

MULTIDIMENSIONAL ANALOGUES OF BOHR'S THEOREM ON POWER SERIES

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ABSTRACT. Generalizing the classical result of Bohr, we show that if an n -variable power series converges in n -circular bounded complete domain D and its sum has modulus less than 1, then the sum of the maximum of the moduli of the terms is less than 1 in the homothetic domain $r \cdot D$, where $r = 1 - \sqrt[n]{2/3}$. This constant is near to the best one for the domain $D = \{z : |z_1| + \dots + |z_n| < 1\}$.

1. PRELIMINARIES

The following formulation of the Bohr's result [5] (due to the work of M. Riesz, I. Schur and F. Wiener) is known.

Theorem 1. *If a power series*

$$(1) \quad \sum_{k=0}^{\infty} c_k z^k$$

converges in the unit disk and its sum has modulus less than 1, then

$$(2) \quad \sum_{k=0}^{\infty} |c_k z^k| < 1$$

in the disk $\{z : |z| < 1/3\}$. Moreover, the constant $1/3$ cannot be improved.

Recently H.P. Boas and D. Khavinson obtained some multidimensional generalizations of this result ([4], see also for more references there).

Denote by K_n the largest number such that if the series

$$(3) \quad \sum_{\alpha} c_{\alpha} z^{\alpha}$$

converges in the unit polydisk $U_1 = \{z : |z_j| < 1, j = 1, \dots, n\}$ and the estimate

$$(4) \quad \left| \sum_{\alpha} c_{\alpha} z^{\alpha} \right| < 1$$

is valid there, then

$$(5) \quad \sum_{\alpha} |c_{\alpha} z^{\alpha}| < 1$$

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holds in $K_n \cdot U_1$; here $\alpha = (\alpha_1, \dots, \alpha_n)$, all α_j are non-negative integers, $z = (z_1, \dots, z_n)$, $z^\alpha = z_1^{\alpha_1}, \dots, z_n^{\alpha_n}$.

Theorem 2 (Boas, Khavinson). *It is true for $n > 1$ that*

$$(6) \quad \frac{1}{3\sqrt{n}} < K_n < \frac{2\sqrt{\log n}}{\sqrt{n}}.$$

Theorem 3 (Boas, Khavinson). *Let the series (3) converge in a complete n -circular domain (Reinhardt domain) D and (4) holds in D . Then (5) is true in the homothetic domain $K_n \cdot D$.*

Notice that Remark 1 from [4], in fact, contains a result stronger than the left part of the inequality (6), namely

$$K_n > \begin{cases} \frac{2}{5\sqrt{n}} & \text{for } n > 1, \\ \frac{1}{2\sqrt{n}} & \text{for large enough } n. \end{cases}$$

2. MAIN RESULTS

We consider some other multidimensional variations of Bohr’s problem. Denote by $B_n(D)$ the largest number r such that if the series (3) converges in a complete n -circular bounded domain D and (4) holds in it, then

$$(7) \quad \sum_{\alpha} \sup_{D_r} |c_{\alpha} z^{\alpha}| < 1,$$

where $D_r = r \cdot D$ is the homothetic transformation of D . If $D = U_1$, then $B_n(D) = K_n$. We point out that our consideration is also a natural generalization of Bohr’s theorem (Theorem 1), because it was shown in [2] that any power series (3), converging in D , converges also in the sense of the left part of (7) for all r , $0 < r < 1$.

Theorem 4. *The inequality*

$$(8) \quad 1 - \sqrt[n]{\frac{2}{3}} < B_n(D)$$

is true for any complete, bounded n -circular domain D .

This estimate can be improved for concrete domains.

Theorem 5. *For the unit ball $D^1 = \{z : |z_1|^2 + \dots + |z_n|^2 < 1\}$ the following estimate is true:*

$$B_n(D^1) > \begin{cases} \frac{2}{5n} & \text{for } n > 1, \\ \frac{1}{2n} & \text{for large enough } n. \end{cases}$$

Theorem 6. *For the unit hypercone $D^\circ = \{z : |z_1| + \dots + |z_n| < 1\}$ the following inequality holds:*

$$(9) \quad B_n(D^\circ) < \frac{0,446663}{n}.$$

Corollary.

$$(10) \quad 1 - \sqrt[n]{\frac{2}{3}} < B_n(D^\circ) < \frac{0.446663}{n}.$$

Remark 1. The asymptotic equality

$$1 - \sqrt[n]{\frac{2}{3}} = \frac{\log 3/2}{n} + O\left(\frac{1}{n^2}\right)$$

is true, where $\log 3/2 \approx 0.405465$. Denoting by B_- the left part of (10) and by B_+ the right one, we get

$$1 < \limsup_{n \rightarrow \infty} \frac{B_+}{B_-} < 1,1016.$$

We now turn our attention to a related problem. Denote by $L_n(D^\circ)$ the biggest number r such that if the series (3) converges in the hypercone D° and (4) holds in it, then

$$(11) \quad \sum_{\alpha} \|c_{\alpha} z^{\alpha}\|_{L^1(\partial D_r^\circ)} < 1,$$

where the L^1 -norm is considered with respect to the measure μ_r . The measure μ_r is the image of the measure

$$d\mu = \frac{(n-1)!}{(2\pi i)^n} d|z_1| \wedge \cdots \wedge d|z_{n-1}| \wedge \frac{dz_1}{z_1} \wedge \cdots \wedge \frac{dz_n}{z_n}$$

by the homothetic transformation $z \rightarrow rz$. Usually the measure $d\mu$ is used for calculating the Szëgo kernel for D° (see [3]); notice that $\mu(\partial D^\circ) = 1$. For $n = 1$ this problem coincides with Bohr's problem. The analogous problem for the polydisk U_1 (with L^1 -norm on its Shilov boundary with respect to usual Lebesgue measure on it) is equivalent to the problem considered in Theorem 2.

Theorem 7. *For the hypercone D° the following estimates are true:*

$$(12) \quad \frac{1}{3e^{1/3}} < L_n(D^\circ) \leq 1/3.$$

Surprisingly, the estimates in (12) do not depend on n . This is different from the results of Theorems 2–6.

We consider now a variation of the multidimensional Bohr problem, dealing with the expansions into a series of homogeneous polynomials, which is also a natural generalization of the power series expansion.

Let Q be a complete circular domain (Cartan's domain) centered at $0 \in Q$. Then any function $f(z)$, holomorphic in Q , can be expanded into the series

$$(13) \quad f(z) = \sum_{k=0}^{\infty} P_k(z),$$

where $P_k(z)$ is a homogeneous polynomial of degree k for every $k \in \mathbf{N}$.

Theorem 8. *If the series (13) converges in the domain Q and the estimate $|f(z)| < 1$ holds in it, then*

$$(14) \quad \sum_{k=0}^{\infty} |P_k(z)| < 1$$

in the homothetic domain $\frac{1}{3}Q$. Moreover, if Q is convex, then $1/3$ is the best possible constant.

Next we consider for the hypercone D° the same problem, which was considered in Theorems 2–3 for U_1 . Denote by $K_n(D^\circ)$ the largest number such that if series (3) converges in D° and estimate (4) is valid there, then (5) holds in $K_n(D^\circ) \cdot D^\circ$.

Theorem 9. For the hypercone D° the following estimates are true:

$$\frac{1}{3e^{1/3}} < K_n(D^\circ) \leq 1/3 .$$

Moreover, if $z \notin 1/3 \cdot D^\circ$, then there exists a series of the form (3) such that it converges in D° and the estimate (4) is valid there, but (5) fails at the point z .

3. PROOFS

Proof of Theorem 4. The following generalization of Cauchy inequalities was considered in [2]: if (4) holds in D , then

$$(15) \quad |c_\alpha| \leq \frac{1}{d_\alpha(D)},$$

where $d_\alpha(D) = \max_D |z^\alpha|$. Using Wiener’s method, it is easy to strengthen the estimates (15):

$$(16) \quad |c_\alpha| \leq (1 - |c_0|^2) \frac{1}{d_\alpha(D)}$$

for $|\alpha| = \alpha_1 + \dots + \alpha_n > 1$. We do not present the proof of (16), because it repeats the proof for polydisk U_1 (see [4]), but deals with inequalities (15) instead of Cauchy inequalities. If (4) holds in D , then, applying (16), we get

$$\begin{aligned} \sum_\alpha \sup_{D_r} |c_\alpha z^\alpha| &= \sum_\alpha |c_\alpha| d_\alpha(D_r) \\ &\leq |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^\infty \frac{d_\alpha(D_r)}{d_\alpha(D)} \\ &= |c_0| + (1 - |c_0|^2) \sum_{k=1}^\infty \binom{n+k-1}{k} r^k \\ &= |c_0| + (1 - |c_0|^2) \left[\frac{1}{(1-r)^n} - 1 \right]. \end{aligned}$$

Now if

$$(17) \quad \frac{1}{(1-r)^n} - 1 \leq \frac{1}{2},$$

then

$$\sum_\alpha \sup_{D_r} |c_\alpha z^\alpha| \leq |c_0| + (1 - |c_0|^2) \frac{1}{2} = 1 - \frac{1}{2}(1 - |c_0|^2) < 1.$$

The condition (17) means that (7) is true if $r \leq 1 - \sqrt[n]{\frac{2}{3}}$.

Proof of Theorem 5. Consider Borel probability measure on ∂D^1 , which is invariant under all unitary transformations of \mathbf{C}^n :

$$d\mu = \frac{(n-1)!}{(2\pi i)^n} d|z_1|^2 \wedge \dots \wedge d|z_{n-1}|^2 \wedge \frac{dz_1}{z_1} \wedge \dots \wedge \frac{dz_n}{z_n}.$$

The monomials z^α are orthogonal with respect to integration by μ and

$$(18) \quad \int_{\partial D^1} |z^{2\alpha}| d\mu = \frac{\alpha_1! \dots \alpha_n! (n-1)!}{(|\alpha| + n - 1)!}.$$

Next, we repeat the proof of Theorem 2 from [4] using Wiener's method, but integrating on the sphere ∂D^1 with respect to the measure μ instead of integrating on the unit torus as in [4]. Thus, we obtain

$$\sum_{|\alpha|=k} |c_\alpha|^2 \frac{\alpha_1! \cdots \alpha_n!(n-1)!}{(|\alpha|+n-1)!} \leq (1-|c_0|^2)^2.$$

Furthermore, notice that from the Schwarz Lemma for the ball D^1 it follows (again by using Wiener's method from [4]) that

$$\sum_{|\alpha|=1} |c_\alpha|^2 \leq (1-|c_0|^2)^2.$$

Then, recalling that

$$d_\alpha(D^1) = \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{|\alpha|}}},$$

where $0^0 = 1$, we get

$$\begin{aligned} \sum_\alpha \sup_{D_r} |c_\alpha z^\alpha| &= |c_0| + \left(\sum_{|\alpha|=1} |c_\alpha| \right) r + \sum_{k=2}^\infty \sum_{|\alpha|=k} |c_\alpha| \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{|\alpha|}}} r^k \\ &\leq |c_0| + (1-|c_0|^2)r\sqrt{n} + (1-|c_0|^2) \sum_{k=2}^\infty \left(\sum_{|\alpha|=k} \frac{(k+n-1)! \alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{k^k \alpha_1! \cdots \alpha_n! (n-1)!} \right)^{1/2} r^k \\ &< |c_0| + (1-|c_0|^2)r\sqrt{n} + (1-|c_0|^2) \sum_{k=2}^\infty \left(\frac{(k+n-1)!}{k!(n-1)!} \sum_{|\alpha|=k} \frac{k!}{\alpha_1! \cdots \alpha_n!} \right)^{1/2} r^k \\ &= |c_0| + (1-|c_0|^2)r\sqrt{n} + (1-|c_0|^2) \sum_{k=2}^\infty \sqrt{\binom{k+n-1}{k}} (r\sqrt{n})^k. \end{aligned}$$

It remains now to apply the estimate, obtained in Remark 1 from [4].

Proof of Theorem 6. Consider the function

$$f_a(z) = \frac{1+a}{2} \frac{1-(z_1+\cdots+z_n)}{1-a(z_1+\cdots+z_n)} = \sum_\alpha c_\alpha z^\alpha,$$

where $0 < a < 1$. Then $|f_a(z)| < 1$ in hypercone D° and

$$\sum_\alpha |c_\alpha z^\alpha| = \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^\infty a^{k-1} \sum_{|\alpha|=k} \frac{k!}{\alpha_1! \cdots \alpha_n!} |z^\alpha|.$$

Since

$$d_\alpha(D^\circ) = \frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{|\alpha|}},$$

it follows that

$$\begin{aligned} \sum_{\alpha} |c_{\alpha}| d_{\alpha}(D_r^{\circ}) &= \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^{\infty} \sum_{|\alpha|=k} a^{k-1} \frac{k! \alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{\alpha_1! \cdots \alpha_n! k^k} r^k \\ &> \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^{\infty} a^{k-1} \frac{k!}{k^k} \left(\sum_{|\alpha|=k} \frac{1}{\alpha_1! \cdots \alpha_n!} \right) r^k \\ &= \frac{1+a}{2} + \frac{1-a^2}{2a} \sum_{k=1}^{\infty} \frac{(anr)^k}{k^k} \geq 1, \end{aligned}$$

if

$$\sum_{k=1}^{\infty} \frac{(anr)^k}{k^k} \geq \frac{a}{1+a}.$$

Let $x_0(a)$ be the root of the equation

$$\sum_{k=1}^{\infty} \frac{x^k}{k^k} = \frac{a}{1+a};$$

then, if $anr \geq x_0(a)$, (7) fails for that r and $D = D^{\circ}$. Now, considering $a \rightarrow 1$, we obtain that (7) is not true for $D = D_{\circ}$ if $r \geq \frac{x_0}{n}$, where $x_0 = x_0(1)$. Notice that x_0 is a root of the equation

$$(19) \quad \sum_{k=1}^{\infty} \frac{x^k}{k^k} = 1/2;$$

hence $B_n(D^{\circ}) \leq x_0/n$. Using the program ‘‘Mathematica 3.0’’ [7], we estimated x_0 from above. We obtained that the equation

$$(20) \quad \sum_{k=1}^p \frac{x^k}{k^k} = 1/2$$

has a root 0.446662 (where the last decimal digit is precise) if p runs from 5 till 25. So, the equation (20) has a root less than 0.446663 if $5 \leq p \leq 25$; hence this estimate is true for x_0 .

Proof of Theorem 7. Notice that by analogy with (18) it is true that

$$\int_{\partial D_r^{\circ}} |z^{\alpha}| d\mu_r = \frac{\alpha_1! \cdots \alpha_n! (n-1)!}{(|\alpha| + n - 1)!} r^{|\alpha|}.$$

Using (16), we get

$$\begin{aligned} &\sum_{\alpha} \|c_{\alpha} z^{\alpha}\|_{L^1(\partial D_r^{\circ})} \\ &\leq |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \sum_{|\alpha|=k} \frac{k^k (n-1)!}{(n+k-1)!} \frac{\alpha_1! \cdots \alpha_n!}{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}} r^{|\alpha|} \\ &< |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \frac{k^k (n-1)!}{(n+k-1)!} \sum_{|\alpha|=k} r^{|\alpha|} \\ &= |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \frac{k^k}{k!} r^k. \end{aligned}$$

Denote by r_0 the root of the equation

$$(21) \quad \sum_{k=1}^{\infty} \frac{k^k}{k!} r^k = \frac{1}{2}.$$

Then (11) holds for $r = r_0$; therefore $L_n(D^\circ) \geq r_0$. The identity

$$(22) \quad \sum_{k=1}^{\infty} \frac{k^k}{k!} x^k e^{-kx} = -1 + \frac{1}{1-x}$$

holds for x in a neighborhood of 0. This can be verified by computing the Maclaurin series coefficients of the left-hand side and observing that they reduce to the value 1 by a standard theorem on sums of binomial coefficients. Now put $x = 1/3$ in equation (22) to recognize the solution of equation (21).

Finally, consider, as in the proof of Theorem 6, the function $f_a(z)$ for which

$$\begin{aligned} \sum_{\alpha} \|c_{\alpha} z^{\alpha}\|_{L^1(\partial D_r^\circ)} &= \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^{\infty} \frac{k!(n-1)!a^{k-1}}{(n+k-1)!} \sum_{|\alpha|=k} r^{|\alpha|} \\ &= \frac{1+a}{2} + \frac{1-a^2}{2a} \sum_{k=1}^{\infty} (ar)^k \geq 1, \end{aligned}$$

if

$$\sum_{k=1}^{\infty} (ar)^k \geq \frac{a}{1+a}.$$

When $a \rightarrow 1$, we get that the inequality (11) fails if $r > 1/3$.

Proof of Theorem 8. In each section of the domain Q by a complex line

$$\alpha = \{z : z_j = a_j t, \quad j = 1, \dots, n; \quad t \in \mathbf{C}\}$$

the series turns into the power series by t

$$f(at) = \sum_{k=0}^{\infty} P_k(a)t^k$$

and, in addition, $|f(at)| < 1$. By Theorem 1

$$\sum_{k=0}^{\infty} |P_k(a)t^k| < 1$$

in the section $\alpha \cap (\frac{1}{3} \cdot Q)$. But it is just (14), since α is an arbitrary complex line passing through the origin. Conversely, let the domain Q be convex; then Q is an intersection of half-spaces

$$Q = \bigcap_{a \in J} \{z : \operatorname{Re}(a_1 z_1 + \dots + a_n z_n) < 1\}$$

for some J . Since Q is circular, we obtain

$$Q = \bigcap_{a \in J} \{z : |a_1 z_1 + \dots + a_n z_n| < 1\}.$$

It is sufficient now to show that the constant $1/3$ cannot be improved for each domain $P_a = \{z : |a_1 z_1 + \dots + a_n z_n| < 1\}$. From Theorem 1 it follows that for any $r > 1/3$ there exists a function $f(z)$, represented by (1) and such that $|f(z)| < 1$ in

the unit disk, but (2) fails in the disk $\{z : |z| < r\}$. To complete the proof we use the functions $f(a_1z_1 + \dots + a_nz_n)$.

Proof of Theorem 9. If $z \in r \cdot D^\circ$, then from (16) it follows

$$\begin{aligned} \sum_{\alpha} |c_{\alpha}z^{\alpha}| &\leq |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^{\infty} \frac{|\alpha|^{|\alpha|}}{\alpha_1^{\alpha_1} \dots \alpha_n^{\alpha_n}} |z^{\alpha}| \\ &= |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^{\infty} \frac{|\alpha|^{|\alpha|}}{|\alpha|!} \frac{\alpha_1! \dots \alpha_n!}{\alpha_1^{\alpha_1} \dots \alpha_n^{\alpha_n}} \frac{|\alpha|!}{\alpha_1! \dots \alpha_n!} |z^{\alpha}| \\ &< |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \frac{k^k}{k!} r^k. \end{aligned}$$

Thus (5) holds for $r > r_0$, where r_0 is the root of equation (21).

Suppose $z \notin 1/3 \cdot D^\circ$; then we have for the function $f_a(z)$ (which was used in the proof of Theorem 6)

$$\sum_{\alpha} |c_{\alpha}z^{\alpha}| = \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^{\infty} a^{k-1} (|z_1| + \dots + |z_n|)^k \geq 1$$

if

$$\sum_{k=1}^{\infty} a^k (|z_1| + \dots + |z_n|)^k \geq \frac{a}{1+a}.$$

Hence inequality (5) fails as $a \rightarrow 1$.

4. FINAL REMARKS

Remark 2. In the proof of Theorem 5 we used the facts that in equality (18) the domain in consideration is the ball D^1 and that the estimates $d_{\alpha}(D^1) \leq 1$ are valid. Therefore, an analogous theorem holds for any complete n -circular domain D if $D^1 \subset D \subset U_1$. But for the hypercone D° this is not true (see Theorem 6).

Remark 3. In the proof of Theorem 6 was used the following quite rough inequality:

$$\sum_{|\alpha|=k} \frac{\alpha_1^{\alpha_1} \dots \alpha_n^{\alpha_n}}{\alpha_1! \dots \alpha_n!} \geq \sum_{|\alpha|=k} \frac{1}{\alpha_1! \dots \alpha_u!}.$$

Therefore, in fact, the estimate from above in the theorem might be decreased. For example, if $n = 2$ (10) implies $B_2(D^\circ) < 0.223332$, but using a PC one can show that $B_2(D^\circ) < 0.191373$. Hence, $0.183502 < B_2(D^\circ) < 0.191373$.

Remark 4. Comparing Theorem 2 and the corollary we can see that $B_n(D)$ depends essentially on the domain D since $B_n(U_1) = K_n$. It seems that it is more natural to consider not a single number in the problem considered in Theorem 2 and Theorem 3, but the largest subdomain D_B of D such that (5) holds. From [4] it follows, for example, that U_{1B} contains the ball $\frac{1}{3} \cdot D^1$, and from Theorem 9 it follows that $D^\circ_B \subset 1/3 \cdot D^\circ$.

Remark 5. Notice that there exist such unbounded n -circular domains D , for which the problems defined by the conditions (5) and (7) are equivalent. Consider, for example, the domains $D = \{z : |z|^\beta < c\}$, where β is a multiindex with coprime components. Bounded holomorphic functions in such domains depend only on one

variable z^β and the exact value of Bohr's radius equals $(\frac{1}{3})^{1/|\beta|}$. Thus there exist n -circular domains with Bohr's radius arbitrarily close to 1. Therefore it is impossible to remove the assumption about convexity in Theorem 8.

Remark 6. Unlike Theorems 1–7, in Theorem 8 the series (13) is not a basis expansion. In [1] it was shown that there exists a basis in the space of all holomorphic functions in Q , consisting of homogeneous polynomials $P_{k,m}(z)$, where k is the degree of the polynomial and $m = 1, \dots, \binom{k+n-1}{k}$. It is reasonable to consider Bohr's problem for such basis expansions, but there are no results yet in this direction. This question is a particular case of a more general problem in Bohr spirit for expansions by an arbitrary basis in a domain $D \subset \mathbf{C}^n$ (under some restrictions on the basis, because, for example, the basis $\{1, (z-1)/2, z^2, z^3, \dots\}$ in the unit disk has no Bohr's constant).

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