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ON THE WITTEN-RESHETIKHIN-TURAEV REPRESENTATIONS OF MAPPING CLASS GROUPS

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ABSTRACT. We consider a central extension of the mapping class group of a surface with a collection of framed colored points. The Witten-Reshetikhin-Turaev TQFTs associated to $SU(2)$ and $SO(3)$ induce linear representations of this group. We show that the denominators of matrices which describe these representations over a cyclotomic field can be restricted in many cases. In this way, we give a proof of the known result that if the surface is a torus with no colored points, the representations have finite image.

Recall that an object in a cobordism category of dimension 2+1 is a closed oriented surface Σ, perhaps with some specified further structure. A morphism M from Σ to Σ' is (loosely speaking) a compact oriented 3-manifold perhaps with some specified further structure, called a cobordism, whose boundary is the disjoint union of $-\Sigma$ and Σ'. A morphism M' from Σ' to Σ'' is composed with a morphism from Σ to Σ' by gluing along Σ', inducing any required extra structure from the structures on M and M'. Also, the extra structure on a 3-manifold must induce the extra structure on the boundary. A TQFT in dimension 2+1 is then a functor from such a cobordism category to the category of modules over some ring R. There are further axioms that are generally required [A], [BHMV], [Q]. One usually denotes the module associated to Σ by $V(\Sigma)$, and denotes the homomorphism associated to cobordism M by $Z(M)$. A TQFT yields a representation of (an extension) of the mapping class group of a surface. An extension is needed if there is some choice in the extra data which may be placed on a mapping cylinder.

We will study a version of the Witten-Reshetikhin-Turaev TQFTs [W], [RT] associated to $SU(2)$ and $SO(3)$ constructed by [BHMV]. In particular, we will use the notation where V_p for $p = 2r$ is a TQFT associated to $SU(2)$. Also, V_p for p odd is a TQFT associated to $SO(3)$. We will assume that $p \geq 3$. We will use a variant of the [BHMV] approach obtained by adapting an idea of Walker [Wa]. We replace, in the definition of the cobordism category, a p_1-structure on a surface Σ with a Lagrangian subspace of $H_1(\Sigma, \mathbb{Z})$, and a p_1-structure on a 3-manifold M by an integer and a Lagrangian subspace for the boundary of M. If one does this, one may work over the ring $r_p = \mathbb{Z}[A_p, \frac{1}{p}, u_p]$, where A_p is a primitive $2p$th root of unity,

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and $u_p^2 = A_p^{-6-\frac{p(p+1)}{2}}$. So u_p plays the role of κ_p^3. This makes no real difference to the arguments below but simplifies some expressions. We prefer not to take further roots of unity than necessary. Let λ_p be a root of unity such that $\mathbb{Z}[A_p, u_p] = \mathbb{Z}[\lambda_p]$. Thus, $r_p = \mathbb{Z}[\lambda_p, \frac{1}{p}]$.

An object in the cobordism category $\mathcal{C}_p$ that we consider will be a closed oriented surface (possibly empty) Σ together with a choice of Lagrangian subspace $\ell \subset H_1(\Sigma, \mathbb{Z})$, and also with a (possibly empty) collection of banded points colored with p-colors. A banded point is an oriented arc through the point. Here a p-color is an integer from zero to $p/2-2$ if p is even. If p is odd, a p-color is an even nonnegative integer less than or equal to $p-3$. A morphism from Σ to Σ' is a compact oriented 3-manifold M whose boundary comes equipped with an identification with $-\Sigma \coprod \Sigma'$ where Σ and Σ' are possibly empty objects. Moreover, the three manifold includes a (possibly empty) colored fat graph. Here a fat graph is an oriented surface which deformation retracts to a trivalent graph which meets the boundary along arcs corresponding to the banded points. A colored fat graph is a fat graph whose core is given a p-admissible coloring [BHMV] with matching colors at the banded points. The 3-manifold is also equipped with an integer $n(M)$ called the weight. One defines $n(M' \circ M)$ to be $n(M') + n(M) + \sigma(N(M)_*(\ell), \ell', N(M')^*(\ell''))$. Here σ denotes Wall's non-additivity function [Wall], [Wa] or (minus) the Maslov index, and $N(M)_* : \Lambda(H_1(\Sigma)) \to \Lambda(H_1(\Sigma')$ and $N(M')^* : \Lambda(H_1(\Sigma'')) \to \Lambda(H_1(\Sigma')$ denote Lagrangian actions induced by M and M' [T]. Also ℓ, ℓ', and ℓ'' denote the Lagrangian subspaces with which Σ,Σ', and Σ'' are equipped.

Suppose M is a closed morphism (i.e. a morphism from $\emptyset$ to $\emptyset$) in $\mathcal{C}_p$. M may be described by framed surgery along a framed link L in S^3 which misses the fat graph G. Let $L(\omega_p)$ denote the linear combination of framed links obtained by replacing each component by the skein element ω_p (we use the notation of [BHMV]). Let $e(G)$ be the linear combination of framed links obtained by expanding the graph on the surface. Then define $\langle M \rangle_p \in r_p$ to be $u_p^{n(M)-\mathrm{Sign}(L)} \eta_p$ times the Kauffman bracket polynomial of $e(G) \coprod L(\omega_p)$ evaluated at $A = A_p$. Here $\mathrm{Sign}(L)$ denotes the signature of the linking matrix associated to the framed link L, and $\eta_p \in r_p$ is the element defined at the beginging of §1. The universal construction in [BHMV] then yields a TQFT on $\mathcal{C}_p$ over the ring r_p. Thus, $V_p(\emptyset) = r_p$, and $Z_p(M) : V_p(\emptyset) \to V_p(\emptyset)$ is multiplication by $\langle M \rangle_p$.

Consider a closed connected oriented surface F with a collection of banded colored points. The mapping class group of F consists of isotopy classes of diffeomorphisms f of F to itself preserving the orientation, the coloring, and the framing on the points. Thus, points may be permuted by f if they happen to be colored the same. Now equip F with a Lagrangian subspace ℓ. Thus, (F, ℓ) may be considered as an object Σ of $\mathcal{C}_p$, as long as the colors on F are p-colors. Consider $\mathcal{M}(\Sigma)$, the central extension of the mapping class group of F whose elements are pairs (f, n), where n is an integer. If $(f_1, n_1) : (F, \ell) \to (F, \ell)$, and $(f_2, n_2) : (F, \ell) \to (F, \ell)$, then $(f_2, n_2) \circ (f_1, n_1)$ is defined by $(f_2 \circ f_1, n_1 + n_2 + \sigma(f_2 \circ f_1(\ell), f_2(\ell), \ell))$.

As an object of $\mathcal{C}_p$, Σ is assigned a free r_p-module $V_p(\Sigma)$, which comes equipped with a unimodular Hermitian form $\langle\ ,\ \rangle_\Sigma$. The mapping cylinder of f weighted by an integer n, $C(f, n)$, is a morphism from Σ to itself in $\mathcal{C}_p$. Thus, $C(f, n)$ induces an endomorphism $Z_p(C(f, n))$ of $V_p(\Sigma)$. As $C(f, n)$ is a mapping cylinder, $Z_p(C(f, n))$ is an isometry of $\langle\ ,\ \rangle_\Sigma$. This would not be true for an arbitrary morphism in the cobordism category. In this way, we obtain a representation ρ_p of $\mathcal{M}(\Sigma)$ into the group of isometries of $V_p(\Sigma)$.

Let H be a handlebody weighted zero with boundary Σ such that the kernel of the map on the first homology induced by the inclusion is the Lagrangian subspace with which Σ is equipped. Let $\mathcal{G}$ be a fat graph in H with boundary the framed points, such that H deformation retracts to $\mathcal{G}$. The p-admissible colorings β_i of $\mathcal{G}$ which extend the given coloring on the boundary describe a basis $\mathcal{B}$ for $V_p(\Sigma)$, [BHMV, 4.11]. This basis is orthogonal with respect to the unimodular form defined on $V_p(\Sigma)$. In the case of a torus with no colored points , $\mathcal{B}$ is orthonormal. We have two overlapping results.

Theorem 1. *Let Σ have genus g, and p be either an odd prime, or twice an odd prime. With respect to $\mathcal{B}$, the matrices for ρ_p have entries lying in $\left(\frac{1}{p}\right)^{\left[\frac{g+1}{2}\right]}\mathbb{Z}[\lambda_p]$.*

Here [ ] denotes the greatest integer function. In the lemmas below, we specify somewhat stronger and more general restrictions.

Theorem 2. *Let Σ be a torus without any colored points and let p be even. With respect to $\mathcal{B}$, the matrices for ρ_p have entries lying in $\frac{1}{p}\mathbb{Z}[\lambda_p]$.*

We find these theorems intriguing as one would expect in general that denominators should increase when multiplying matrices.

We obtain the following Corollary which M. Kontsevich informs us is known. Kontsevich observed it in 1988. It is also known in the conformal field theory community. We thank M. Kontsevich and G. Masbaum for pointing out an error in a previous version of this paper. We also thank the referee for useful comments.

Corollary. *If Σ is a torus without any colored points, then ρ_p has finite image in the isometries of $V_p(\Sigma)$.*

Proof. Using [BHMV, 1.5], one may deduce the result for p odd from the result for p even. So we assume p is even. $\mathbb{Z}[\lambda_p]$ maps to a discrete subgroup of $\mathbb{C}^{r_2(\mathbb{Z}[\lambda_p])}$ under the canonical embedding [S, 4.2] of $\mathbb{Z}[\lambda_p]$. Here $r_2(\mathbb{Z}[\lambda_p])$ is the number of pairs of complex conjugate embeddings of $\mathbb{Z}[\lambda_p]$ in $\mathbb{C}$. So $\frac{1}{p}\mathbb{Z}[\lambda_p]$ also maps to a discrete subgroup. $Z_p(f,n)$ is represented by matrices whose entries lie in $\frac{1}{p}\mathbb{Z}[\lambda_p]$ and it is an isometry of the form $\langle\ ,\ \rangle_\Sigma$ which is positive definite under each complex embedding. It follows that each column in one of these matrices under each complex embedding must have specified norm with respect to $\langle\ ,\ \rangle_\Sigma$. There can only be a finite number of vectors which meet these conditions. Thus, there are only a finite number of possible matrices which can describe isometries in the image of this representation. □

Funar [F] has proved that the above corollary is false for surfaces of higher genus. The reason the above proof does not extend to higher genus or colored points is that [BHMV] the form on $\langle\ ,\ \rangle_\Sigma$, is not, in general, positive definite under all complex embeddings.

The figure eight knot is a genus one fibered knot. Thus, zero framed surgery along this knot is a fiber bundle over a circle with fiber a torus. In [G1], [G2] we studied the map, $Z_p(F8)$, induced by the monodromy of this bundle for $p \leq 20$. We found that in every case $Z_p(F8)$ had finite order, even though the monodromy itself is nonperiodic. Here is the list of the periods for p from 3 to 20: 1, 1, 10, 6, 4, 3, 12, 30, 5, 12, 14, 12, 20, 12, 18, 12, 9, 60. We conjectured that $Z_p(F8)$ would have finite period for all p. Our attempts to prove it lead to this paper. Of course, it follows from the Corollary whose truth was unknown to us.

We mention some other related results. Wright has studied the projective version of these representations for $p = 8$ [Wr1] (with no colored points). One result of her work is that the image of this representation is finite. In [Wr2], Wright studies the $p = 12$ case for genus two surfaces (without colored points) and we can again conclude the image of this representation is finite.

In §1, we prove Theorem 1. This result follows easily from the integrality of an associated quantum invariant of closed manifolds with links when p is either an odd prime, or twice an odd prime. For manifolds without links this is due to H. Murakami [M1], [M2], although he states the result only for rational homology spheres. Masbaum and Roberts [MR1] have given an elegant proof of this result including the case where the manifolds contain colored links.

In §2, we prove Theorem 2, by studying Jeffrey's explicit formula [J] for a related representation on $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in SL(2,\mathbb{Z})$, in terms of a, b, c, and d.

§1. Proof of Theorem 1

Recall $\eta_p = u_p A_p^3 \frac{A_p^2 - A_p^{-2}}{p} G_p(A_p)$, where $G_p(A_p) = \frac{1}{2}\sum_{m=1}^{2p}(-A_p)^{-m^2} \in \mathbb{Z}[A_p]$ [BHMV], [MR1]. Recall that our u_p plays the role of κ_p^3. So $\eta_p \in \frac{1}{p}\mathbb{Z}[\lambda_p]$. We also note that $\eta_p^2 = -\frac{(A_p^2 - A_p^{-2})^2}{p} \in \frac{1}{p}\mathbb{Z}[\lambda_p]$. Thus, $\eta_p^g \in \left(\frac{1}{p}\right)^{\left[\frac{g+1}{2}\right]}\mathbb{Z}[\lambda_p]$.

Let r be an odd prime. Let p be either r or $2r$. Let M be a closed morphism in $\mathcal{C}_p$. As in [MR1], define $\mathcal{I}_p(M) = \eta_p^{-1} Z_p(M)$. If $n(M) = 0$, then by [MR1] $\mathcal{I}_p(M) \in \mathbb{Z}[A_p]$. This is stated for the case that the colored fat graph is a colored framed link. However, it follows immediately for colored fat graphs as well since we may expand a graph into linear combination of links over $\mathbb{Z}[A_p]$, as the idempotents are defined over $\mathbb{Z}[A_p]$ [MR1]. Changing $n(M)$ multiplies $I_p(M)$ by a power of u_p. So if we allow $n(M)$ to be nonzero, we have $\mathcal{I}_p(M) \in \mathbb{Z}[\lambda_p]$. Thus, $\langle M \rangle_p = \eta_p I_p(M) \in \eta_p \mathbb{Z}[\lambda_p]$.

Lemma 1. *Suppose Σ is a connected nonempty object in $\mathcal{C}_p$ of genus g. If N is endomorphism in $\mathcal{C}_p$ from Σ itself Σ, the matrix for $Z_p(N)$ with respect to $\mathcal{B}$ has entries in ${\eta_p}^g\mathbb{Z}[\lambda_p] \subset \left(\frac{1}{p}\right)^{\left[\frac{g+1}{2}\right]}\mathbb{Z}[\lambda_p]$.*

Proof. By [BHMV, 4.11], each $\langle \beta_i, \beta_j \rangle_\Sigma$ is $\delta_i^j \eta_p^{1-g}$ times a unit in $\mathbb{Z}[\lambda_p]$. Perhaps the special case where Σ is a 2-sphere with no colored points should be discussed separately. In this case $\mathcal{G}$ is empty, there is only one coloring β, and $\langle \beta, \beta \rangle_\Sigma = \eta_p$.

Let s_i denote $(\langle \beta_i, \beta_i \rangle_\Sigma)^{-1}$. Thus, $s_i \in \eta_p^{g-1}\mathbb{Z}[\lambda_p]$. Then

$$Z_p(N)(\beta_i) = \sum_j s_j \langle (H, \beta_i) \cup N \cup (-H, \beta_j) \rangle_p (\beta_j).$$

It follows that the matrix for $Z_p(N)$ with respect to $\mathcal{B}$ has entries in $\eta_p^g\mathbb{Z}[\lambda_p]$. ☐

If Σ' is another connected object in $\mathcal{C}_p$, let $\mathcal{B}'$ be a basis for $V_p(\Sigma)$ described as above. The same proof also proves:

Lemma 2. *For every morphism N from Σ' to Σ, the matrix for $Z_p(N)$ with respect to the bases $\mathcal{B}'$ and $\mathcal{B}$ has entries in $\eta_p^g\mathbb{Z}[\lambda_p]$.*

Let g' denote the genus for Σ'. We remark that the roles of the genus of g and g' are not symmetric. If $\tilde{N}$ from Σ to Σ' is the morphism obtained by reversing the orientation on N and the choice of incoming and outgoing manifolds, then $Z_p(\tilde{N})$ will, with respect to above bases, have entries in $\eta_p^{g'}\mathbb{Z}[\lambda_p]$, but $Z_p(N)$ may not.

§2. Proof of Theorem 2

Assume $p = 2r$. Let Σ be the boundary of a fixed solid handlebody H of genus one in S^3. We assign it the Lagrangian subspace spanned by the (homology class of) a meridian. We may let $\mathcal{G}$ be a core of the solid torus fattened with framing zero. For $0 \le l \le r-2$ let b_l denote H with $\mathcal{G}$ colored l. Let β_l denote the element of $V_p(\Sigma)$ represented by b_l. Then $\mathcal{B} = \{\beta_l\}_{0\le l\le r-2}$.

We identify the mapping class group of a torus without colored points with $SL(2,\mathbb{Z})$ by looking at the map on homology with respect to a meridian, longitude basis. Let $S = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, $T = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$.

From the framing relation, one has $\rho_p(T,0)(\beta_l) = \mu_l\beta_l$ where $\mu_l = (-A_p)^{l^2+2l} = (-A_p)^{(l+1)^2-1}$. Also $\langle \rho_p(C(S,0))(\beta_l), \beta_j\rangle_\Sigma$ is $\langle\ \rangle_p$ of S^3 weighted zero containing a zero framed Hopf link with components colored l and j, which we denote by $h_{l,j}$. Since $\mathcal{B}$ is orthonormal, we have $\rho_p(C(S,0))$ with respect to the basis $\mathcal{B}$ is given by the matrix $h_{l,j}$. According to Morton and Strickland [MS], the bracket evaluation of this Hopf link is $(-1)^{l+j}[(l+1)(j+1)]$. Here $[n]$ denotes $\frac{A_p^{2n} - A_p^{-2n}}{A_p^2 - A_p^{-2}}$. We remark that the derivation in [MS] may be mimicked completely on the skein theory level without reference to representation theory. Thus, $h_{l,j} = \eta_p(-1)^{l+j}[(l+1)(j+1)]$. Also, one has that $\rho_p(f,n) = (u_p)^n\rho_p(f,0)$.

It is now convenient to consider the basis $\mathbb{B}$ with elements: $\beta'_l = (-1)^{l-1}b_{l-1}$ for $1 \le l \le r-1$. With respect to $\mathbb{B}$, $\rho_p(T,0)$ is given by $\hat{S} = \delta_l^j(-A_p)^{l^2-1}$, and $\rho_p(S,0)$ is given by $\hat{S} = \eta_p[lj]$.

We adopt the notation $\zeta_t = e^{\frac{2\pi i}{t}}$. Consider the embedding [MR2, note p.134] $\psi : \mathbb{Q}[\lambda_p] \to \mathbb{C}$ which sends A_p to $-\alpha$ where α denotes ζ_{2p} and sends u_p to $\zeta_8^3\alpha^{-3}$. Then $\psi(\eta_p) = -\frac{a^2-a^{-2}}{\sqrt{2r}}i$. Also, $\psi(\hat{T}) = \delta_l^j\alpha^{l^2-1}$, and $\psi(\hat{S}) = \frac{-i}{\sqrt{2r}}\left(\alpha^{lj} - \alpha^{-lj}\right)$. Let $s = 4r$ if r is even and $s = 8r$ if r is odd. Then $\mathbb{Z}[\zeta_s] = \psi(\mathbb{Z}[\lambda_p])$.

Jeffrey [J] studies a matrix representation $\mathcal{R}_p$ of $SL(2,\mathbb{Z})$, coming from conformal field theory, which Witten uses in [W]. This is defined by $\mathcal{R}_p(S) = \psi(\hat{S})$, and $\mathcal{R}_p(T) = \alpha\zeta_8^{-1}\psi(\hat{T}) = \delta_l^j\zeta_8\alpha^{l^2}$. Since $\alpha^{-1}\zeta_8 \in \mathbb{Z}[\zeta_s]$, the theorem will be proved if we can show that $p\mathcal{R}_p\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \prec \mathbb{Z}[\zeta_s]$. Here we let the symbol $\prec$ mean "has all entries lying in." Jeffery's expression for $\mathcal{R}\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ [J, 2.7] is

$$\mathcal{R}_p\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)_{j,l} = -\frac{i\zeta_8^{-\Phi(U)}\operatorname{sign}(c)}{\sqrt{p|c|}}\zeta_{4rc}^{dl^2} \sum_{\substack{\gamma \pmod{2rc} \\ \gamma = j \pmod{2r}}} \zeta_{4rc}^{a\gamma^2}\left(\zeta_{2rc}^{\gamma l} - \zeta_{2rc}^{-\gamma l}\right),$$

Φ denotes the Rademacher Φ-function with values in $\mathbb{Z}$. Thus, one has that

$$\mathcal{R}_p\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) \prec \frac{1}{pc}\mathbb{Z}[\zeta_{s|c|}].$$

To see this, we need to observe that $\sqrt{t} \in \mathbb{Z}[\zeta_{4t}]$ using a quadratic Gauss sum. We also have

$$\mathcal{R}_p(\begin{pmatrix} c & d \\ -a & -b \end{pmatrix}) \prec \frac{1}{a\sqrt{p}}\mathbb{Z}[\zeta_{s|a|}] \quad \text{and} \quad \mathcal{R}_p(\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}) \prec \frac{1}{\sqrt{p}}\mathbb{Z}[\zeta_s].$$

As $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)\left(\begin{smallmatrix} c & d \\ -a & -b \end{smallmatrix}\right)$, we have

$$\mathcal{R}_p(\begin{pmatrix} a & b \\ c & d \end{pmatrix}) \prec \frac{1}{pa}\mathbb{Z}[\zeta_{s|a|}].$$

As a,c are relatively prime, $\mathbb{Q}[\zeta_{s|a|}] \cap \mathbb{Q}[\zeta_{s|c|}] = \mathbb{Q}[\zeta_s]$. So $\mathcal{R}_p\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \prec \mathbb{Q}[\zeta_s]$. So $pa\mathcal{R}_p\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \prec \mathbb{Z}[\zeta_s]$, and $pc\ \mathcal{R}_p\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \prec \mathbb{Z}[\zeta_s]$. Again, since a,c are relatively prime, $p\mathcal{R}_p\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \prec \mathbb{Z}[\zeta_s]$.

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