

**A REFLEXIVITY PROBLEM
CONCERNING THE C^* -ALGEBRA $C(X) \otimes \mathcal{B}(\mathcal{H})$**

LAJOS MOLNÁR

(Communicated by David R. Larson)

ABSTRACT. Let X be a compact Hausdorff space and let \mathcal{H} be a separable Hilbert space. We prove that the group of all order automorphisms of the C^* -algebra $C(X) \otimes \mathcal{B}(\mathcal{H})$ is algebraically reflexive.

1. INTRODUCTION

Reflexivity problems concerning subspaces of the algebra $\mathcal{B}(\mathcal{H})$ of all bounded linear operators acting on a Hilbert space \mathcal{H} is one of the most active research areas in operator theory. In the past decade, similar questions concerning some important subsets of linear transformations acting on Banach algebras (not on Hilbert spaces) have also attracted attention. The originators of the research in this direction are Kadison and Larson. In the papers [3], [7] the reflexivity of the Lie algebra of derivations was treated. In [6, Some concluding remarks (5), p. 298] Larson raised the reflexivity problem for automorphism groups. This was investigated together with the similar problem for isometry groups in a series of papers [1, 8, 10, 11, 12, 13]. For example, in [10] we proved that the automorphism group and the isometry group of the suspension of $\mathcal{B}(\mathcal{H})$, \mathcal{H} being a separable infinite dimensional Hilbert space, are algebraically reflexive. As for the automorphism group, this result was a consequence of [10, Theorem 2] stating that for any locally compact Hausdorff space X , if the automorphism group of $C_0(X)$ (the algebra of all continuous complex valued functions on X vanishing at infinity) is reflexive, then so is the automorphism group of $C_0(X) \otimes \mathcal{B}(\mathcal{H})$. The main theorem of the present paper gives a similar result for the group of all order automorphisms in case X is compact. The C^* -algebra $C(X) \otimes \mathcal{B}(\mathcal{H})$ is of importance because of several remarkable results concerning the structure of its derivations and automorphisms [5] (see also [4]).

Let us fix the concepts and the notation that we use throughout. Let \mathcal{X} be a Banach space (in fact, we shall be mostly interested in the case when it is a

Received by the editors November 16, 1998 and, in revised form, May 3, 1999.

1991 *Mathematics Subject Classification*. Primary 47B48, 47B49.

Key words and phrases. Reflexivity, order automorphism, C^* -algebra.

This research was supported from the following sources: 1) Joint Hungarian-Slovene research project supported by OMFb in Hungary and the Ministry of Science and Technology in Slovenia, Reg. No. SLO-2/96, 2) Hungarian National Foundation for Scientific Research (OTKA), Grant No. T-030082 F-019322, 3) a grant from the Ministry of Education, Hungary, Reg. No. FKFP 0304/1997.

©2000 American Mathematical Society

C^* -algebra) and for any subset $\mathcal{E} \subset \mathcal{B}(X)$ we define

$$\text{ref}_{al} \mathcal{E} = \{T \in \mathcal{B}(X) : Tx \in \mathcal{E}x \text{ for all } x \in X\}.$$

The elements of $\text{ref}_{al} \mathcal{E}$ can be described as those linear transformations which are locally in \mathcal{E} . The set \mathcal{E} is called algebraically reflexive if $\text{ref}_{al} \mathcal{E} = \mathcal{E}$, that is, when, so to say, every linear transformation locally in \mathcal{E} is globally in \mathcal{E} . We use the expression “algebraically reflexive” since there is an analogous concept called “topological reflexivity”.

If \mathcal{A} is a C^* -algebra, denote by \mathcal{A}^+ the cone of all positive elements in \mathcal{A} . The linear bijection $\phi : \mathcal{A} \rightarrow \mathcal{A}$ is called an order automorphism of \mathcal{A} if $\phi(a)$ is positive if and only if a is positive ($a \in \mathcal{A}$). Clearly, every linear map ψ on \mathcal{A} which is a local order automorphism (meaning that ψ is locally in the set of all order automorphisms of \mathcal{A}) preserves the positivity in both directions ($\psi(a) \geq 0$ if and only if $a \geq 0$) and it is injective. Therefore, to prove that the group of order automorphisms of \mathcal{A} is algebraically reflexive, the only thing which needs verification is that every local order automorphism of \mathcal{A} is surjective.

If \mathcal{R} and \mathcal{R}' are $*$ -algebras, then the linear map $\phi : \mathcal{R} \rightarrow \mathcal{R}'$ is called a Jordan $*$ -homomorphism if it satisfies

$$\phi(x^2) = \phi(x)^2, \quad \phi(x^*) = \phi(x)^* \quad (x \in \mathcal{R}).$$

The definition of Jordan $*$ -isomorphisms and that of Jordan $*$ -automorphisms should be clear. The set of all Jordan $*$ -automorphisms of the $*$ -algebra \mathcal{R} is denoted by $\text{Jaut}(\mathcal{R})$.

Let X be a compact Hausdorff space and let \mathcal{H} be a Hilbert space. Consider the tensor product $C(X) \otimes \mathcal{B}(\mathcal{H})$ which is meant in the C^* -algebra sense. It is well-known that this C^* -algebra is isomorphic and isometric to the algebra $C(X, \mathcal{B}(\mathcal{H}))$ of all continuous functions from X into $\mathcal{B}(\mathcal{H})$ and that the linear span of the elements fA ($f \in C(X)$, $A \in \mathcal{B}(\mathcal{H})$) is dense in $C(X, \mathcal{B}(\mathcal{H}))$. The unit of $C(X)$ is denoted by 1 while the unit of $\mathcal{B}(\mathcal{H})$ is denoted by I .

2. THE RESULTS

Our first theorem describes the order automorphisms of $C(X, \mathcal{B}(\mathcal{H}))$.

Theorem 2.1. *Let \mathcal{H} be a separable Hilbert space and let X be a compact Hausdorff space. If ϕ is an order automorphism of $C(X, \mathcal{B}(\mathcal{H}))$, then ϕ is of the form*

$$(1) \quad \phi(f)(x) = b(x)([\tau(x)](f(\varphi(x))))b(x)^* \quad (f \in C(X, \mathcal{B}(\mathcal{H})), x \in X),$$

where $b \in C(X, \mathcal{B}(\mathcal{H}))$ is such that its values are invertible positive operators, $\tau : X \rightarrow \text{Jaut}(\mathcal{B}(\mathcal{H}))$ is a function such that $x \mapsto \tau(x)$ and $x \mapsto \tau(x)^{-1}$ are strongly continuous and $\varphi : X \rightarrow X$ is a homeomorphism.

Proof. First suppose that ϕ is unital, that is, $\phi(1I) = 1I$. According to a well-known theorem of Kadison [2, Corollary 5] every unital order automorphism of a C^* -algebra is a Jordan $*$ -automorphism. Thus ϕ is a Jordan $*$ -automorphism of $C(X, \mathcal{B}(\mathcal{H}))$. Since the closed Jordan ideals of a C^* -algebra coincide with its closed associative ideals, following the arguments used in the proofs of [10, Lemma 2.1 and Theorem 1], we obtain the form (1) with $b = 1I$. Otherwise, define $c(x) = \sqrt{\phi(1I)(x)}^{-1}$. We show that the unital map $\psi : C(X, \mathcal{B}(\mathcal{H})) \rightarrow C(X, \mathcal{B}(\mathcal{H}))$ defined by

$$\psi(f)(x) = c(x)\phi(f)(x)c(x) \quad (f \in C(X, \mathcal{B}(\mathcal{H})), x \in X)$$

is an order automorphism. To see this, we only have to prove that $c : X \rightarrow \mathcal{B}(\mathcal{H})$ is continuous. But this follows from the continuity of the inverse and the square-root operations. It is now trivial to complete the proof. \square

Remark. In fact, (1) can be written in a more explicit form if one recalls that the Jordan *-automorphisms of the algebra $\mathcal{B}(\mathcal{H})$ are exactly the maps

$$A \longmapsto UAU^*, \quad A \longmapsto VA^{tr}V^*,$$

where U, V are unitary operators on \mathcal{H} and tr denotes the transpose with respect to an arbitrary but fixed orthonormal basis in \mathcal{H} .

Concerning the reflexivity of the group of all order automorphisms of the function algebra $C(X)$ we have the following result. Using the above mentioned result of Kadison, for example, one can easily verify that the order automorphisms of $C(X)$ are precisely the maps

$$f \longmapsto h \cdot f \circ \varphi,$$

where $h \in C(X)$ is everywhere positive and $\varphi : X \rightarrow X$ is a homeomorphism.

Theorem 2.2. *Let X be a first countable compact Hausdorff space. Then the group of all order automorphisms of $C(X)$ is algebraically reflexive.*

Proof. Let $\phi : C(X) \rightarrow C(X)$ be a linear map which is a local order automorphism of $C(X)$. Clearly, we may suppose that $\phi(1) = 1$. We claim that in this case ϕ is an algebra endomorphism of $C(X)$. To see this, let $x \in X$ be fixed and consider the linear functional $\phi_x(f) = \phi(f)(x)$ ($f \in C(X)$). Since this is a positive linear functional, by the Riesz representation theorem we have a regular probability measure μ on the Borel sets of X such that

$$\phi_x(f) = \int_X f d\mu \quad (f \in C(X)).$$

Suppose that there are two disjoint closed sets A, B in X which are of positive measure. Then, by Urysohn's lemma, we can choose continuous functions $f, g : X \rightarrow [0, 1]$ with disjoint support such that $f|_A = 1, f|_B = 0$ and $g|_A = 0, g|_B = 1$. Consider the function $f + ig$. Clearly, its integral is a complex number with nonzero real and imaginary parts. On the other hand, from the local property of ϕ it follows that $\phi_x(f + ig)$ should be a positive scalar multiple of one of the values of $f + ig$. Since the range of $f + ig$ lies in $\mathbb{R} \cup i\mathbb{R}$, we arrive at a contradiction. This shows that either $\mu(A) = 0$ or $\mu(B) = 0$. Since this holds for every pair of disjoint closed sets of X , by regularity we obtain that every Borel set in X has measure either 0 or 1. This implies that μ is a Dirac measure. In fact, it follows that the integral (as a number) of every simple function is contained in the range of that function. Then, approximating continuous functions by simple ones, we find that the integral of every continuous function $f \in C(X)$ belongs to its range. Therefore, ϕ_x is a linear functional on $C(X)$ which sends 1 to 1 and has the property that $\phi_x(f)$ belongs to the spectrum of f . By the famous Gleason-Kahane-Żelazko theorem it follows that ϕ_x is multiplicative and, therefore, a point-evaluation which gives us that its representing measure is a Dirac measure. Hence we obtain that ϕ is a unital endomorphism of $C(X)$. But the form of such morphisms of $C(X)$ is well-known. Namely, there is a continuous function $\varphi : X \rightarrow X$ such that

$$\phi(f) = f \circ \varphi \quad (f \in C(X)).$$

It remains to prove that φ is bijective. The surjectivity of φ follows from the injectivity of ϕ . To the injectivity of φ observe that using Urysohn's lemma, by the first countability of X , we have a nonnegative function f_0 on X which vanishes at exactly one point. If φ was noninjective, we would obtain that $\phi(f_0)$ vanishes at more than one point. But this is a contradiction due to the local property of ϕ . This completes the proof. \square

Remark. We show that in our previous result the condition of first countability is essential. The following example was communicated to us by Félix Cabello Sánchez on the occasion of another reflexivity problem. Consider the compact Hausdorff space $\mathbb{N}^* = \beta\mathbb{N} \setminus \mathbb{N}$, where $\beta\mathbb{N}$ stands for the Stone-Čech compactification of the positive integers. Let $A, B \subset \mathbb{N}$ be disjoint infinite sets. Then, by [15, 3.10, 3.14, 3.15], the sets $A^* = cl_{\beta\mathbb{N}}A \setminus A$ and $B^* = cl_{\beta\mathbb{N}}B \setminus B$ are disjoint clopen sets in \mathbb{N}^* and they are both homeomorphic to \mathbb{N} . This gives us the existence of a continuous surjective and noninjective function $\varphi : \mathbb{N}^* \rightarrow \mathbb{N}^*$. Consider the map $\phi : f \mapsto f \circ \varphi$. It is an interesting property of \mathbb{N}^* that if two functions $f, g \in C(\mathbb{N}^*)$ have the same range, then there is a homeomorphism $h : \mathbb{N}^* \rightarrow \mathbb{N}^*$ such that $f = g \circ h$ [15, first section on p. 83]. It then follows that for every $f \in C(\mathbb{N}^*)$ there exists a homeomorphism $h : \mathbb{N}^* \rightarrow \mathbb{N}^*$ such that $f \circ \varphi = f \circ h$. Consequently, ϕ is a local order automorphism of $C(\mathbb{N}^*)$ which is not surjective.

The following result shows that there are compact Hausdorff spaces which are not first countable but have our reflexivity property.

Proposition 2.3. *The group of all order automorphisms of $C(\beta\mathbb{N}) \cong \ell_\infty$ is algebraically reflexive.*

Proof. Let $\phi : C(\beta\mathbb{N}) \rightarrow C(\beta\mathbb{N})$ be a linear map which is a local order automorphism. As before, we can suppose that $\phi(1) = 1$. By the proof of Theorem 2.2 we obtain that there is a surjective function $\varphi : \beta\mathbb{N} \rightarrow \beta\mathbb{N}$ such that $\phi(f) = f \circ \varphi$ ($f \in C(\beta\mathbb{N})$). We assert that $\varphi(\mathbb{N}) \subset \mathbb{N}$. Let c_0 denote the space of all complex sequences converging to 0. Observe that, under the identification of ℓ_∞ and $C(\beta\mathbb{N})$, c_0 can be considered as a subalgebra of $C(\beta\mathbb{N})$. Since the set of all isolated points of $\beta\mathbb{N}$ is exactly \mathbb{N} , every homeomorphism of $\beta\mathbb{N}$ maps \mathbb{N} onto \mathbb{N} . Let $f \in C(\beta\mathbb{N})$ be defined by $f(n) = 1/n$ ($n \in \mathbb{N}$). By the local property of ϕ it follows that the set of all nonzeros of $\phi(f)$ is \mathbb{N} . On the other hand, since $\phi(f) = f \circ \varphi$, we obtain that the set of all nonzeros of $\phi(f)$ is the preimage of the set of nonzeros of f under φ . Consequently, we have $\mathbb{N} = \varphi^{-1}(\mathbb{N})$.

Clearly, ϕ can be considered as a map from ℓ_∞ into itself. We obtain

$$\phi((\lambda_n)) = (\lambda_{\varphi(n)}) \quad ((\lambda_n) \in \ell_\infty).$$

Here φ is treated as a function from \mathbb{N} into itself. Since ϕ is injective, we obtain that φ maps onto \mathbb{N} . We are done if we prove that φ is injective as well. To see this, observe that, for any sequence (λ_n) with entries all zero but one, $\phi((\lambda_n))$ must have the same property. The proof is now complete. \square

In the proof of our main result we use the following easy lemma.

Lemma 2.4. *The only nontrivial subspaces in $\mathbb{C}\mathcal{B}(\mathcal{H})^+$ which contain an invertible operator are one-dimensional.*

Proof. Let \mathcal{M} be a nontrivial subspace with the above property. Clearly, we can assume that $I \in \mathcal{M}$. Let $A \in \mathcal{M}$ be arbitrary. Since $A + \lambda I$ is a scalar multiple of a

positive operator, it follows that, for every $\lambda \in \mathbb{C}$, the spectrum $\sigma(A) + \lambda$ of $A + \lambda I$ lies on a straight line of the plane going through 0. This trivially implies that the spectrum of A consists of one element, which, by the normality of A , gives us that A is a scalar. \square

Remark. We note that the conclusion in the above lemma remains valid even if we do not assume that the subspace in question contains an invertible element. In that case the proof is not so trivial and can be based on an observation of Radjavi and Rosenthal [14, Remark iii, p. 691] stating that every subspace of normal operators is commutative. However, the above lemma is sufficient for our purposes.

Theorem 2.5. *Let \mathcal{H} be a separable Hilbert space. If the group of all order automorphisms of $C(X)$ is algebraically reflexive, then so is the group of all order automorphisms of $C(X) \otimes \mathcal{B}(\mathcal{H})$.*

Proof. Let $\phi : C(X, \mathcal{B}(\mathcal{H})) \rightarrow C(X, \mathcal{B}(\mathcal{H}))$ be a linear map which is a local order automorphism of $C(X, \mathcal{B}(\mathcal{H}))$. Clearly, we may assume that $\phi(1I) = 1I$.

Fix $x \in X$ and consider the map

$$f \mapsto \phi(fI)(x)$$

on $C(X)$. By Theorem 2.1, this is a linear map whose values are lying in $\mathbb{C}\mathcal{B}(\mathcal{H})^+$. Since $\phi(1I) = 1I$, from Lemma 2.4 it follows that the map $f \mapsto \phi(fI)$ can be considered as a linear transformation on $C(X)$ into itself. We assert that it is an automorphism of $C(X)$. By the local form of ϕ , for every $f \in C(X)$ there exist a function $h_f \in C(X, \mathcal{B}(\mathcal{H}))$ whose values are invertible and positive, and a homeomorphism $\varphi_f : X \rightarrow X$ such that

$$\phi(fI) = f \circ \varphi_f \cdot h_f.$$

Since the values of $\phi(fI)$ are scalars, we easily obtain that h_f can be chosen to be scalar valued as well. So, the map

$$f \mapsto \phi(fI)$$

can be considered as a local order automorphism of $C(X)$. By our assumption on $C(X)$ it follows that this map is an order automorphism of $C(X)$. Since we have supposed that $\phi(1I) = 1I$, we infer that the above map is in fact an automorphism of $C(X)$; that is, there exists a homeomorphism $\varphi : X \rightarrow X$ for which

$$\phi(fI) = (f \circ \varphi)I \quad (f \in C(X)).$$

Let us fix a point $x \in X$ and consider the map

$$A \mapsto \phi(1A)(x)$$

on $\mathcal{B}(\mathcal{H})$. By the local form of ϕ , the above map is a local order automorphism of $\mathcal{B}(\mathcal{H})$. By [9, Theorem 2], every local order automorphism of $\mathcal{B}(\mathcal{H})$ is an order automorphism. Therefore, it follows that there exist a positive invertible operator $b(x) \in \mathcal{B}(\mathcal{H})$ and a Jordan *-automorphism $\tau(x)$ of $\mathcal{B}(\mathcal{H})$ such that

$$\phi(1A)(x) = b(x)[\tau(x)](A)b(x) \quad (x \in X).$$

Since ϕ is unital, we infer that $b(x)^2 = I$ which gives us that $b(x) = I$. Hence, there is a function $\tau : X \rightarrow \text{Jaut}(\mathcal{B}(\mathcal{H}))$ such that

$$\phi(1A)(x) = [\tau(x)](A) \quad (x \in X).$$

Clearly, τ is strongly continuous.

We claim that

$$\phi(fA) = f(\varphi(x))([\tau(x)](A)) \quad (f \in C(X), A \in \mathcal{B}(\mathcal{H}), x \in X).$$

To see this, pick $x \in X$. Let $A \in \mathcal{B}(\mathcal{H})$ be positive and invertible. Consider the linear map

$$f \mapsto \phi(fA)(x).$$

The image of this map is a linear subspace in $\mathbb{C}\mathcal{B}(\mathcal{H})^+$ containing the invertible operator $\phi(1A)(x) = [\tau(x)](A)$. So, by Lemma 2.4 it follows that every member of the above linear subspace is a scalar multiple of $[\tau(x)](A)$. Fixing f for a moment and considering any other positive invertible operator B , we have constants $\alpha, \beta, \gamma \in \mathbb{C}$ such that

$$\begin{aligned} \phi(fA)(x) &= \alpha[\tau(x)](A), \\ \phi(fB)(x) &= \beta[\tau(x)](B), \\ \phi(f(A+B))(x) &= \gamma[\tau(x)](A+B). \end{aligned}$$

By the additivity of ϕ , in case A, B are linearly independent, we find that $\alpha = \beta$. Clearly, this is the case also when A, B are linearly dependent. This yields that for every $f \in C(X)$ and $x \in X$, there is a constant $\lambda_{f,x}$ such that

$$\phi(fA)(x) = \lambda_{f,x}[\tau(x)](A)$$

holds true for every positive invertible operator A . Putting $A = I$ we obtain

$$f(\varphi(x))I = \phi(fI)(x) = \lambda_{f,x}[\tau(x)](I) = \lambda_{f,x}.$$

This means that

$$(2) \quad \phi(fA)(x) = f(\varphi(x))[\tau(x)](A)$$

holds for every $f \in C(X)$, $x \in X$, and positive invertible $A \in \mathcal{B}(\mathcal{H})$. Since every operator in $\mathcal{B}(\mathcal{H})$ is a linear combination of positive invertible operators, it follows that (2) holds true for every operator $A \in \mathcal{B}(\mathcal{H})$. This gives us that

$$\phi(fA)(x) = [\tau(x)]((fA)(\varphi(x)))$$

for every $f \in C(X)$, $x \in X$ and $A \in \mathcal{B}(\mathcal{H})$. Since the set of all elements of the form fA ($f \in C(X)$, $A \in \mathcal{B}(\mathcal{H})$) is dense in $C(X, \mathcal{B}(\mathcal{H}))$, by the continuity of ϕ we obtain

$$(3) \quad \phi(f)(x) = [\tau(x)](f(\varphi(x)))$$

for every $f \in C(X, \mathcal{B}(\mathcal{H}))$. Up to this point, everything was done in order to be able to prove that ϕ is surjective. This is now easy. In fact, we first verify that $x \mapsto \tau(x)^{-1}$ is strongly continuous. Since every Jordan *-automorphism of a C^* -algebra is an isometry, this follows from the equality

$$\begin{aligned} \|\tau(x)^{-1}(A) - \tau(x_0)^{-1}(A)\| &= \|A - \tau(x)(\tau(x_0)^{-1}(A))\| \\ &= \|\tau(x_0)(\tau(x_0)^{-1}(A)) - \tau(x)(\tau(x_0)^{-1}(A))\| \end{aligned}$$

and from the strong continuity of τ . To show the surjectivity of ϕ , it is now enough to check that, for every $g \in C(X, \mathcal{B}(\mathcal{H}))$, the function $x \mapsto [\tau(x)^{-1}](g(x))$ belongs

to $C(X, \mathcal{B}(\mathcal{H}))$. But this follows from the inequality

$$\begin{aligned} & \|\tau(x)^{-1}(g(x)) - \tau(x_0)^{-1}(g(x_0))\| \\ &= \|\tau(x)^{-1}(g(x) - g(x_0))\| + \|\tau(x)^{-1}(g(x_0)) - \tau(x_0)^{-1}(g(x_0))\| \\ &\leq \|g(x) - g(x_0)\| + \|\tau(x)^{-1}(g(x_0)) - \tau(x_0)^{-1}(g(x_0))\| \end{aligned}$$

using the continuity of g and the strong continuity of $x \mapsto \tau(x)^{-1}$. The proof is now complete. \square

REFERENCES

1. C.J.K. Batty and L. Molnár, *On topological reflexivity of the groups of *-automorphisms and surjective isometries of $B(H)$* , Arch. Math. **67** (1996), 415–421. MR **97f**:47034
2. R.V. Kadison, *A generalized Schwarz inequality and algebraic invariants for operator algebras*, Ann. of Math. **56** (1952), 494–503. MR **14**:481c
3. R.V. Kadison, *Local derivations*, J. Algebra **130** (1990), 494–509. MR **91f**:46092
4. R.V. Kadison and J. Ringrose, *Derivations and automorphisms of operator algebras*, Comm. Math. Phys. **4** (1967), 32–63. MR **34**:6552
5. E.C. Lance, *Automorphisms of certain operator algebras*, Amer. J. Math. **91** (1969), 160–174. MR **39**:3324
6. D.R. Larson, *Reflexivity, algebraic reflexivity and linear interpolation*, Amer. J. Math. **110** (1988), 283–299. MR **89d**:47096
7. D.R. Larson and A.R. Sourour, *Local derivations and local automorphisms of $B(X)$* , in Proc. Sympos. Pure Math. 51, Part 2, Providence, Rhode Island 1990, 187–194. MR **91k**:47106
8. L. Molnár, *The set of automorphisms of $B(H)$ is topologically reflexive in $B(B(H))$* , Studia Math. **122** (1997), 183–193. MR **98e**:47068
9. L. Molnár and P. Šemrl, *Order isomorphisms and triple isomorphisms of operator ideals and their reflexivity*, Arch. Math. **69** (1997), 497–506. MR **99a**:47054
10. L. Molnár and M. Györy, *Reflexivity of the automorphism and isometry groups of the suspension of $\mathcal{B}(\mathcal{H})$* , J. Funct. Anal. **159** (1998), 568–586. CMP 99:04
11. L. Molnár and B. Zalar, *Reflexivity of the group of surjective isometries on some Banach spaces*, Proc. Edinb. Math. Soc. **42** (1999), 17–36. CMP 99:09
12. L. Molnár and B. Zalar, *On local automorphisms of group algebras of compact groups*, Proc. Amer. Math. Soc. **128** (2000), 93–99. CMP 98:16
13. L. Molnár, *Reflexivity of the automorphism and isometry groups of C^* -algebras in BDF theory*, Arch. Math. (to appear)
14. H. Radjavi and P. Rosenthal, *On invariant subspaces and reflexive algebras*, Amer. J. Math. **91** (1969), 683–692. MR **40**:4796
15. R. C. Walker, *The Stone-Čech Compactification*, Springer, 1974. MR **52**:1595

INSTITUTE OF MATHEMATICS, LAJOS KOSSUTH UNIVERSITY, 4010 DEBRECEN, P.O. BOX 12, HUNGARY

E-mail address: molnar1@math.klte.hu