ON TIGHTNESS AND DEPTH
IN SUPERATOMIC BOOLEAN ALGEBRAS

SAHARON SHELAH AND OTMAR SPINAS

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Abstract. We introduce a large cardinal property which is consistent with $L$ and show that for every superatomic Boolean algebra $B$ and every cardinal $\lambda$ with the large cardinal property, if tightness$^+(B) \geq \lambda^+$, then depth$^+(B) \geq \lambda$. This improves a theorem of Dow and Monk.

In [DM, Theorem C], Dow and Monk have shown that if $\lambda$ is a Ramsey cardinal (see [J, p.328]), then every superatomic Boolean algebra with tightness at least $\lambda^+$ has depth at least $\lambda$. Recall that a Boolean algebra $B$ is superatomic iff every homomorphic image of $B$ is atomic. The depth of $B$ is the supremum of all cardinals $\lambda$ such that there is a sequence $(b_\alpha : \alpha < \lambda)$ in $B$ with $b_\beta < b_\alpha$ for all $\alpha < \beta < \lambda$ (a well-ordered chain of length $\lambda$). Then depth$^+$ of $B$ is the first cardinal $\lambda$ such that there is no well-ordered chain of length $\lambda$ in $B$. The tightness of $B$ is the supremum of all cardinals $\lambda$ such that $B$ has a free sequence of length $\lambda$, where a sequence $(b_\alpha : \alpha < \lambda)$ is called free provided that if $\Gamma$ and $\Delta$ are finite subsets of $\lambda$ such that $\alpha < \beta$ for all $\alpha \in \Gamma$ and $\beta \in \Delta$, then

$$\bigcap_{\alpha \in \Gamma} -b_\alpha \cap \bigcap_{\beta \in \Delta} b_\beta \neq 0$$

By tightness$^+(B)$ we denote the first cardinal $\lambda$ for which there is no free sequence of length $\lambda$ in $B$.

For $b \in B$ we sometimes write $b^0$ for $-b$ and $b^1$ for $b$.

We improve Theorem C from [DM] in two directions. We introduce a large cardinal property which is much weaker than Ramseyness and even consistent with $L$ (the constructible universe) and show that in Theorem C from [DM] it suffices to assume that $\lambda$ has this property. Moreover we show that it suffices to assume tightness$^+(B) \geq \lambda^+$ instead of tightness$(B) \geq \lambda^+$ to conclude that depth$(B) \geq \lambda$. In particular we get:

Theorem 1. Suppose that $\Theta^*$ exists. Let $B$ be a superatomic Boolean algebra in the constructible universe $L$, and let $\lambda$ be an uncountable cardinal in $V$. Then in $L$ it is true that tightness$^+(B) \geq \lambda^+$ implies that depth$^+(B) \geq \lambda$.

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Lemma 3. Assume we have that $(R, \lambda, \gamma)$ is a limit cardinal in $L$; hence it suffices to show that, in $L$, $\text{depth}(B) \geq \kappa$ for all cardinals $\kappa < \lambda$. As was the case with the proof of Theorem C of [DM], we can’t show that, under the assumptions of Theorem 1, $\text{depth}(B) = \lambda$ is attained, i.e. that there is a well-ordered chain of length $\lambda$.

For the proof we consider the following large cardinal property:

Definition 2. Let $\lambda, \kappa, \theta$ be infinite cardinals, and let $\gamma$ be an ordinal. The relation $R_\gamma(\lambda, \kappa, \theta)$ is defined as follows:

For every $c : [\lambda]^{<\omega} \to \theta$ there exists $A \subseteq \lambda$ of order-type $\gamma$, such that for every $u \in [A]^{<\omega}$ there exists $B \subseteq \lambda$ of order-type $\kappa$ such that $\forall w \in [B]^{\omega} \ c(w) = c(u)$.

Lemma 3. Assume $R_\gamma(\lambda, \kappa, \theta)$, where $\gamma$ is a limit ordinal. For every $c : [\lambda]^{<\omega} \to \theta$ there exists $A \subseteq \lambda$ as in the definition of $R_\gamma(\lambda, \kappa, \theta)$ such that additionally $c([A]^n)$ is constant for every $n < \omega$.

Proof. Define $c'$ on $[\lambda]^{<\omega}$ by

$$c'(\beta_0, \ldots, \beta_{n-1}) = \{(v, c(\beta_i : i \in v)) : v \subseteq n\}.$$ 

As $\theta$ is infinite we can easily code the values of $c'$ as ordinals in $\theta$ and therefore apply $R_\gamma(\lambda, \kappa, \theta)$ to it. We get $A \subseteq \lambda$ of order-type $\gamma$. We shall prove that $c([A]^n)$ is constant, for every $n < \omega$. Fix $w_1, w_2 \in [A]^n$. Since $\gamma$ is a limit, without loss of generality we may assume that $\max(w_1) < \min(w_2)$. Let $w = w_1 \cup w_2$. By Definition 2 there exists $B \subseteq \lambda$, o.t.$B = \kappa$, such that $c([B]^{2n})$ is constant with value $c'(w)$. Let $(\beta_{\nu} : \nu < \kappa)$ be the increasing enumeration of $B$. We have

$$c'(\beta_0, \ldots, \beta_{2n-1}) = c'(\beta_n, \ldots, \beta_{3n-1}).$$

By the definition of $c'$ we get

$$c(\beta_0, \ldots, \beta_{n-1}) = c(\beta_n, \ldots, \beta_{2n-1}) =: c_0.$$ 

This information is coded in $c'(\beta_0, \ldots, \beta_{2n-1})$, i.e.

$$\{(0, \ldots, n-1), c_0\}, \{(n, \ldots, 2n-1), c_0\} \in c'(\beta_0, \ldots, \beta_{2n-1}).$$

As $c'(\beta_0, \ldots, \beta_{2n-1}) = c'(w)$, we conclude $c(w_1) = c(w_2) = c_0$. 

Theorem 4. Assume $R_\gamma(\lambda, \kappa, \omega)$, where $\gamma$ is a limit ordinal. If $B$ is a Boolean algebra and $(a_\nu : \nu < \lambda)$ is a sequence in $B$, then one of the following holds:

(a) there exists $A \subseteq \lambda$, o.t.$(A) = \gamma$, such that $(a_\nu : \nu \in A)$ is independent;

(b) there exist $n < \omega$ and a strictly increasing sequence $(\beta_\nu : \nu < \kappa)$ in $\lambda$ such that, letting

$$b_\nu = \bigcup_{k<n} \bigcap_{l<n} a_{\beta_n^2 \nu + n k + l},$$

we have that $(b_\nu : \nu < \kappa)$ is constant;

(c) there exists a strictly decreasing sequence in $B$ of length $\kappa$.

Corollary 5. Assume $R_\gamma(\lambda, \kappa, \omega)$, where $\gamma$ is a limit ordinal. If $B$ is a superatomic Boolean algebra, then tightness$^+(B) > \lambda$ implies Depth$^+(B) > \kappa$.
Proof of Corollary 5. Let \((a_\nu : \nu < \lambda)\) be a free sequence in \(B\). As a superatomic Boolean algebra does not have an infinite independent subset, (a) is impossible. Suppose (b) were true. Define \(b_\nu\) as in (\#). Clearly we have
\[-b_\nu \geq \bigcap_{k,l<n} a_{\nu+2k+2l+1}^0,\]
and
\[b_\nu \geq \bigcap_{k,l<n} a_{\alpha_n^2
u+2k+2l+1}.\]
Hence if \(\nu < \mu\) and \(b_\nu = b_\mu\), we obtain
\[0 = -b_\nu \cap b_\mu \geq \bigcap_{k,l<n} a_{\nu+2k+2l+1}^0 \cap \bigcap_{k,l<n} a_{\mu+2k+2l+1}.\]
This contradicts freeness of \((a_\nu : \nu < \kappa)\). We conclude that (c) must hold. \(\square\)

Proof of Theorem 4. Define \(c : [\lambda]^{<\omega} \rightarrow [\omega^2]^{<\omega}\) by
\[c\{\beta_0 < \cdots < \beta_{n-1}\} = \{\eta \in \omega^2 : \bigcap_{i<n} a_{\eta_i}^\eta = 0\}.
\]
Note that \(c\{\beta_0 < \cdots < \beta_{n-1}\} = c\{a_0 < \cdots < a_{n-1}\}\) implies that \(\{a_\beta_0, \ldots, a_\beta_{n-1}\}\) and \(\{a_{\alpha_0}, \ldots, a_{\alpha_{n-1}}\}\) have the same quantifier-free diagram, i.e., for every quantifier-free formula \(\phi(x_0, \ldots, x_{n-1})\) in the language of Boolean algebra,
\[B \models \phi[a_{\beta_0}, \ldots, a_{\beta_{n-1}}] \iff B \models \phi[a_{\alpha_0}, \ldots, a_{\alpha_{n-1}}].\]

Let \(A \subseteq \lambda\) be as guaranteed for \(c\) by \(R_c(\lambda, \kappa, \omega)\). By Lemma 3 we may assume that \(c[A]^\kappa\) is constant, for every \(n < \omega\).

If \((a_\alpha : \alpha \in A\) is independent, we are done. Therefore we may assume that this is false. For \(m < \omega\) define
\[\Gamma_m = \{\eta \in \omega^2 : \exists (\beta_0 < \cdots < \beta_{m-1}) \subseteq A \bigcap_{i<m} a_{\eta_i}^\eta = 0\}\]
By assumption, in the definition of \(\Gamma_m\) the existential quantifier can be replaced by a universal one to give the same set. There exists \(m < \omega\) such that \(\Gamma_m \neq \emptyset\). Define
\[\Gamma'_m = \{\eta \in \Gamma_m : \text{no proper subsequence of } \eta \text{ belongs to } \bigcup_{k<m} \Gamma_k\}.\]

By Kruskal’s Theorem [K], we have that \(\bigcup_{m<\omega} \Gamma'_m\) is finite. Let \(n^*\) be minimal such that \(\bigcup_{m<\omega} \Gamma'_m = \bigcup_{m<n^*} \Gamma'_m\). Then clearly we have that, for every \(m < \omega\) and \(\eta \in \Gamma_m, \eta\) has a subsequence in \(\bigcup_{k<n^*} \Gamma_k\). Let \(m^* = (n^*)^2\), and let
\[\tau(x_0, \ldots, x_{m^*+1}) = \bigcup_{l<n^*} \bigcap_{k<n^*} x_{n^*l+k}.
\]

Claim 1. If \(\eta \in m^*[2], t \in \{0,1\}, \text{ and } \tau[\eta(0), \ldots, \eta(m^*-1)] = t \in \text{ in the Boolean algebra } \{0,1\}, \text{ then } |\{i < m^* : \eta(i) = t\}| \geq n^*\). \(\square\)

Let \((b_\nu : \nu < \gamma)\) be the strictly increasing enumeration of \(A\), and define
\[b_\nu = \tau[a_{b_{\gamma^0,\nu}}, a_{b_{\gamma^1,\nu+1}}, \ldots, a_{b_{\gamma^{m^*},\nu+m^*}}],\]
for every \(\nu < \gamma\), where the evaluation of \(\tau\) takes place in \(B\), of course. It is easy to see that the sequence \((b_\nu : \nu < \gamma)\) inherits from \((a_\beta_\nu : \nu < \gamma)\) the property that any two finite subsequences of same length have the same quantifier-free diagram.
Claim 2. If \( \eta \in \Gamma_n \), then \( \bigcap_{i<n} b_{q(i)}^i = 0 \).

Proof of Claim 2. Otherwise there exists an ultrafilter \( D \) on \( B \) such that \( \bigcap_{i<n} b_{q(i)}^i \in D \). Define \( \zeta \in \Gamma_{nm^*} \) by \( \zeta(i) = 1 \) iff \( a_{\beta_i} \in D \). Then \( \bigcap_{i<n} a_{\beta_i}^{q(i)} \in D \), and hence \( \zeta \notin \Gamma_{nm^*} \). Let \( h : B \rightarrow B/D = \{0,1\} \) be the canonical homomorphism induced by \( D \). We calculate
\[
1 = h(\bigcap_{i<n} b_{q(i)}^i) = \bigcap_{i<n} h(b_i)^{q(i)} = \bigcap_{i<n} \tau[h(a_{\beta_{(i+1)}}), \ldots, h(a_{\beta_{m^*(i+1)}})]^{q(i)}
\]
We conclude that \( \tau[\zeta(m^*i), \ldots, \zeta(m^*(i+1) - 1)] = \eta(i) \), for all \( i < n \), and hence by Claim 1 we can choose \( j_i \in [m^*i, m^*(i+1)] \) such that \( \zeta(j_i) = \eta(i) \). Clearly \( i_0 < i_1 \) implies that \( j_{i_0} < j_{i_1} \). But this implies \( \zeta \in \Gamma_{nm^*} \), a contradiction.

Claim 3. If \( t < \omega, \eta \in \Gamma_{n} \), \( 0 = k_0 < k_1 < \cdots < k_t = n \), and \( \eta \models [k_i, k_{i+1}] \) is constant for all \( i < t \), and if \( \rho \in \mathcal{P} \) is defined by \( \rho(i) = \eta(k_i) \), then \( \bigcap_{i<t} b_{q(i)}^i = 0 \).

Proof of Claim 3. Wlog we may assume that \( \eta \in \Gamma'_n \) for some \( n < n^* \). Indeed, otherwise we can find \( m < n^* \), \( \eta' \in \Gamma'_n \) and some increasing \( h : m \rightarrow n \) such that \( \eta'(i) = \eta(h(i)) \), for all \( i < m \). Then \( \{h^{-1}[k_i, k_{i+1}] : i < t \} \) equals \( \{l_i, l_{i+1} : i < s \} \) for some \( l_0 = 0 < l_1 < \cdots < l_{s-1} = m \). Note that \( \eta' \models [l_i, l_{i+1}] \) is constant, and letting \( \rho' \in \mathcal{P} \) be defined by \( \rho'(i) = \eta'(l_i) \), we have \( \rho'(i) = \rho(h(i)) \). Hence \( \bigcap_{i<s} b_{q(i)}^i = 0 \) implies \( \bigcap_{i<t} b_{q(i)}^i = 0 \).

Therefore we assume \( \eta \in \Gamma'_n \), for some \( n < n^* \). Suppose we had \( \bigcap_{i<t} b_{q(i)}^i > 0 \). Let \( D \) be an ultrafilter on \( B \) containing \( \bigcap_{i<t} b_{q(i)}^i \). Let \( h : B \rightarrow B/D \) be the canonical homomorphism. Define \( \zeta \in \Gamma_{nm^*} \) such that \( \zeta(i) = 1 \) iff \( a_i \in D \). Hence \( \zeta \notin \Gamma_{nm^*} \). We get
\[
h(\bigcap_{i<t} b_{q(i)}^i) = \bigcap_{i<t} \tau[\zeta(im^*), \ldots, \zeta((i+1)m^* - 1)]^{q(i)} = 1.
\]
Hence by Claim 1,
\[
\forall i < t \exists a_i \in \{[im^*, \ldots, (i+1)m^* - 1]\}^n \forall j \in a_i \quad \zeta(j) = \rho(i).
\]
Define \( \mu \in \Gamma_{nm^*} \) by \( \mu(j) = \rho(i) \) iff \( j \in [im^*, (i+1)n^* \). Then \( \mu \) is a subsequence of \( \zeta \) and therefore \( \mu \notin \Gamma_{nm^*} \). But also \( \eta \) is a subsequence of \( \mu \), and hence \( \eta \notin \Gamma_{n} \), a contradiction.

Claim 4. Suppose \( \rho \in \mathcal{P} \) and \( \bigcap_{i<t} b_{q(i)}^i = 0 \). Let \( \zeta \in \mathcal{P} \) be defined such that \( \zeta(m^*i) = \rho(i) \) and \( \zeta(m^*(i+1)) \) is constant for every \( i < t \). Then \( \zeta \in \Gamma_{m^*t} \).

Proof of Claim 4. Otherwise, \( \bigcap_{i<m^*} a_{q(i)}^i > 0 \). Let \( D \) be an ultrafilter containing \( \bigcap_{i<m^*} a_{q(i)}^i \). Let \( h : B \rightarrow B/D \) be the canonical homomorphism. We have
\[
h(\bigcap_{i<t} b_{q(i)}^i) = \bigcap_{i<t} \tau[\zeta(m^*i), \ldots, \zeta(m^*(i+1) - 1)]^{q(i)} = \bigcap_{i<t} \tau[\rho(i), \ldots, \rho(i)]^{q(i)} = 1.
\]
This is a contradiction.

Since we assume that \( (a_\alpha : \alpha \in A) \) is not independent, by Claim 2 we can find \( k^* < \omega \) minimal such that for some \( \rho^* \in \mathcal{P} \), \( \bigcap_{i<k^*} b_{q(i)}^i = 0 \). Note that \( \rho^*(i+1) \neq \rho^*(i) \) for every \( i < k^* - 1 \). Indeed, otherwise let \( \zeta \in m^*k^* \) be defined
as in Claim 4. So \( \zeta \in \Gamma_{m^*k^*} \). By Claim 3 we can find \( \rho' \) of shorter length than \( \rho^* \) such that \( \cap_{i \in c} b_i^{\rho(i)} = 0 \), contradicting the minimal choice of \( k^* \).

Suppose first that \( k^* = 1 \). We conclude that \((b_\nu : \nu < \gamma)\) either is constantly 1 or 0. The main part of the definition of \( R_\gamma(\lambda, \kappa, \omega) \) then gives a sequence of length \( \kappa \) as desired in (b) of Theorem 4.

Second suppose \( k^* > 1 \). If \( \cap_{i < k^* - 2} b_i^{\rho'(i)} \cap b_{k^* - 2}^{\rho'(i)} = 0 \) and \( \cap_{i < k^* - 2} b_i^{\rho'(i)} \cap b_{k^* - 1}^{\rho'(i)} = 0 \), then \( \cap_{i < k^* - 2} b_i^{\rho'(i)} \cap b_{k^* - 2}^{\rho'(i)} \cap b_{k^* - 1}^{\rho'(i)} = \cap_{i < k^* - 2} b_i^{\rho'(i)} \cap b_{k^* - 1}^{\rho'(i)} \), and an application of the main part of the definition of \( R_\gamma(\lambda, \kappa, \omega) \) gives a sequence as desired in (b).

Otherwise, if \( \rho'(k^* - 2) = 1 \) and \( \rho'(k^* - 1) = 0 \), then

\[
\cap_{i < k^* - 2} b_i^{\rho'(i)} \cap b_{k^* - 2}^{\rho'(i)} < \cap_{i < k^* - 2} b_i^{\rho'(i)} \cap b_{k^* - 1}^{\rho'(i)},
\]

and applying the definition gives (c). A similar argument applies if \( \rho'(k^* - 2) = 0 \) and \( \rho'(k^* - 1) = 1 \).

**Theorem 6.** Assume the following:

1. \( \mathcal{B} \) exists,
2. \( V \models \lambda \) is an uncountable cardinal,
3. \( \kappa, \theta < \lambda \), and \( L \models \kappa \) is a regular cardinal.

Then \( L \models R_\omega(\lambda, \kappa, \theta) \).

**Proof.** Let \( c : [\lambda]^{<\omega} \to \theta, c \in L \), be arbitrary.

Let \( Y \) be the set of all \( w \in [\lambda]^{<\omega} \) such that for every \( n \leq |w| \) and \( u \in [w]^n \) there exists \( B \subseteq \lambda \) of order-type \( \kappa \) in \( L \) such that \( \forall v \in [B]^n \ c(u) = c(v) \). Clearly \( Y \in L \).

**Claim 1.** If in \( V \) there exists \( A \subseteq [\lambda]^{<\omega} \subseteq Y \), then \( L \models R_\omega(\lambda, \kappa, \theta) \).

**Proof of Claim 1.** Let \( T \) be the set of all one-to-one sequences \( \rho \in <\omega \lambda \) with \( \text{ran}(\rho) \in Y \), ordered by extension. Then \( T \) is a tree and by assumption, \( T \) has an \( \omega \)-branch in \( V \). By absoluteness, \( T \) has an \( \omega \)-branch \( b \) in \( L \). Then \( \text{ran}(b) \) (or some subset) witnesses \( L \models R_\omega(\lambda, \kappa, \theta) \).

Let \((i_\nu : \nu < \lambda^+)\) be the increasing enumeration of the club of indiscernibles of \( L_{\lambda^+} \). Then \((i_\nu : \nu < \lambda)\) is the club of indiscernibles of \( L_{\lambda} \). As \( c \in L_{\lambda^+} \) there exist ordinals \( \xi_0 < \cdots < \xi_p < \xi < \xi_{p+1} < \lambda^+ \) and a Skolem term \( t_c \) such that

\[ L_{\lambda^+} \models c = t_c[i_{\xi_0}, \ldots, i_{\xi_{p+1}}]. \]

By indiscernibility and remarkability (see [J, p.345]) it easily follows that if \( \alpha^* = \max\{\xi_p - 1, \theta\} + 1 \), then \( c[\{i_\nu : \alpha^* \leq \nu < \lambda\}]^n \) is constant for every \( n \leq \omega \), say with value \( c_n \). Let \( n < \omega \) be arbitrary. Let \( \delta_0 = i_{\alpha^* + \kappa}, \delta_1 = i_{\alpha^* + \kappa + 1}, \ldots, \delta_n - 1 = i_{\alpha^* + \kappa + n - 1} \).

**Claim 2.** For every \( \alpha < \delta_0 \) there exists a limit \( \delta, \alpha < \delta < \delta_0 \), such that for all \( \beta_0 < \cdots < \beta_{n-2} < \delta \) the following hold:

1. \( c[\delta, \beta_0, \ldots, \beta_{n-2}, \delta] = c[\delta_0, \ldots, \delta_{n-1}] \)
2. \( c[\delta, \beta_0, \beta_1, \ldots, \beta_{n-2}, \delta] = c[\delta_0, \delta_1, \ldots, \delta_{n-1}] \)
3. \( c[\delta, \beta_0, \beta_1, \beta_2, \ldots, \beta_{n-2}, \delta] = c[\delta_0, \beta_1, \beta_2, \ldots, \delta_{n-1}] \)
4. \( c[\delta, \beta_0, \beta_1, \beta_2, \beta_3, \ldots, \beta_{n-2}, \delta] = c[\delta_0, \beta_1, \beta_2, \beta_3, \ldots, \delta_{n-1}] \)
5. \( \vdots \)
6. \( c[\delta, \beta_0, \beta_1, \beta_2, \ldots, \beta_{n-2}, \delta] = c[\delta_0, \ldots, \beta_{n-2}, \delta_{n-1}] \).
Proof of Claim 2. Let \( \alpha < \delta_0 \) be arbitrary. Choose \( \gamma < \kappa \) such that \( \gamma \) is a limit and \( i_{\alpha^*+\gamma} > \alpha \), and let \( \delta = i_{\alpha^*+\gamma} \).

Then clearly \((*)_0\) holds.

In order to prove \((*)_1\), let \( \beta < \delta \) be arbitrary. There exist ordinals \( \nu_0 < \cdots < \nu_{k-1} < \alpha^* + \gamma \) and a Skolem term \( t_\beta \) such that

\[
t^b_\beta [i_{\nu_0}, \ldots, i_{\nu_{k-1}}] = \beta.
\]

Moreover there exist ordinals \( \mu_0 < \cdots < \mu_{l-1} < \alpha^* \) and a Skolem term \( t \) such that

\[
L_\lambda^+ \models t[i_{\mu_0}, \ldots, i_{\mu_{l-1}}] = t_c[i_{\xi_0}, \ldots, i_{\xi_{q-1}}] \{t_\beta[i_{\nu_0}, \ldots, i_{\nu_{k-1}}], \delta_1, \ldots, \delta_{n-1}\}.
\]

Note that all indices of occurring indiscernibles, except for \((+)\)

\[
\text{Order-type } \kappa \text{ such that } t \kappa \in Y \\ \Longrightarrow \text{Obtain } \gamma \nu \\
\text{Here } \gamma \nu \in [B]^n \quad c(\nu) = c\nu. \text{ Fix } n < \omega. \text{ Working in L, we construct } B \text{ inductively as } \{\gamma_\nu : \nu < \kappa\}.
\]

Fix \( \delta_0 < \delta_1 < \cdots < \delta_{n-2} < \lambda \) as above. Apply Claim 2 in \( \lambda = 0 \) and obtain \( \gamma_0 \in (0, \delta_0) \). Suppose we have gotten \( \gamma_{\nu_0} < \cdots < \gamma_{\nu_{n-1}} \) be arbitrary. We have

\[
(\gamma_{\nu_0})_{n-1} c(\gamma_{\nu_0}, \ldots, \gamma_{\nu_{n-2}}, \delta_{n-1})
\]

\[
= \cdots
\]

\[
= c(\gamma_{\nu_0}, \delta_1, \ldots, \delta_{n-1})
\]

\[
= (\nu_0)_n. \quad \Box
\]

References


Institute of Mathematics, Hebrew University, Givat Ram, 91904 Jerusalem, Israel

E-mail address: shelah@math.huji.ac.il

Mathematik, ETH-Zentrum, 8092 Zürich, Switzerland

E-mail address: spinas@math.ethz.ch