PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 128, Number 1, Pages 93–99
S 0002-9939(99)05108-4
Article electronically published on June 30, 1999

ON LOCAL AUTOMORPHISMS OF GROUP ALGEBRAS OF COMPACT GROUPS

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(Communicated by David R. Larson)

Abstract. We show that with few exceptions every local isometric automorphism of the group algebra $L^p(G)$ of a compact metric group G is an isometric automorphism.

In the last decade considerable work has been done concerning certain local maps of operator algebras. The originators of this research are Kadison and Larson. In [Kad], Kadison studied local derivations on a von Neumann algebra $\mathcal{R}$. A continuous linear map on $\mathcal{R}$ is called a local derivation if it agrees with some derivation at each point (the derivations possibly differring from point to point) in the algebra. This investigation was motivated by the study of Hochschild cohomology of operator algebras. It was proved in [Kad] that in the above setting, every local derivation is a derivation. Independently, Larson and Sourour proved in [LaSo] that the same conclusion holds true for local derivations of the full operator algebra $\mathcal{B}(\mathcal{X})$, where $\mathcal{X}$ is a Banach space. For other results on local derivations of various algebras see, for example, [Bre, BrSe1, Cri, Shu, ZhXi]. Besides derivations, there is at least one additional very important class of transformations on Banach algebras which certainly deserves attention. This is the group of automorphisms. In [Lar, Some concluding remarks (5), p. 298], from the view-point of reflexivity, Larson raised the problem of local automorphisms (the definition should be self-explanatory) of Banach algebras. In his joint paper with Sourour [LaSo], it was proved that if $\mathcal{X}$ is an infinite dimensional Banach space, then every surjective local automorphism of $\mathcal{B}(\mathcal{X})$ is an automorphism (see also [BrSe1]). For a separable infinite dimensional Hilbert space $\mathcal{H}$, it was shown in [BrSe2] that the above conclusion holds true without the assumption on surjectivity, i.e. every local automorphism of $\mathcal{B}(\mathcal{H})$ is an automorphism. For other results on local automorphisms of various operator algebras, we refer to [BaMo, Mol2, Mol3, Mol4].

Received by the editors March 4, 1998.
1991 *Mathematics Subject Classification.* Primary 43A15, 43A22, 46H99.
Key words and phrases. Compact group, group algebra, isometric automorphism, local isometric automorphism.
This research was supported by the Joint Hungarian-Slovene research project supported by OMFB in Hungary and the Ministry of Science and Technology in Slovenia, Reg. No. SLO-2/96. The first author was supported in part by the Hungarian National Foundation for Scientific Research (OTKA), Grant No. T–016846 F–019322, and by a grant from the Ministry of Education, Hungary, Reg. No. FKFP 0304/1997. The second author was supported in part by a grant from the Ministry of Science and Technology, Slovenia.

In this note we investigate a similar problem for the L^p-algebras (convolution algebras) of compact groups which are of fundamental importance in harmonic analysis. For the local isometric automorphisms of $L^p(G)$ (i.e. linear maps on $L^p(G)$ which agree with some isometric automorphism at each point in the algebra) we obtain the following result.

Theorem. *Let G be a first countable compact group and let $1 \leq p \leq \infty$. Then in the following cases every local isometric automorphism of $L^p(G)$ is an isometric automorphism.*

(i) *$p \neq 2$ and either G is commutative, or G is finite, or G has an at least three-dimensional irreducible representation.*
(ii) *$p = 2$ and G does not have any two-dimensional irreducible representation.*

Remark. Concerning the topological condition in our result we note that by [HeRo, (8.3) Theorem] every first countable compact group is metrizable.

It is our approach to study first the restriction of our local isometric automorphism ψ, acting on $L^p(G)$, to the subalgebra $C(G)$ of all continuous complex valued functions on G. We equip $C(G)$ with the usual supremum norm. In the proof of our theorem we use the following

Lemma. *Let X be a first countable compact Hausdorff space. If $\psi : C(X) \to C(X)$ is a local surjective isometry, then it is a surjective isometry.*

Proof. By the Banach-Stone theorem, every surjective isometry of $C(X)$ is of the form $f \mapsto \tau \cdot f \circ \varphi$, where $\tau : X \to \mathbb{C}$ is a continuous function of modulus 1 and $\varphi : X \to X$ is a homeomorphism. It is now apparent that ψ sends continuous functions of modulus 1 to functions of the same kind. Therefore, ψ preserves the unitary elements of the C^*-algebra $C(X)$. A result of Russo and Dye [RuDy, Corollary 2] says that in that case ψ is a Jordan *-homomorphism followed by multiplication by a fixed unitary element. Without any loss of generality we may assume that $\psi(1) = 1$. Thus, we obtain that ψ is an endomorphism of $C(X)$. It is a folk result that every endomorphism of $C(X)$ which sends 1 to 1 is of the form

$$f \longmapsto f \circ \varphi_0$$

where $\varphi_0 : X \to X$ is a continuous function. Since ψ is an isometry, it readily follows from Urysohn's lemma that φ_0 is surjective. It remains to prove that φ_0 is injective as well. To this end, suppose on the contrary that there are different points $x, y \in X$ such that $\varphi_0(x) = \varphi_0(y) = z$. We construct a continuous function $f : X \to \mathbb{C}$ as follows. Let (U_n) be a monotone decreasing sequence of open sets in X such that $\bigcap_n U_n = \{z\}$. By Urysohn's lemma, for every n we have a continuous function $f_n : X \to [0, 1]$ such that

$$f_n(z) = 1 \text{ and } f_n(t) = 0 \ \ (t \in X \setminus U_n).$$

Let

$$f = 1 - \sum_{n=1}^{\infty} \frac{1}{2^n} f_n.$$

Clearly, f is a continuous function and $f(t) = 0$ if and only if $t = z$. Since ψ is a local surjective isometry, there exist a continuous function $\tau : X \to \mathbb{C}$ of modulus 1 and a homeomorphism $\varphi : X \to X$ such that

$$f \circ \varphi_0 = \psi(f) = \tau \cdot f \circ \varphi.$$

It follows that

$$\{x, y\} \subset (f \circ \varphi_0)^{-1}(0) = (f \circ \varphi)^{-1}(0) = \varphi^{-1}(z).$$

Since φ is bijective, this is a contradiction. Consequently, φ_0 is injective which implies the surjectivity of ψ. $\square$

For the proof of our theorem we also need the following well-known facts from harmonic analysis (see [HeRo, Sections 27,28,31]).

Let X be the dual object of G. For every $\sigma \in \mathsf{X}$ let d_σ be the dimension of the irreducible representations of G belonging to the equivalence class σ. Let $\mathfrak{C}_0(\mathsf{X})$ denote the subset of $\prod_{\sigma\in\mathsf{X}} M_{d_\sigma}(\mathbb{C})$ consisting of those elements $(E_\sigma)_{\sigma\in\mathsf{X}}$ which vanish at infinity (here we consider the operator norm of matrices). Clearly, $\mathfrak{C}_0(\mathsf{X})$ is a C^*-algebra with the pointwise operations and the sup-norm. It is well-known that the Fourier transform is an injective *-homomorphism of $L^1(G)$ into $\mathfrak{C}_0(\mathsf{X})$ whose range contains the subalgebra $\mathfrak{C}_{00}(\mathsf{X})$ of all cofinite elements. A similar statement holds true for any of the convolution algebras $C(G), L^p(G)$ $(1 \le p \le \infty)$. For a $\sigma_0 \in \mathsf{X}$ let

$$\mathfrak{I}_{\sigma_0} = \{(E_\sigma)_{\sigma\in\mathsf{X}} \,:\, E_\sigma = 0 \ (\sigma \neq \sigma_0)\}.$$

Under the Fourier transform, the minimal ideals of any of the previously mentioned convolution algebras are in a one-to-one correspondence with the ideals $\mathfrak{I}_\sigma$ $(\sigma \in \mathsf{X})$. Therefore, the minimal ideals are isomorphic to full matrix algebras and they are algebraically orthogonal to each other (i.e. the product of any two of them is $\{0\}$). The structure theory of group algebras tells us that in $C(G), L^p(G)$ $(1 \le p < \infty)$ the algebraic direct sum of the minimal ideals is dense in the corresponding norm topology (see [HeRo, (28.39) Theorem and (27.39) Theorem]).

Proof of the Theorem. Let us suppose first that $p \neq 2$. In this case the form of isometric automorphisms of $L^p(G)$ is well-known. For every such automorphism ϕ there exist a continuous group character $\tau : G \to \mathbb{T}$ and a (bi)continuous group automorphism $\varphi : G \to G$ such that

$$\phi(f)(x) = \tau(x) f(\varphi(x)) \qquad (x \in G, f \in L^p(G)) \tag{1}$$

(see [Str, Theorems 2,3] and note that by the uniqueness of the Haar measure every continuous automorphism of G is measure preserving). In particular, it follows that ϕ maps $C(G)$ onto $C(G)$ and ϕ is a surjective isometry with respect to the sup-norm.

Let $\psi : L^p(G) \to L^p(G)$ be a local isometric automorphism of $L^p(G)$. By (1), $\psi_{|C(G)}$ is a local surjective isometry of $C(G)$ and our Lemma yields that it is a surjective isometry. Using the Banach-Stone theorem, we have a continuous function $t : G \to \mathbb{T}$ and a homeomorphism $g : G \to G$ such that

$$\psi(f)(x) = t(x) f(g(x)) \qquad (x \in G, f \in C(G)). \tag{2}$$

Considering the local form of ψ at the function $f = 1$, we obtain that t is a character. Pick different points $x, y \in G \setminus \{1\}$. Then x, y, xy are pairwise different and so are $g(x), g(y), g(xy)$. It is not hard to construct a nonnegative continuous function f on G with the property that $f^{-1}(0) = \{g(x), g(y)\}$, $f^{-1}(1) = \{g(xy)\}$. By the local form of ψ it follows that there exist a continuous character $\tau : G \to \mathbb{T}$ and a continuous group automorphism $\varphi : G \to G$ such that

$$t \cdot f \circ g = \tau \cdot f \circ \varphi. \tag{3}$$

Taking absolute value we arrive at $f \circ g = f \circ \varphi$ and then we deduce $\{g(x), g(y)\} = \{\varphi(x), \varphi(y)\}$ and $g(xy) = \varphi(xy)$. Since φ is an automorphism, we have either $g(xy) = g(x)g(y)$ or $g(xy) = g(y)g(x)$. Similarly, we can prove that $g(1) = 1$ and $g(x^2) = g(x)^2$. Therefore, the homeomorphism $g : G \to G$ has the property that for every $x, y \in G$ we have either $g(xy) = g(x)g(y)$ or $g(xy) = g(y)g(x)$. By [Sco, Theorem 2] it follows that g is either an automorphism or an antiautomorphism of G. Apparently, this implies that ψ is either an automorphism or an antiautomorphism of $C(G)$.

Let $p < \infty$. Since in this case $C(G)$ is dense in $L^p(G)$ and g, being a continuous automorphism or antiautomorphism of G, is measure preserving, it follows easily that the formula (2) holds true also for the elements of $L^p(G)$. If $p = \infty$, then we can arrive at the same conclusion by using the fact that ψ is an isometry with respect to any p-norm and $L^\infty(G) \subset L^p(G)$.

Let us stop here for a while and deal with the case $p = 2$. In this case we do not have the form (1) of isometric automorphisms but we can use Plancherel's theorem instead. It says that via the Fourier transform, $L^2(G)$ is isometric and isomorphic to

$$\mathfrak{C}_2(\mathsf{X}) = \{(A_\sigma)_{\sigma\in\mathsf{X}} : (\sum_{\sigma\in\mathsf{X}} d_\sigma \|A_\sigma\|_2^2)^{1/2} < \infty\}. \tag{4}$$

Here, $\|.\|_2$ denotes the Hilbert-Schmidt norm of matrices and on $\mathfrak{C}_2(\mathsf{X})$ we consider the norm suggested in (4). For what remains we need to know what the isometric Jordan automorphisms of the H^*-algebra $\mathfrak{C}_2(\mathsf{X})$ look like. Recall that a linear map ϕ between algebras $\mathcal{A}$ and $\mathcal{B}$ is called a Jordan homomorphism if it satisfies

$$\phi(x^2) = \phi(x)^2 \qquad (x \in \mathcal{A}),$$

or equivalently

$$\phi(xy + yx) = \phi(x)\phi(y) + \phi(y)\phi(x) \qquad (x, y \in \mathcal{A}).$$

From the proof of [Mol1, Theorem] it follows that for any isometric Jordan automorphism ϕ of $\mathfrak{C}_2(\mathsf{X})$ there are a bijection $\alpha : \mathsf{X} \to \mathsf{X}$ and unitary matrices $U_\sigma \in M_{d_\sigma}(\mathbb{C})$ such that ϕ is of the form

$$\phi((A_\sigma)_\sigma) = (U_{\alpha(\sigma)} A^{[T]}_{\alpha(\sigma)} U^*_{\alpha(\sigma)})_\sigma. \tag{5}$$

Here, $^{[T]}$ denotes that on the corresponding "coordinate" space we might have to take transpose. Now, let ψ be a local isometric automorphism of $\mathfrak{C}_2(\mathsf{X})$. Clearly, ψ sends idempotents to idempotents. Like on matrix algebras, it is now a standard argument to show that ψ is a Jordan homomorphism on the subalgebra $\mathfrak{C}_{00}(\mathsf{X})$ of $\mathfrak{C}_2(\mathsf{X})$ consisting of all cofinite elements (see, e.g., [Mol2, Theorem 2]). By the continuity of ψ and the density of $\mathfrak{C}_{00}(\mathsf{X})$ in $\mathfrak{C}_2(\mathsf{X})$ we infer that ψ is a Jordan homomorphism. Let $\sigma_0 \in \mathsf{X}$. The preimage of the minimal ideal $\mathfrak{I}_{\sigma_0}$ under ψ is a finite dimensional Jordan ideal. Suppose that $\psi^{-1}(\mathfrak{I}_{\sigma_0}) \neq \{0\}$. Since the full matrix algebras are simple Jordan algebras, it follows easily that this preimage is of the form

$$\psi^{-1}(\mathfrak{I}_{\sigma_0}) = \{(A_\sigma)_{\sigma\in\mathsf{X}} : A_\sigma = 0 \ (\sigma \notin \{\sigma_1, \dots, \sigma_n\})\}. \tag{6}$$

By the local form of ψ we get that in (6) we have $d_{\sigma_1} = \ldots = d_{\sigma_n} = d_{\sigma_0}$ and $n = 1$. We obtain $\psi(\psi^{-1}(\mathfrak{I}_{\sigma_0})) = \mathfrak{I}_{\sigma_0}$. To sum up, if a minimal ideal contains a nonzero element of the range of ψ, then this minimal ideal is included in the range. Now, the result [HeRo, (28.2) Theorem] says that if G is infinite, then the cardinality of

X is equal to the topological weight of G. Since G is metrizable and compact, it follows that it is second countable. We have $\mathsf{X} = \{\sigma_n\}_n$. For every n, let $x_n \in \mathfrak{I}_{\sigma_n}$ be its unit. Consider the element

$$x = \sum_n \frac{1}{n\sqrt{d_{\sigma_n}}} x_n$$

and pick a minimal ideal, say $\mathfrak{I}_{\sigma_1}$. By the local form of ψ we clearly have $\psi(x)\mathfrak{I}_{\sigma_1} \neq \{0\}$. This implies that there is an index n such that $\psi(x_n)\mathfrak{I}_{\sigma_1} \neq \{0\}$. But by the local form of ψ once again, it follows that $\psi(x_n)$ is the unit of some minimal ideal. This results in $\psi(x_n) \in \mathfrak{I}_{\sigma_1} \neq \{0\}$. By what we have proved before, we conclude that $\mathfrak{I}_{\sigma_1}$ is included in the range of ψ. Since this minimal ideal was arbitrary and the range of ψ is closed, we obtain that ψ is surjective and, hence, it is an isometric Jordan automorphism of $\mathfrak{C}_2(\mathsf{X})$.

Now, we are in a position to complete the proof of our theorem. Let $p \neq 2$. Since ψ is an isometric automorphism or antiautomorphism of $L^p(G)$, we are done in the commutative case. If G is finite, then G is discrete and hence there is an injective nonnegative continuous function on G. If we put this function into (3), we obtain that g is an automorphism of G which implies that ψ is an isometric automorphism of $L^p(G)$. Next suppose that $d_{\sigma_n} \geq 3$ for some $\sigma_n \in \mathsf{X}$. If ψ is an antiautomorphism, then by its form given in (2), ψ can be extended from $C(G)$ to an isometric antiautomorphism of $L^2(G)$. By the local property of ψ we obtain that at the elements of $C(G)$, ψ agrees with some isometric automorphism of $L^p(G)$ which, keeping its form in mind, can be extended or restricted to an isometric automorphism of $L^2(G)$. The form of isometric automorphisms and antiautomorphisms of $L^2(G)$ can be easily obtained from (5). In fact, in the former case there is no transpose in (5) at all, while in the latter one we must take transpose on every "coordinate" space. Therefore, assuming that ψ is an antiautomorphism we would obtain that every matrix $A \in M_{d_{\sigma_n}}(\mathbb{C})$ is unitarily equivalent to its transpose. But this is a contradiction. Indeed, one can verify quite easily that the matrix $A = (a_{ij})$, where $a_{i+1,i} = i$ $(i = 1, \dots, d_{\sigma_0} - 1)$ and the other entries are all zero, is not unitarily equivalent to its transpose. Finally, in the case when $p = 2$, the proof can be completed in the same way observing that in one dimension the transpose does not matter. $\square$

Remark. We conclude the paper with some comments on our result.

First, consider the topological condition in the Theorem, i.e. the first countability of G. We suspect that it is essential in the case $p \neq 2$ as well, but as for $p = 2$ we have the following counterexample. If G is a compact group with weight greater than $\aleph_0$, then by [HeRo, (28.2) Theorem] we have a dimension d which appears uncountably many times among the d_σ's. Without serious loss of generality we can restrict our attention to the corresponding part of $\mathfrak{C}_2(\mathsf{X})$. So, let us consider the H^*-algebra

$$\{f : \Lambda \to M_d(\mathbb{C}) \,:\, (\sum_{\lambda \in \Lambda} \|f(\lambda)\|_2^2)^{1/2} < \infty\}$$

of all Hilbert-Schmidt functions, where Λ is an uncountable set. Let Λ' be a proper subset of Λ with a bijection $\alpha : \Lambda' \to \Lambda$. Define $\psi(f)$ by

$$\psi(f)(\lambda) = \begin{cases} f(\alpha(\lambda)) & \text{if} \quad \lambda \in \Lambda', \\ 0 & \text{if} \quad \lambda \in \Lambda \setminus \Lambda'. \end{cases}$$

If f is a function from our collection, then it takes nonzero values only on a countable set $\{\lambda_n\}_n$. Let β be a bijection from $\Lambda \setminus \{\alpha^{-1}(\lambda_n)\}_n$ onto $\Lambda \setminus \{\lambda_n\}_n$. Define a bijection $\gamma : \Lambda \to \Lambda$ by

$$\gamma(\lambda) = \begin{cases} \alpha(\lambda) & \text{if} \quad \lambda \in \{\alpha^{-1}(\lambda_n)\}_n, \\ \beta(\lambda) & \text{if} \quad \lambda \in \Lambda \setminus \{\alpha^{-1}(\lambda_n)\}_n. \end{cases}$$

It is easy to see that $\psi(f) = f \circ \gamma =: \phi(f)$, and ϕ is an isometric isomorphism of the algebra of all Hilbert-Schmidt functions. Therefore, ψ is a nonsurjective local isometric automorphism.

Our second remark is the following. In the proof of our theorem we have seen that if $p \neq 2$, every local isometric automorphism ψ of $L^p(G)$ is of the form

$$\psi(f) = t \cdot f \circ g \qquad (f \in L^p(G)) \tag{7}$$

where $t : G \to \mathbb{T}$ is a continuous character and $g : G \to G$ is either a continuous automorphism or a continuous antiautomorphism. This implies that ψ is either an automorphism or an antiautomorphism of $L^p(G)$ regardless of the possible additional properties of G. It is a natural question whether the conditions listed in (i) cover every possibility. We mean that if a compact metric group is noncommutative, infinite and has only one- and two-dimensional irreducible representations, does it follow that there is a local isometric automorphism of $L^p(G)$ which is not an automorphism? Unfortunately (or fortunately), the answer to this question is negative. To see this, pick, for example, an arbitrary noncommutative finite group H and consider the product $G = \mathbb{T} \times H$. If H has only one- and two-dimensional irreducible representations (e.g., H is the symmetric group $\mathfrak{S}_3$), then the same holds true for $\mathbb{T} \times H$ as well. Let ψ be a local isometric automorphism of $L^p(G)$ which is necessarily of the form (7). If $\varphi : \mathbb{T} \times H \to \mathbb{T} \times H$ is a continuous automorphism, then an easy argument using the connectedness of $\mathbb{T}$ and the fact that none of its closed subspaces is homeomorphic to $\mathbb{T}$ shows that φ can be written in the form

$$\varphi(z, x) = (\alpha(z, x), \beta(x)) \qquad (z \in \mathbb{T}, x \in H), \tag{8}$$

where β is an automorphism of H. Clearly, we have a similar form for the antiautomorphism of H. Now, by (3) we know that for every $f \in L^p(G)$ there exist a continuous character $\tau : G \to \mathbb{T}$ and a continuous automorphism $\varphi : G \to G$ such that

$$t \cdot f \circ g = \tau \cdot f \circ \varphi. \tag{9}$$

Let f be the function defined by $f(z, x_i) = i$, where $z \in \mathbb{T}$ is arbitrary and x_i is the ith element of H. Considering suitable τ and φ and comparing the two sides of (9) we deduce that the second coordinate function in the decomposition of g, which can be given similarly to (8), is an automorphism of G. Since H is noncommutative, we obtain that from the two possibilities, that g is either an automorphism or an antiautomorphism, the former one is true. Therefore, we have proved that on $L^p(\mathbb{T} \times H)$ every local isometric automorphism is an automorphism. So there is a small gap in the $p \neq 2$ part of our theorem which is not the case with $p = 2$ as we shall see immediately. Our last remark concerning the case $p \neq 2$ is that we would clearly obtain the conclusion of the theorem if there were an injective nonnegative function in $C(G)$. More precisely, we would be done even if for every subset of G with three elements we could guarantee the existence of a nonnegative function

which is injective when restricted to this subset. Unfortunately, this is not the case even with the nicest groups like $\mathbb{T}$.

Finally, as for the case $p = 2$, we show that the condition that G does not have any two-dimensional irreducible representation is not only necessary but also sufficient to reach the desired conclusion. To verify this, we note that every 2×2 complex matrix is unitarily equivalent to its transpose as it was proved in [BaMo, Remark]. Now, if we suppose that at least one of the d_σ's, say d_{σ_0} is 2, then considering the map ψ on $\mathfrak{C}_2(\mathsf{X})$ which acts as the identity on the factors $M_{d_\sigma}(\mathbb{C})$, $\sigma \neq \sigma_0$ and acts as the transposition on $M_{d_{\sigma_0}}(\mathbb{C})$, we have a local isometric automorphism of $L^2(G)$ which is not an automorphism.

References

[BaMo] C.J.K. Batty and L. Molnár, *On topological reflexivity of the groups of *-automorphisms and surjective isometries of $B(H)$*, Arch. Math. **67** (1996), 415–421. MR **97f:**47034

[Bre] M. Brešar, *Characterizations of derivations on some normed algebras with involution*, J. Algebra **152** (1992), 454–462. MR **94e:**46098

[BrSe1] M. Brešar and P. Šemrl, *Mappings which preserve idempotents, local automorphisms, and local derivations*, Canad. J. Math. **45** (1993), 483–496. MR **94k:**47054

[BrSe2] M. Brešar and P. Šemrl, *On local automorphisms and mappings that preserve idempotents*, Studia Math. **113** (1995), 101–108. MR **96i:**47058

[Cri] R.L. Crist, *Local derivations on operator algebras*, J. Funct. Anal. **135** (1996), 76–92. MR **96m:**46128

[HeRo] E. Hewitt and K.A. Ross, *Abstract Harmonic Analysis I., II.*, Springer Verlag, 1963, 1970. MR **28:**158; MR **41:**7378

[Kad] R.V. Kadison, *Local derivations*, J. Algebra **130** (1990), 494–509. MR **91f:**46092

[Lar] D.R. Larson, *Reflexivity, algebraic reflexivity and linear interpolation*, Amer. J. Math. **110** (1988), 283–299. MR **89d:**47096

[LaSo] D.R. Larson and A.R. Sourour, *Local derivations and local automorphisms of $B(X)$*, in Proc. Sympos. Pure Math. 51, Part 2, Providence, Rhode Island, 1990, 187–194. MR **91k:**47106

[Mol1] L. Molnár, *Algebraic difference between p-classes of an H^*-algebra*, Proc. Amer. Math. Soc. **124** (1996), 169–175. MR **96d:**46072

[Mol2] L. Molnár, *The set of automorphisms of $B(H)$ is topologically reflexive in $B(B(H))$*, Studia Math. **122** (1997), 183–193. MR **98e:**47068

[Mol3] L. Molnár, *Reflexivity of the automorphism and isometry groups of C^*-algebras in BDF theory*, Arch. Math., to appear.

[Mol4] L. Molnár and M. Győry, *Reflexivity of the automorphism and isometry groups of the suspension of $B(H)$*, J. Funct. Anal. **159** (1998), 568–586. CMP 99:04

[RuDy] B. Russo and H.A. Dye, *A note on unitary operators in C^*-algebras*, Duke Math. J. **33** (1966), 413–416. MR **33:**1750

[Sco] W.R. Scott, *Half-homomorphisms of groups*, Proc. Amer. Math. Soc. **8** (1957), 1141–1144. MR **20:**2388

[Shu] V.S. Shul'man, *Operators preserving ideals in C^*-algebras*, Studia Math. **109** (1994), 67–72. MR **95b:**46097

[Str] R.S. Strichartz, *Isomorphisms of group algebras*, Proc. Amer. Math. Soc. **17** (1966), 858–862. MR **33:**1751

[ZhXi] J. Zhu and C. Xiong, *Bilocal derivations of standard operator algebras*, Proc. Amer. Math. Soc. **125** (1997), 1367–1370. MR **97g:**46091

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