrete space ω is l-embedded into X, that is, there exists a continuous linear mapping

$$\varphi : C_p(\omega) = R^\omega \longrightarrow C_p(X)$$

such that $g = \varphi(g)_{|\omega}$ for every $g \in R^\omega$ (see [5]). Hence, R^ω is a retract of $C_p(X)$ (see [4]). Therefore, for the dual mapping $\widetilde{\varphi} : R^{C(X)} \longrightarrow R^{R^\omega}$, we have

$$\widetilde{\varphi}(C(C_p(X))) = C(R^\omega).$$

Now, ω^ω is closed in the separable metrizable space R^ω; hence, using Lemma 2.1, for the projection p_{ω^ω} of R^{R^ω} into R^{ω^ω} we have $p_{\omega^\omega}(C(R^\omega)) = C(\omega^\omega)$. Hence,

$$p_{\omega^\omega} \circ \widetilde{\varphi}(C(C_p(X))) = C(\omega^\omega).$$

Since $C_p(\omega^\omega)$ is not K-analytic-framed in R^{ω^ω} (by Corollary 2.6), it follows that $C_pC_p(X)$ is not K-analytic-framed in $R^{C(X)}$.

A space X is called *a P-space*, if every G_δ-subset of X is open in X. If M is a subspace of X, we denote by $C_p(M|X)$ the subspace of $C_p(M)$, which is the image of $C_p(X)$ under the restriction mapping.

In the proof of Theorem 3.3, we need the next theorem which is slightly more general than Theorem 1.2.2 in [3] and is proved by the same argument. (See also assertions 9.2, 9.6 and 9.12 in [2].)

Theorem 3.2. *If $C_p(M|X)$ is σ-countably compact, then M is a P-space (in itself), and M is bounded in X.*

Theorem 3.3. *If $C_pC_p(X)$ is K-analytic-framed in $R^{C(X)}$, then every countable subspace M of X is discrete.*

Proof. The restriction mapping of $C_p(X)$ onto $C_p(M|X)$ is surjective and continuous. Therefore, by the dual mapping, $R^{C(M|X)}$ topologically embeds as a closed subspace into $R^{C(X)}$ in such a way, that $C_pC_p(M|X)$ embeds as a subspace into $C_pC_p(X)$.

Let Z be a K-analytic subspace of $R^{C(X)}$ such that $C_pC_p(X) \subset Z$. Then the intersection of Z and $R^{C(M|X)}$ is a closed subspace F of Z such that $C_pC_p(M|X) \subset F$. It follows that F is K-analytic and $C_pC_p(M|X)$ is K-analytic-framed in $R^{C(M|X)}$. Since M has a countable network, it follows that $C_p(M|X)$ also has a countable network (see [3]). Thus, by Theorem 2.4, $C_p(M|X)$ is σ-compact. This implies by Theorem 3.2 that M is a P-space. Since M is countable, it follows that M is discrete.

Corollary 3.4. *If $C_pC_p(X)$ is K-analytic-framed in $R^{C(X)}$, and Y is a subspace of X such that Y is countably compact in X, then Y is finite.*

Corollary 3.5. *If $C_pC_p(X)$ is K-analytic-framed in $R^{C(X)}$, and Y is a dense subspace of X such that Y is countably compact in X, then X is finite.*

Corollary 3.6. *If $C_pC_p(X)$ is K-analytic-framed in $R^{C(X)}$, and X is countably compact, then X is finite.*

Remark. The last result cannot be generalized to the class of pseudocompact spaces. Indeed, in [2], Proposition 9.31, a space X constructed by D. Shakhmatov is described. The space X is infinite, pseudocompact, and the space $C_p(X)$ is σ-bounded (even σ-pseudocompact) and, therefore, projectively σ-compact. By virtue of Okunev's result in [18], the space $C_pC_p(X)$ is K-analytic-framed in $R^{C(X)}$.

Problem 1. Let X be a Tychonoff space such that $C_p(X)$ is regularly subanalytic. Is X then σ-compact? Is the space $C_p(\omega^\omega)$ regularly subanalytic? Is $C_pC_p(\omega^\omega)$ regularly subanalytic?

This problem is a version of Christensen's problem (see [11]), in which no restrictions on separation axioms were explicitly stated. Note that A. Bešlagić [5] and J. Calbrix [6] proved that every regular cosmic space can be embedded in a (Hausdorff) analytic space, while there exists a Hausdorff cosmic space which cannot be embedded in any (Hausdorff) analytic space. It seems that the descriptive structure of $C_p(X)$ tends to become more and more complicated when we take iterated function spaces, so they may be useful in the search for a counterexample to the first part of Problem 1.

Problem 2. Let $C_p(X)$ be K-analytic-framed ($K_{\sigma\delta}$-framed) in R^X. Is X then σ-bounded?

The positive answer to this question would mean that the conditions 2), 3), and 4) (2) and 3)) in Theorem 2.3 are equivalent.

Problem 3. Let X be a Lindelöf space such that $C_p(X)$ is K-analytic. Is X then σ-compact?

Problem 4. Let X be a Lindelöf space such that $C_p(X)$ is K-analytic-framed. Is X then σ-compact?

Problem 5. Characterize X as being analytic by a natural topological property of $C_p(X)$.

That this might be possible is suggested by the following result of Okunev [19]: If X is analytic, and $C_p(X)$ and $C_p(Y)$ are homeomorphic, then Y is also analytic.

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