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BEHAVIOR OF POSITIVE RADIAL SOLUTIONS FOR QUASILINEAR ELLIPTIC EQUATIONS

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Abstract. We establish a necessary and sufficient condition so that positive radial solutions to

$$-\mathrm{div}(A(|\nabla u|)\nabla u) = f(u), \quad \text{in } B_R(0)\setminus\{0\},\ R>0,$$

having an isolated singularity at $x = 0$, behave like a corresponding *fundamental* solution. Here, $A : \mathbb{R}\setminus\{0\} \to \mathbb{R}$ and $f : [0,\infty) \to [0,\infty)$ are continuous functions satisfying some mild growth restrictions.

1. Introduction

Let $\phi : \mathbb{R} \to \mathbb{R}$ be an increasing odd homeomorphism from $\mathbb{R}$ onto itself and let $A : \mathbb{R}\setminus\{0\} \to \mathbb{R}$ be defined by $A(s) = \frac{\phi(s)}{s}$. In this paper we study the behavior of positive radial solutions in a neighborhood of zero for the strongly nonlinear problem

$$(D) \qquad -\mathrm{div}(A(|\nabla u(x)|)\nabla u(x)) = f(u(x)), \qquad x \in \Omega^*,$$

where $\Omega^* := B_R(0)\setminus\{0\} \subset \mathbb{R}^N$, $N > 1$. Here $f \in C(\mathbb{R}^+,\mathbb{R}^+)$, with $\mathbb{R}^+ = [0,+\infty)$.

Radial solutions to (D) satisfy

$$(P) \qquad -(r^{N-1}\phi(u'))' = r^{N-1}f(u), \quad r \in (0,R),$$

where $r = |x|$, $x \in \Omega^*$, and $'$ denotes $\frac{d}{dr}$.

By a solution to (P) we mean a function $u \in C^1(0,R)$ and such that $\phi(u') \in C^1(0,R)$ which satisfies the equation in (P).

Problem (D) includes the class of equations containing the m-Laplacian operator,

$$(D_H) \qquad -\mathrm{div}(|\nabla u|^{m-2}\nabla u) = f(u), \quad 1 < m < N.$$

In case that $f(u) = |u|^{\delta-1}u$, the behavior of positive solutions to (D_H) near zero is well known; see [2], [4].

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In particular, if $1 < m < N$ and $m - 1 < \delta$, by setting $\delta^* := \frac{N(m-1)}{N-m}$, the *Serrin number*, and $h(r) := r^{-\frac{N-m}{m-1}}$, the *fundamental solution* for the m-Laplacian operator, we have that the following criterium for the behavior of positive radial solutions to (D_H) relative to h can be obtained from [2]:

(i) if $\delta < \delta^*$, then any positive radial singular solution to (D_H) satisfies $\lim_{r\to 0} \frac{u(r)}{h(r)} = c$, for some positive constant c,

(ii) If $\delta \geq \delta^*$, then any radial nonnegative solution u to (D_H) satisfies $\lim_{r\to 0} \frac{u(r)}{h(r)} = 0$.

In case (i), we say the singular solution is of *fundamental* type. Clearly from (i) and (ii), a positive radial singular solution u of (D_H) (with f a power) satisfies:

$$\lim_{r\to 0} \frac{u(r)}{h(r)} \neq 0 \qquad \text{if and only if} \qquad \delta < \delta^*, \tag{1.1}$$

and note that this is equivalent to

$$\lim_{r\to 0} \frac{u(r)}{h(r)} \neq 0 \qquad \text{if and only if} \qquad \int_0 s^{N-1} f(h(s))ds < \infty, \tag{1.2}$$

with $f(u) = |u|^{\delta-1}u$. In this direction, and for the case $m = 2$, we refer to Remark 2 in [3].

In [1] the study of the behavior and existence of singular solutions for problem (P) was initiated. Following [1], solutions to

$$-(r^{N-1}\phi(h'))' = 0, \qquad r > 0,$$

are called *fundamental solutions* to (P). By direct integration they have the form

$$h_C(r) \equiv h_C(r, R) := \int_r^R \phi^{-1}(Ct^{1-N})\, dt, \tag{1.3}$$

with $C > 0$. Since we are interested in singular solutions we assume as in [1] that

$$\lim_{r\to 0^+} h_C(r) = +\infty. \tag{1.4}$$

Also from [1] if a solution u to (P) satisfies $\liminf_{r\to 0} \frac{u(r)}{h_C(r)} > 0$, we will say that u is of fundamental type. Let us set

$$\bar{p} - 1 = \liminf_{x\to\infty} \frac{\log(\phi(x))}{\log(x)} \quad \text{and} \quad \overline{\delta} = \limsup_{x\to\infty} \frac{\log(f(x))}{\log(x)}.$$

Under the assumptions (1.4) and

$$1 < \bar{p} < N, \quad \bar{p} - 1 < \overline{\delta} < \frac{N(\bar{p}-1)}{N-\bar{p}}, \tag{1.5}$$

with f ultimately increasing, we proved in [1] (see Theorem 3.2) that any positive singular solution to (P) is of fundamental type. Further, (1.5) implies that

$$\int_0 s^{N-1} f(h_C(s))ds < \infty \text{ for any } C > 0, \tag{1.6}$$

but not conversely.

All these facts together motivated the following question: Does a result like (1.2) hold for problem (P) within the generality of [1]? In this paper, by imposing suitable conditions on ϕ and f, we give a positive answer to this question (see Theorem 2.1 below). As a consequence of this result, in the last section of this paper we will give an example for which all the conditions of Theorem 3.2 in [1] are satisfied except the last inequality in (1.5) which we replace by an equality. Clearly then we cannot use Theorem 3.2 to classify singular solutions. Nevertheless we will see that Theorem 2.1 can be used to obtain this classification.

Now we introduce some notation. Throughout the paper we will set

$$\Phi(x)=\int_0^x \phi(t)dt, \quad \Phi^*(x)=\int_0^x \phi^{-1}(s)ds, \quad F(x)=\int_0^x f(t)dt, \tag{1.7}$$

and

$$p^\Phi := \limsup_{x\to+\infty} \frac{x\phi(x)}{\Phi(x)}, \quad \delta_F := \liminf_{x\to+\infty} \frac{xf(x)}{F(x)} - 1. \tag{1.8}$$

Furthermore we will assume

$$1 < p^\Phi < \infty, \qquad 0 < \delta_F < \infty. \tag{H}$$

In the next section we state and prove our main result. The proof of it is based on the use of appropriate supersolutions.

2. Main result

In this section we will prove our following main result which gives a characterization of positive singular solutions to (P) of fundamental type.

Theorem 2.1. *Let ϕ be an odd increasing homeomorphism from $\mathbb{R}$ onto itself satisfying* (1.4), *and let $f \in C(\mathbb{R}^+)$ be an ultimately increasing function. Assume that ϕ and f satisfy (H) and the superlinearity type of condition*

$$p^\Phi < \delta_F + 1. \tag{2.1}$$

Let u be a positive singular solution to (P). Then the following statements are equivalent:

(i) *There exists $C>0$ such that $\liminf_{r\to 0} \frac{u(r)}{h_C(r)} > 0$ (i.e. u is of fundamental type).*

(ii) *There exists $C>0$ such that $\int_0 s^{N-1} f(h_C(s))ds < +\infty$.*

Remark 2.2. It follows from Theorem 4.1 in [1] that if $f \in C(\mathbb{R}^+)$ is ultimately increasing, $\liminf_{x\to\infty} \frac{x\phi(x)}{\Phi(x)} > 1$, and (ii) above holds, then positive solutions to (P) satisfying (i) in Theorem 2.1 indeed exist.

The proof of Theorem 2.1 is based on the use of supersolutions and on Lemma 2.4 below. Let us define first what we mean by a supersolution to (P).

Definition 2.3. We say that a positive function $w : (0, r_0) \mapsto (0, +\infty)$ is a supersolution to (P) if it satisfies

$$-(r^{N-1}\phi(w'))' \geq r^{N-1} f(w(r)), \quad r \in (0, r_0), \tag{2.2}$$

and

$$\lim_{r\to 0} r^{N-1}\phi(w') \leq 0. \tag{2.3}$$

We note that in this definition w need not be defined at 0.

For an example of a supersolution let us consider the case where $\phi(s) = s$ and $f(s) = s^\delta$, $N > 2$, in (2.3). Then, for $\frac{2}{N-2} < \delta < \frac{N}{N-2}$, we have that $w(r) = Cr^{2-\delta(N-2)}$ is a supersolution to the corresponding problem (P). Indeed, in this case

$$-(r^{N-1}w')' = C(\delta(N-2)-2)(N-\delta(N-2))r^{N-1}(r^{2-N})^\delta \geq r^{N-1}w^\delta$$

for $r \in (0, r_0(C))$ and also $\lim_{r\to 0} r^{N-1}w' = 0$.

Our next lemma is a key tool in proving our main result.

Lemma 2.4. *Let ϕ be an odd increasing homeomorphism of $\mathbb{R}$ onto $\mathbb{R}$ and let $f \in C(\mathbb{R}^+, \mathbb{R}^+)$ be ultimately increasing. Suppose that ϕ and f satisfy (H) and (2.1) and assume that there exists a supersolution w to (P). Then any nonnegative solution u to (P) satisfying*

$$\lim_{r\to 0^+} r^{N-1}\phi(u'(r)) = 0 \quad \textit{and} \quad \lim_{r\to 0^+} \frac{u(r)}{w(r)} = 0 \tag{2.4}$$

is a regular solution to (P).

Proof. Let u, w satisfy the hypotheses of the lemma, and assume by contradiction that u is not bounded. Then the second assertion in (2.4) implies that w is not bounded. We will next fix some constants that will be used throughout the proof. Let $\mu > 0$ be such that

$$\tilde{\delta}_F := \delta_F - \mu > p^\Phi + \mu - 1 := \tilde{p}^\Phi - 1 > 0.$$

By the definition of δ_F and p^Φ, there exists $x_1(\mu) = x_1 > 0$ such that

$$\tilde{\delta}_F + 1 < \frac{xf(x)}{F(x)} \quad \text{and} \quad \frac{x\phi(x)}{\Phi(x)} < \tilde{p}^\Phi \tag{2.5}$$

for all $x \geq x_1$, and thus, using the identity $x\phi(x) = \Phi(x) + \Phi^*(\phi(x))$, we find that for $x \geq \phi(x_1)$

$$\frac{x\phi^{-1}(x)}{\Phi^*(x)} > \frac{\tilde{p}^\Phi}{\tilde{p}^\Phi - 1} := \tilde{p}^\Phi_*.$$

Hence, if $x_0 = \max\{x_1, \phi(x_1)\}$, then the functions $x^{-\tilde{\delta}_F-1}F(x)$ and $x^{-\tilde{p}^\Phi_*}\Phi^*(x)$ are increasing for $x \geq x_0$. Also, since ϕ is increasing, we obtain that $\Phi^*(2x) \geq \int_x^{2x}\phi^{-1}(t)dt \geq x\phi^{-1}(x)$ for all $x \geq 0$. Similarly, by the ultimate monotonicity of f, we have that $F(2x) \geq xf(x)$ for all x sufficiently large. Hence, combining with (2.5), we obtain that

$$\phi^{-1}(x) \leq A\Big(\frac{x}{y}\Big)^{\tilde{p}^\Phi_*-1}\phi^{-1}(y), \quad f(x) \leq B\Big(\frac{x}{y}\Big)^{\tilde{\delta}_F} f(y), \tag{2.6}$$

for all x, y such that $y \geq 2x$ and $x \geq x_0$, where $A := 2^{\tilde{p}^\Phi_*-1}$ and $B := \frac{1}{\tilde{\delta}_F+1}$. For later use, we also set

$$q := (\tilde{p}^\Phi_* - 1)\tilde{\delta}_F, \quad C := 2^{q-1}AB^{\tilde{p}^\Phi_*-1}, \tag{2.7}$$

and we note that from (2.1), $q > 1$. From the first inequality in (2.6) we observe that

$$\phi^{-1}(x) \leq A\Big(\frac{x}{y}\Big)^{\tilde{p}^\Phi_*-1}\phi^{-1}(y) + \phi^{-1}(x_0) \tag{2.8}$$

for all $x,\ y > 0$ such that $2x \le y$. We will assume that x_0 is such that f is increasing for $x \ge x_0$. By using that $q > 1$, the unboundedness of u and w near zero, and the second assertion in (2.4), we can now choose $\varepsilon_1 \in (0, 1/4)$, $b \in (0, 1/4)$ and $r_* > 0$ such that

$$Cb^{q-1} \quad < \quad q-1, \tag{2.9}$$

$$r_*\phi^{-1}(x_0) + u(r_*) \quad < \quad \Big(b - \frac{Cb^q}{q-1}\Big)w(r_*), \quad x_0 \le u(r_*), \tag{2.10}$$

$$u(r) \quad \le \quad \varepsilon_1 w(r), \quad r \in (0, r_*), \tag{2.11}$$

and set $c := bw(r_*)$. Then, since ε_1 and b are less than $1/4$, and using that w is nonincreasing, we get that for all $s \in (0, r_*)$,

$$2\varepsilon_1 w(s) + 2c \le \frac{w(s)}{2} + \frac{w(r_*)}{2} \le w(s), \quad s \in (0, r_*), \tag{2.12}$$

and thus, from the second inequality in (2.6) with $x = \varepsilon_1 w(s) + c$ and $y = w(s)$, it holds that

$$f(\varepsilon_1 w(s) + c) \le B\Big(\varepsilon_1 + \frac{c}{w(s)}\Big)^{\tilde{\delta}_F} f(w(s)) \tag{2.13}$$

for all $s \in (0, r_*)$. By integrating the equation in (P) over $(0, r)$, $r \le r_*$, using the first assertion in (2.4), (2.13), and the fact that f is increasing for $x \ge x_0$, we obtain that

$$\begin{aligned} r^{N-1}\phi(|u'(r)|) &\le \int_0^r s^{N-1} f(u(s))ds && (2.14)\\ &\le \int_0^r s^{N-1} f(\varepsilon_1 w(s) + c)ds \\ &\le B\int_0^r s^{N-1}\Big(\varepsilon_1 + \frac{c}{w(s)}\Big)^{\tilde{\delta}_F} f(w(s))ds. && (2.15) \end{aligned}$$

Then, again using that w is nonincreasing, we have that $\dfrac{c}{w(s)} \le \dfrac{c}{w(r)}$ for all $0 < s \le r \le r_*$. Thus, since w is a supersolution to (P), we find that

$$\begin{aligned} r^{N-1}\phi(|u'(r)|) &\le B\Big(\varepsilon_1 + \frac{c}{w(r)}\Big)^{\tilde{\delta}_F} \int_0^r s^{N-1} f(w(s))ds \\ &\le B\Big(\varepsilon_1 + \frac{c}{w(r)}\Big)^{\tilde{\delta}_F} r^{N-1}\phi(|w'(r)|). \end{aligned}$$

Hence, we have that

$$|u'(r)| \le \phi^{-1}\Big(B\Big(\varepsilon_1 + \frac{c}{w(r)}\Big)^{\tilde{\delta}_F}\phi(|w'(r)|)\Big), \tag{2.16}$$

for all $r \in (0, r_*)$. Now, using that $B < 1$ and (2.12), we may apply (2.8) with $x = B\Big(\varepsilon_1 + \frac{c}{w(r)}\Big)^{\tilde{\delta}_F}\phi(|w'(r)|)$ and $y = \phi(|w'(r)|)$ to obtain from (2.16) that

$$|u'(r)| \le AB^{\tilde{p}_*^{\Phi}-1}\Big(\varepsilon_1 + \frac{c}{w(r)}\Big)^q |w'(r)| + \phi^{-1}(x_0),$$

where q is as defined in (2.7). Moreover, from the convexity of the function t^q, $q > 1$, and the definition of C in (2.7), we also get

$$|u'(r)| \le C\varepsilon_1^q |w'(r)| + Cc^q (w(r))^{-q}|w'(r)| + \phi^{-1}(x_0).$$

Integrating this inequality over (r, r_*) and using that $|w'(s)| = -w'(s)$, we obtain

$$u(r) - u(r_*) \le C\varepsilon_1^q w(r) - Cc^q \int_r^{r_*} (w(r))^{-q} w'(s) ds + r_* \phi^{-1}(x_0),$$

and thus taking into account that $q > 1$, we have

$$u(r) \le C\varepsilon_1^q w(r) + \frac{Cc^q}{(q-1)w(r_*)^q} w(r_*) + r_* \phi^{-1}(x_0) + u(r_*).$$

Now, from the definition of $c = bw(r_*)$ and the above bound for u, we get from (2.9) and (2.10)

$$u(r) \le C\varepsilon_1^q w(r) + \frac{Cb^q}{q-1} w(r_*) + bw(r_*) - \frac{Cb^q}{q-1} w(r_*).$$

Hence, we conclude that

$$u(r) \le \varepsilon_2 w(r) + c, \quad \text{for all} \quad r \in (0, r_*),$$

where $\varepsilon_2 = C\varepsilon_1^q$. Repeating this process n times, we find that

$$u(r) \le \varepsilon_n w(r) + c, \quad \text{for all} \quad r \in (0, r_*), \tag{2.17}$$

where the sequence $\{\varepsilon_n\}$ is recursively defined by $\varepsilon_{n+1} = C\varepsilon_n^q$, i.e., $\varepsilon_{n+1} = C^n \varepsilon_1^{q^n}$. Then, since

$$\frac{\varepsilon_{n+1}}{\varepsilon_n} = C\varepsilon_1^{q^{n-1}(q-1)} \to 0 \text{ as } n \to \infty,$$

it follows that the sequence $\{\varepsilon_n\}$ tends to 0 as n tends to infinity, leading in this way to the contradiction that u is bounded. □

Remark 2.5. We note that the result is the same if instead of assuming that f is ultimately increasing, we assume that f satisfies the additional condition

$$\limsup_{x \to +\infty} \frac{xf(x)}{F(x)} < +\infty.$$

Indeed, in this case it can be shown that given $\mu > 0$ there exist x_0 and a positive constant d such that

$$f(s) \le df(t) \quad \text{for all } t \ge s \ge x_0, \tag{2.18}$$

and hence the same argument is valid by a suitable modification of the constants involved.

We are now in position to prove Theorem 2.1.

Proof. Let the assumptions of Theorem 2.1 hold and let u be a positive singular solution to (P).

(i) ⇒ *(ii)*. As in the proof of Theorem 2.1 in [1], $\liminf_{r \to 0} \frac{u(r)}{h_{\tilde{C}}(r)} > 0$ for some $\tilde{C} > 0$ implies that there exist $C > 0$ and $r_* > 0$ such that $u(r) \ge h_C(r)$ for all $r \in (0, r_*)$. Since in this case the limit $\lim_{r \to 0} r^{N-1} \phi(u'(r)) < 0$, then by integrating the equation in (P) over (r, r_*) and using that f is ultimately increasing we obtain

$$-r_*^{N-1} \phi(u'(r_*)) \ge \int_r^{r_*} s^{N-1} f(h_C(s)) ds,$$

and hence $(i) \Rightarrow (ii)$.

(ii) $\Rightarrow$ *(i)*. Now we assume that $\int_0 s^{N-1}f(h_{\tilde{C}}(s))ds < \infty$ for some positive constant $\tilde{C}$ and continue our argument by contradiction. Thus we assume that u is a positive singular solution to (P) such that $\liminf_{r\to 0}\frac{u(r)}{h_C(r)} = 0$ for all $C > 0$. Then, by Theorem 2.1 in [1], it follows that

$$\lim_{r\to 0}\frac{u(r)}{h_C(r)} = 0 \quad \text{for all } C > 0. \tag{2.19}$$

Hence, by taking $C = \tilde{C}$ in (2.19), and using that u is singular we obtain that $x_0 \le 2u(r) \le h_{\tilde{C}}(r)$ for all r small. Thus by the second inequality in (2.6), it holds that

$$\frac{f(u(r))}{f(h_{\tilde{C}}(r))} \le K\Big(\frac{u(r)}{h_{\tilde{C}}(r)}\Big)^{\tilde{\delta}_F} \tag{2.20}$$

for all r sufficiently small. Moreover, since $\lim_{r\to 0} r^{N-1}\phi(u'(r)) = 0$, by dividing both sides of the equation in (P) by $r^{N-1}f(h_{\tilde{C}}(r))$, using L'Hôpital's rule, (2.19), and (2.20), we obtain

$$\lim_{r\to 0^+}\frac{-r^{N-1}\phi(u'(r))}{\int_0^r s^{N-1}f(h_{\tilde{C}}(s))ds} = \lim_{r\to 0^+}\frac{f(u(r))}{f(h_{\tilde{C}}(r))} = 0,$$

and thus

$$\lim_{r\to 0^+}\frac{|u'(r)|}{\phi^{-1}\Big(r^{1-N}\int_0^r s^{N-1}f(h_{\tilde{C}}(s))ds\Big)} = 0. \tag{2.21}$$

Now set

$$w(r) := \int_r^{r_0}\phi^{-1}\Big(t^{1-N}\int_0^t s^{N-1}f(h_{\tilde{C}}(s))ds\Big).$$

Then w is nonnegative and strictly decreasing in $(0, r_0)$ and thus $\lim_{r\to 0^+} w(r) = L$ exists and it is either finite or $+\infty$. If it is finite, then by (2.21) u is bounded and thus regular. If $L = +\infty$, we can again apply L'Hôpital's rule in (2.21) to find that

$$\lim_{r\to 0^+}\frac{u(r)}{w(r)} = 0. \tag{2.22}$$

Next let us show that w is a supersolution to (P). Indeed, a direct computation shows that

$$-(r^{N-1}\phi(w'(r)))' = r^{N-1}f(h_{\tilde{C}}(r)),$$

and since by our assumption (ii) we have

$$\int_0^r s^{N-1}f(h_{\tilde{C}}(s))ds \le \tilde{C}$$

for r small, from the definition of w we conclude that $h_{\tilde{C}}(r) \ge w(r)$ for r small enough. Hence, we obtain that w satisfies (2.2) for r_0 small enough. Also, since w is a positive decreasing function, we have that (2.3) holds and thus w is a supersolution to (P). Thus, by (2.22), we conclude from Lemma 2.4 that u is regular, which is a contradiction. $\square$

3. An example

Let us consider problem (P), where the homeomorphism ϕ is implicitly given by

$$\phi^{-1}(x) = x^{\frac{1}{p-1}}(\log(x+x_0))^{-\frac{2(N-p)}{N(p-1)}}, \quad x \geq 0,$$

where $1 < p < N$ and $x_0 > \exp(2(N-p)/N)$. For the function f we take

$$f(x) = x^{\frac{N(p-1)}{N-p}}, \quad x \geq 0.$$

With the notation of the introduction, it can be verified that

$$p^{\Phi} = \bar{p} = p \quad \text{and} \quad \delta_F = \overline{\delta} = \frac{N(p-1)}{N-p}.$$

Hence, Theorem 3.2 in [1] cannot be applied.

However, by observing that

$$\lim_{s \to 0} \frac{h_1(s)}{s^{\frac{p-N}{p-1}}|\log(s)|^{-\frac{2(N-p)}{N(p-1)}}} = \frac{p-1}{(N-p)(N-1)^{\frac{2(N-p)}{N(p-1)}}},$$

we find that

$$\int_0 s^{N-1}(h_1(s))^{\frac{N(p-1)}{N-p}} ds \leq C \int_0 s^{-1}|\log(s)|^{-2} ds < \infty,$$

for some positive constant C and thus our Theorem 2.1 applies. We conclude from it that for ϕ and f as above, any positive singular solution to

$$-(r^{N-1}\phi(u'))' = r^{N-1}f(u), \quad r \in (0, R),$$

is of fundamental type.

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