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GLOBAL EXISTENCE AND BLOWUP OF SOLUTIONS FOR A PARABOLIC EQUATION WITH A GRADIENT TERM

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Abstract. The author discusses the semilinear parabolic equation $u_t = \Delta u + f(u) + g(u)|\nabla u|^2$ with $u|_{\partial\Omega} = 0,\ u(x,0) = \phi(x)$. Under suitable assumptions on f and g, he proves that, if $0 \le \phi \le \lambda\psi$ with $\lambda < 1$, then the solutions are global, while if $\phi \ge \lambda\psi$ with $\lambda > 1$, then the solutions blow up in a finite time, where ψ is a positive solution of $\Delta\psi + f(\psi) + g(\psi)|\nabla\psi|^2 = 0$, with $\psi|_{\partial\Omega} = 0$.

We study the solutions of the following semilinear parabolic problem:

$$
\begin{aligned}
u_t &= \Delta u + f(u) + g(u)|\nabla u|^2, && t > 0,\ x \in \Omega,\\
u(x,t) &= 0, && t > 0,\ x \in \partial\Omega,\\
u(x,0) &= \phi(x), && x \in \Omega,
\end{aligned} \tag{1}
$$

where $\Omega \subset R^n$ is a bounded domain with smooth boundary $\partial\Omega$.

Kawohl and Peletier [KP] discussed the blowup behaviors when $f(u) = |u|^{p-1}u$ and $g(u) \equiv c$. Galaktionov [G] obtained exact solutions of $u_t - u_{xx} = u^2 + |u_x|^2$ on $R^+ \times R$. Many authors (see [KP], [CW], [S], [F], and [D]) discussed the following problem:

$$
\begin{aligned}
u_t &= \Delta u + |u|^{p-1}u - a|\nabla u|^q, && t > 0,\ x \in \Omega,\\
u(x,t) &= 0, && t > 0,\ x \in \partial\Omega,\\
u(x,0) &= \phi(x), && x \in \Omega.
\end{aligned} \tag{2}
$$

Existence of global solutions of (2) depends upon the balance between the power of the damping term and that of the source nonlinearity. In one dimension, Deng [D] and Fila [F] proved that under suitable assumptions on p and q, the solutions of (2) blow up in a finite time if $\phi(x) \ge \psi(x)$, $\forall x \in \Omega$, where ψ is the unique steady state of (2). Deng [D] also proved that if $\phi < \psi$, the solutions of (2) approaches zero as $t \to 0$.

In the case where there is no gradient term, Brezis *et al.* [BC] discussed the relations between the existence of global, classical solutions and the existence of weak solutions of the corresponding stationary problem. Under suitable assumptions on $f(u)$, they concluded for (1) with $g(u) = 0$ that:

(a) If there exists a global, classical solution of (1), then there exists a weak solution of the stationary problem.

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(b) If there is no weak solution of the stationary problem, then for any positive initial value the solution of (1) blows up in finite time.

(c) If there is a weak solution w of the stationary problem, then for any $u_0 \in L^\infty(\Omega)$ with $0 \le u_0 \le w$, the solution of (1) with $u(0) = u_0$ is global.

Chen and Derrick [CD] considered the system

$$(u_i)_t = \Delta u_i + f_i(u_1, \dots, u_m), \quad t > 0,\ x \in \Omega,$$

with $u_i|_{\partial\Omega} = 0$, $u_i(x, 0) = \phi_i(x)$ and $m \ge 1$. Under suitable assumptions on the nonlinear terms f_i, they proved that, if $0 \le \phi_i \le \lambda\psi_i$ with $\lambda < 1$, then the solutions are global, while if $\phi_i \ge \lambda\psi_i$ with $\lambda > 1$, then the solutions blow up in a finite time, where ψ_i are positive solutions of $\Delta\psi_i + f_i(\psi_1, \dots, \psi_m) = 0$, with $\psi_i|_{\partial\Omega} = 0$.

The purpose of this paper is to obtain similar results to those of [CD] for the problem (1). We assume that

(i) $g \in C^1$, $g \ge 0$ and $g' \ge 0$ for $u > 0$ and $ug'(u)/g(u)$ is bounded as $u \to 0^+$.

(ii) ψ is a positive solution of $\Delta\psi + f(\psi) + g(\psi)|\nabla\psi|^2 = 0$, with $\psi|_{\partial\Omega} = 0$.

(iii) $\phi(x) \in C^{2+\beta}(\overline{\Omega})$, for $\beta \in (0, 1)$, with $\phi|_{\partial\Omega} = 0$.

(iv) f is locally Lipschitz continuous, $f(u) = o(u)$ as $u \to 0^+$ and $f(u)/u > f(v)/v$ for any $u > v > 0$.

(v) $f(u)/u^\sigma \ge c_0 > 0$ for some $\sigma > 1$ and $u > 0$.

Our result is:

Theorem 1. *If* $0 \le \phi \le \lambda\psi$ *with* $\lambda < 1$ *and the assumptions* (i)-(iv) *hold, then the solution* u *of* (1) *exists for all* $t > 0$ *and decays exponentially in* t. *If* $\phi \ge \lambda\psi$ *with* $\lambda > 1$ *and the assumptions* (i)-(v) *hold, then the solution* u *of* (1) *must blow up in a finite time.*

Proof. From standard parabolic PDE theory, there exists a unique solution $u(x, t) \in C^{1,0}(\overline{\Omega} \times [0, \tau]) \bigcap C^{2,1}(\overline{\Omega} \times (0, \tau])$ for some $\tau > 0$ (see [A1]). By the maximum principle, since $v = \lambda\psi$ is a supersolution of (1) if $\phi \le \lambda\psi$, we have

$$0 \le u \le \lambda\psi, \tag{3}$$

and since $v = \lambda\psi$ is a subsolution of (1) if $\phi \ge \lambda\psi$, we have

$$u \ge \lambda\psi. \tag{4}$$

We first prove the blowup property. Let T^* be the maximal time such that $u(x, t)$ exists. For any number n, set

$$h_n(t) = \int_\Omega \frac{\psi^{n+2}(x)}{u^n(x, t)}\, d\Omega \tag{5}$$

for $t \in [0, T^*)$. Differentiating (5), substituting in the equation (1) and integrating by parts (notice that the boundary values are always zero), we have

$$\begin{aligned}\frac{d}{dt}h_n(t) &= -n\int_\Omega \frac{\psi^{n+2}}{u^{n+1}}\left(\Delta u + f(u) + g(u)|\nabla u|^2\right) d\Omega \\ &= -n(n+1)\int_\Omega \frac{\psi^{n+2}}{u^{n+2}}|\nabla u|^2\, d\Omega + n(n+2)\int_\Omega \frac{\psi^{n+1}}{u^{n+1}}\nabla u \nabla\psi\, d\Omega \\ &\quad -n\int_\Omega \frac{\psi^{n+2}}{u^{n+1}} f(u)\, d\Omega - n\int_\Omega \frac{\psi^{n+2}}{u^{n+1}} g(u)|\nabla u|^2\, d\Omega. \end{aligned}\tag{6}$$

To simplify (6), we write

$$\psi^2|\nabla u|^2 = |\psi\nabla u - u\nabla\psi|^2 + 2u\psi\nabla u\nabla\psi - u^2|\nabla\psi|^2.$$

It follows from (6) that

$$\begin{aligned}\frac{d}{dt}h_n(t) &= -n\int_\Omega \frac{\psi^n}{u^{n+2}}|\psi\nabla u - u\nabla\psi|^2\left[n+1+\left(1-\frac{1}{n}\right)ug(u)\right]\,d\Omega\\ &\quad -n^2\int_\Omega \frac{\psi^{n+1}}{u^{n+1}}\nabla u\nabla\psi\ d\Omega + n(n+1)\int_\Omega \frac{\psi^n}{u^n}|\nabla\psi|^2\ d\Omega\\ &\quad -\int_\Omega \frac{\psi^{n+2}}{u^{n+1}}g(u)|\nabla u|^2\ d\Omega + (n-1)\int_\Omega \frac{\psi^n}{u^{n-1}}g(u)|\nabla\psi|^2\ d\Omega\\ &\quad -2(n-1)\int_\Omega \frac{\psi^{n+1}}{u^n}g(u)\nabla u\nabla\psi\ d\Omega - n\int_\Omega \frac{\psi^{n+2}}{u^{n+1}}f(u)\ d\Omega.\end{aligned}\tag{7}$$

Using the identity

$$\begin{aligned}-n^2\int_\Omega \frac{\psi^{n+1}}{u^{n+1}}\nabla u\nabla\psi\ d\Omega &= n\int_\Omega \psi^{n+1}\nabla\Big(\frac{1}{u^n}\Big)\nabla\psi\ d\Omega\\ &= -n(n+1)\int_\Omega \frac{\psi^n}{u^n}|\nabla\psi|^2\ d\Omega - n\int_\Omega \frac{\psi^{n+1}}{u^n}\Delta\psi\ d\Omega,\end{aligned}$$

and $\Delta\psi = -f(\psi) - g(\psi)|\nabla\psi|^2$, we obtain from (7)

$$\begin{aligned}\frac{d}{dt}h_n(t) &\le -n\int_\Omega \frac{\psi^{n+1}}{u^n}\Delta\psi\ d\Omega - \int_\Omega \frac{\psi^{n+2}}{u^{n+1}}g(u)|\nabla u|^2\ d\Omega\\ &\quad -(n+3)\int_\Omega \frac{\psi^n}{u^{n-1}}g(u)|\nabla\psi|^2\ d\Omega - 2\int_\Omega \frac{\psi^{n+1}}{u^{n-1}}g'(u)\nabla u\nabla\psi\ d\Omega\\ &\quad -2\int_\Omega \frac{\psi^{n+1}}{u^{n-1}}g(u)\Delta\psi\ d\Omega - n\int_\Omega \frac{\psi^{n+2}}{u^{n+1}}f(u)\ d\Omega\\ &= -n\int_\Omega \frac{\psi^{n+2}}{u^n}\left(\frac{f(u)}{u} - \frac{f(\psi)}{\psi}\right)d\Omega + n\int_\Omega \frac{\psi^{n+1}}{u^n}g(\psi)|\nabla\psi|^2\ d\Omega\\ &\quad -\int_\Omega \frac{\psi^n}{u^{n+1}}g(u)\left|\psi\nabla u + \frac{u^2g'(u)}{g(u)}\,\nabla\psi\right|^2 d\Omega\\ &\quad -(n+3)\int_\Omega \frac{\psi^n}{u^{n-1}}g(u)|\nabla\psi|^2\ d\Omega + 2\int_\Omega \frac{\psi^{n+1}}{u^{n-1}}f(\psi)g(u)\,d\Omega\\ &\quad +2\int_\Omega \frac{\psi^{n+1}}{u^{n-1}}g(\psi)g(u)|\nabla\psi|^2\ d\Omega + \int_\Omega \frac{\psi^n}{u^{n-3}}\frac{g'^2(u)}{g(u)}\,|\nabla\psi|^2 d\Omega\\ &\le -n\int_\Omega \frac{\psi^n}{u^{n-1}}g(u)|\nabla\psi|^2\left(1 - \frac{\psi g(\psi)}{ug(u)} - \frac{2}{n}\psi g(\psi) - \frac{u^2g'^2(u)}{ng^2(u)}\right)d\Omega\\ &\quad -n\int_\Omega \frac{\psi^{n+1}}{u^n}f(\psi)\left(\frac{f(u)/u}{f(\psi)/\psi} - 1 - \frac{2}{n}ug(u)\right)d\Omega.\end{aligned}\tag{8}$$

Using (4) and assumptions (i) and (iv), we can take n sufficiently large that for $t\in[0,T^*-\delta]$ with $\delta>0$

$$\frac{f(u)/u}{f(\psi)/\psi} - 1 - \frac{2}{n}ug(u) \ge \epsilon_n > 0,\tag{9}$$

$$1 - \frac{\psi g(\psi)}{ug(u)} - \frac{2}{n}\psi g(\psi) - \frac{u^2g'^2(u)}{ng^2(u)} \ge 0.\tag{10}$$

Thus we obtain $h_n'(t) \leq 0$ or $h_n(t) \leq h_n(s)$ for $0 \leq s < t \leq T^* - \delta$. Taking nth roots and letting $n \to \infty$, we have

$$\max_{\overline{\Omega}} \frac{\psi(x)}{u(x,t)} \leq \max_{\overline{\Omega}} \frac{\psi(x)}{u(x,s)},$$

which implies that

$$v(t) = \sup_{\Omega} \frac{\psi(x)}{u(x,t)}$$

is decreasing.

To prove that $u(x,t)$ blows up in a finite time we first show that $T^* < \infty$. Suppose instead that $T^* = \infty$ and $\lim_{t\to\infty} v(t) = b$. For any $\epsilon > 0$, let $\Omega_\epsilon = \{x \in \Omega \,|\, \mathrm{dist}(x, \partial\Omega) < \epsilon\}$. If $b > 0$, then $u(x,t)$ is bounded in $\Omega - \Omega_\epsilon$. It follows from (8), (10) and assumption (v) that

$$(11) \qquad h_n'(t) \quad \leq \quad -n\epsilon_n \int_\Omega \frac{\psi^{n+1}}{u^n} f(\psi)\, d\Omega \leq -n\epsilon^n \epsilon_n c_0 \int_{\Omega-\Omega_\epsilon} \psi^{1+\sigma}\, d\Omega,$$

for sufficiently large but fixed n, where ϵ satisfies

$$\frac{\psi(x)}{u(x,t)} \geq \epsilon,$$

for $x \in \Omega - \Omega_\epsilon$ (notice that we temporarily assume u is bounded). Integrating (11) from 0 to t yields

$$h_n(t) + n\epsilon^n \epsilon_n c_0 t \int_{\Omega-\Omega_\epsilon} \psi^{1+\sigma}\, d\Omega \leq h_n(0).$$

Letting $t \to \infty$, we get a contradiction. If $b = 0$, then for any $N > 1$, we have

$$\frac{\psi(x)}{u(x,t_1)} \leq v(t_1) < \frac{1}{N} \quad \text{or} \quad u(x,t_1) \geq N\psi(x),$$

for sufficiently large t_1. If we use this $u(x,t_1)$ as the initial value to solve the problem (1), then the solution must blow up. Since the solution of (1) is unique by the maximum principle, we still have a contradiction. Thus $T^* < \infty$. If u is bounded in $(0, T^*)$, then, by [A2], so is $|\nabla u|$, and u can be extended to a time greater than T^*, which is impossible from the definition of T^*. Hence $u(x,t)$ blows up as $t \to T^*$.

Now we prove that $u(x,t)$ decays exponentially if $\phi \leq \lambda\psi$ with $\lambda < 1$. Similar to the argument above, setting

$$z_n(t) = \int_\Omega \frac{u^{n+2}(x,t)}{\psi^n(x)}\, d\Omega,$$

we have

$$
\begin{aligned}
\frac{d}{dt}z_n(t) &= (n+2)\int_\Omega \frac{u^{n+1}}{\psi^n}\Big(\Delta u + f(u) + g(u)|\nabla u|^2\Big)\ d\Omega \\
&= -(n+2)\int_\Omega \frac{u^n}{\psi^n}|\nabla u|^2\left(n+1-\frac{n+3}{n+2}ug(u)\right)\ d\Omega \\
&\quad +n(n+2)\int_\Omega \frac{u^{n+1}}{\psi^{n+1}}\nabla u\nabla\psi\ d\Omega \\
&\quad -\int_\Omega \frac{u^{n+1}}{\psi^n}g(u)|\nabla u|^2\ d\Omega + (n+2)\int_\Omega \frac{u^{n+1}}{\psi^n}f(u)\ d\Omega \\
&= (n+2)\int_\Omega \frac{u^{n+1}}{\psi^n}f(u)\ d\Omega - \int_\Omega \frac{u^{n+1}}{\psi^n}g(u)|\nabla u|^2\ d\Omega \\
&\quad -(n+2)\int_\Omega \frac{u^n}{\psi^{n+2}}|\psi\nabla u - u\nabla\psi|^2\left(n+1-\frac{n+3}{n+2}ug(u)\right)\ d\Omega \\
&\quad +(n+2)\int_\Omega \frac{u^{n+2}}{\psi^{n+1}}\Delta\psi\ d\Omega - (n+3)\int_\Omega \frac{u^{n+3}}{\psi^{n+2}}g(u)|\nabla\psi|^2\ d\Omega \\
&\quad +2(n+3)\int_\Omega \frac{u^{n+2}}{\psi^{n+1}}g(u)\nabla u\nabla\psi\ d\Omega \\
&= -(n+2)\int_\Omega \frac{u^{n+2}}{\psi^{n+1}}f(\psi)\left(1-\frac{f(u)/u}{f(\psi)/\psi}-\frac{2}{n+2}ug(u)\right)\ d\Omega \\
&\quad -(n+2)\int_\Omega \frac{u^n}{\psi^{n+2}}|\psi\nabla u - u\nabla\psi|^2\left(n+1-\frac{n+3}{n+2}ug(u)\right)\ d\Omega \\
&\quad -(n+2)\int_\Omega \frac{u^{n+2}}{\psi^{n+1}}g(\psi)|\nabla\psi|^2\Bigg(1-\frac{ug(u)}{\psi g(\psi)} \\
&\qquad -\frac{2}{n+2}ug(u) - \frac{u^3g'^2(u)}{(n+2)\psi g(\psi)g(u)}\Bigg)\ d\Omega \\
&\quad -\int_\Omega \frac{u^{n+1}}{\psi^{n+2}}g(u)\left|\psi\nabla u + \frac{u^2g'(u)}{g(u)}\nabla\psi\right|^2\ d\Omega \\
&\quad -3\int_\Omega \frac{u^{n+3}}{\psi^{n+2}}g(u)|\nabla\psi|^2\ d\Omega. \qquad (12)
\end{aligned}
$$

Using (3) and assumption (iv), we can take n sufficiently large such that for sufficiently small $\epsilon_n > 0$

$$1-\frac{f(u)/u}{f(\psi)/\psi}-\frac{2}{n+2}ug(u) \geq \epsilon_n,$$

$$n+1-\frac{n+3}{n+2}ug(u) \geq 0,$$

$$1-\frac{ug(u)}{\psi g(\psi)}-\frac{2}{n+2}ug(u)-\frac{u^3g'^2(u)}{(n+2)\psi g(\psi)g(u)} \geq 0.$$

Define Ω_ϵ as before. Using (12), we obtain

$$\begin{aligned}\frac{d}{dt}z_n(t) &\leq -(n+2)\epsilon_n \int_\Omega \frac{u^{n+2}}{\psi^{n+1}} f(\psi)\, d\Omega \\ &\leq -(n+2)\,\epsilon_1\,\epsilon_n \int_{\Omega-\Omega_\epsilon} \frac{u^{n+2}(x,t)}{\psi^n(x)}\, d\Omega \equiv -(n+2)\,\epsilon_1\,\epsilon_n\, z_n^\epsilon(t),\end{aligned}$$

where $\epsilon_1 = \min\ f(\psi)/\psi$ on $\Omega - \Omega_\epsilon$. Then

$$z_n^\epsilon(t) < z_n(t) \leq z_n(0) - (n+2)\,\epsilon_1\,\epsilon_n \int_0^t z_n^\epsilon(\tau) d\tau,$$

which implies for fixed n that

$$\frac{1}{\max \psi^n} \int_{\Omega-\Omega_\epsilon} u^{n+2}(x,t)\, d\Omega < z_n^\epsilon(t) \to 0, \quad \text{as} \quad t \to \infty.$$

Since $u = 0$ on the boundary and $u(x,t) \leq \psi(x)$, we have $\int_\Omega u^{n+2}\, d\Omega \to 0$ as $t \to \infty$. By [H], $u(x,t)$ decays exponentially.

Example. For $n = 1$, the problem

$$\begin{aligned} &u_t = u_{xx} + (u^2 + u_x^2)u \quad \text{in}\, (0,\pi), \\ &u(0,t) = u(\pi,t) = 0, \end{aligned}$$

with the initial value $\phi(x) = \sin x/\sqrt{1+c}$ has exact solution

$$u(x,t) = \frac{1}{\sqrt{1+ce^{2t}}} \sin x.$$

The corresponding steady state is $\psi(x) = \sin(x)$ in $(0,\pi)$. If $c > 0$, then $\phi < \psi$ and the solution is global and exponentially decays. If $-1 < c < 0$, then $\phi > \psi$ and the solution blows up in a finite time.

Remark. If the problem $\Delta\psi + f(\psi) + a\psi^p|\nabla\psi|^2 = 0$, with $\psi|_{\partial\Omega} = 0$, has two positive solutions, then they must have intersection points by the results of Theorem 1. In fact, assume $\psi_1(x)$ and $\psi_2(x)$ are solutions with $\psi_1(x) > \psi_2(x)$ in Ω. We first show that

$$\sup_\Omega \frac{\psi_2(x)}{\psi_1(x)} < 1. \tag{13}$$

If (13) it not true, then there is a point $x_0 \in \partial\Omega$ such that

$$1 = \lim_{x\to x_0} \frac{\psi_2(x) - \psi_2(x_0)}{\psi_1(x) - \psi_1(x_0)} = \frac{\partial\psi_2(x_0)/\partial n}{\partial\psi_1(x_0)/\partial n},$$

where $\partial\psi_1(x_0)/\partial n < 0$ by the strong maximum principle. Let $\psi = \psi_1 - \psi_2$. Then ψ satisfies

$$\begin{aligned}\Delta\psi + \int_0^1 \left[f'(\psi_2 + \theta(\psi_1 - \psi_2)) + g'(\psi_2 + \theta(\psi_1 - \psi_2))|\nabla\psi_1|^2\right] d\theta\psi \\ + g(\psi_2)(\nabla\psi_1 + \nabla\psi_2)\nabla\psi = 0,\end{aligned}$$

$\psi|_{\partial\Omega} = 0$ and $\partial\psi(x_0)/\partial n = 0$. But the strong maximum principle says $\partial\psi(x)/\partial n < 0$ for all $x \in \partial\Omega$, which is a contradiction. Thus (13) is true. Now we can choose two numbers $\lambda_1 < 1 < \lambda_2$ and the initial value $\phi(x)$ such that $\lambda_2\psi_2(x) < \phi(x) < \lambda_1\psi_1(x)$, leading to a contradiction by Theorem 1. Hence the two solutions must have intersection points in Ω.

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