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ON RESIDUALLY S_2 IDEALS AND PROJECTIVE DIMENSION ONE MODULES

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ABSTRACT. We prove that certain modules are faithful. This enables us to draw consequences about the reduction number and the integral closure of some classes of ideals.

1. INTRODUCTION

This work has been motivated by a recent paper of C. Huneke about cancellation theorems for special ideals in local Gorenstein rings. In its simplest form any such theorem says that if I, J and L are ideals in a Noetherian local ring R such that $LI \subset JI$, then $L \subset J$. Using the so-called determinant trick, in general one can only conclude that $L \subset JI\colon I \subset \overline{J}$, where $\overline{J}$ denotes the integral closure of J. We recall that the *integral closure* $\overline{J}$ of an ideal J is the set (ideal, to be precise) of all elements integral over J. An element $x \in R$ is *integral* over J if x satisfies a monic equation of the form $x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0$, where $a_i \in J^i$ for $1 \leq i \leq n$.

To say that cancellation holds for every such ideal L is equivalent to saying that the R/J-module I/JI is faithful, that is to say, $JI\colon I = J$. There are not many known instances when cancellation happens. Besides the ones described in [5], we would like to single out [3, Theorem 2.4 and Theorem 3.1].

More generally, one can ask for which ideals I and J and which positive integers t the equality $JI^t\colon I^t = J$ holds. This kind of question is particularly interesting when the ideal J is a (minimal) reduction of the ideal I. We say that a subideal J of I is a *reduction* of I if $I^{r+1} = JI^r$ for some positive integer r. The smallest such r is called the *reduction number* of I with respect to J. *Minimal reductions* are reductions minimal with respect to containment. If the residue field is infinite, their minimal number of generators does not depend on the minimal reduction of the ideal. This number is called the *analytic spread* of I, in symbols $\ell = \ell(I)$. It equals the dimension of the fiber cone of I and it is always greater than or equal to the height of I. If $I^{r+1} = JI^r$, one has that $I \subset JI^r\colon I^r$. Thus, conditions of the

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kind $JI^t \colon I^t = J$, with J a reduction of I and t some integer, give a severe bound on the reduction number of ideals which admit proper reductions.

Our main results, in the ideal case, are Theorem 2.2 and Theorem 2.3. They answer the above questions in the case of ideals satisfying local bounds on the minimal number of generators up to a given codimension and some other residual properties in the sense of [2, 4, 6, 8, 9, 11, 12]. In the same spirit as [5], immediate consequences of Theorem 2.3 are drawn about the reduction number and the integral closure of an ideal I. The proofs of Theorem 2.2 and Theorem 2.3 use, though, techniques coming essentially from [11] (and its subsequent refinements).

We also prove a similar cancellation theorem in the case of certain modules of projective dimension one (see Theorem 2.6). The motivations and the techniques we use come from [10], where the authors study Rees algebras of modules via Bourbaki ideals.

It is worth pointing out the analogy between the classes of ideals and modules presently studied and the ones recently studied in [4]: some of the theorems proved in [4] are also sort of cancellation theorems.

2. The main results

2.1. **The ideal case.** We first recall some additional definitions that are essential in stating our results. For a more comprehensive treatment, we refer the reader to [11, 2].

Definition 2.1. Let R be a local Cohen–Macaulay ring, let I be an R-ideal of grade g, and let $s \geq g$ be an integer.

1. We say that I satisfies property G_s, if $\mu(I_\mathfrak{p}) \leq \dim R_\mathfrak{p}$ for any prime ideal $\mathfrak{p}$ containing I with $\dim R_\mathfrak{p} \leq s-1$.
2. An s-residual intersection of I is a proper ideal $K = \mathfrak{a} \colon I$ where $\mathfrak{a}$ is a subideal of I with $\mu(\mathfrak{a}) \leq s \leq \operatorname{height}(K)$.
3. An s-residual intersection K of I is called a geometric s-residual intersection if $\operatorname{height}(I+K) \geq s+1$.
4. We say that I is s-residually S_2 if for every $g \leq i \leq s$ and every i-residual intersection K of I, R/K satisfies Serre's condition S_2.
5. We say that I is weakly s-residually S_2 if for every $g \leq i \leq s$ and every geometric i-residual intersection K of I, R/K satisfies Serre's condition S_2.

Theorem 2.2. *Let R be a local Cohen–Macaulay ring. Let I be an unmixed R-ideal of grade g satisfying G_s for some $s \geq g+1$. Let $K = \mathfrak{a} \colon I$ be an s-residual intersection. Then there exists a generating sequence $a_1, \ldots, a_s$ of $\mathfrak{a}$ such that with $\mathfrak{a}_i = (a_1, \ldots, a_i)$ then*

(*a*) $\mathfrak{a}_i I^t \colon I^t = \mathfrak{a}_i$ *if* $1 \leq i \leq g$ *and for every* t*;*

(*b*) $\mathfrak{a}_i I^t \colon I^t \subset I$ *if* $g \leq i \leq s$ *and for every* t*.*

Suppose further that I is weakly $(s-1)$-residually S_2. Then

(*c*) $\mathfrak{a}_i I^t \colon I^t = \mathfrak{a}_i$ *if* $g+1 \leq i \leq s-1$ *or if* $i = s$ *and K is a geometric s-residual intersection, and for every t.*

Proof. Pick the elements a_i as in [11, Corollary 1.6(a)].

(a) Clearly, $\mathfrak{a}_i \subset \mathfrak{a}_i I^t \colon I^t$ for all t. It is enough to prove the statement at the associated primes of $R/\mathfrak{a}_i$ which all have height i. Let $\mathfrak{p}$ be any such prime. If $i < g$, then $(\mathfrak{a}_i)_\mathfrak{p} I^t_\mathfrak{p} \colon I^t_\mathfrak{p} = (\mathfrak{a}_i)_\mathfrak{p} : R_\mathfrak{p} = (\mathfrak{a}_i)_\mathfrak{p}$. If $i = g$, we either have $I_\mathfrak{p} = (\mathfrak{a}_g)_\mathfrak{p}$ or $I_\mathfrak{p} = R_\mathfrak{p}$. In both cases we conclude that $(\mathfrak{a}_g)_\mathfrak{p} I^t_\mathfrak{p} \colon I^t_\mathfrak{p} = (\mathfrak{a}_g)_\mathfrak{p}$.

(b) It is enough to show the inclusion at the associated primes of I, which all have height g since I is unmixed. Let $\mathfrak{p}$ be any such prime. Then $I_\mathfrak{p} = (\mathfrak{a}_g)_\mathfrak{p}$ and $(\mathfrak{a}_i)_\mathfrak{p} I_\mathfrak{p}^t \colon I_\mathfrak{p}^t = (\mathfrak{a}_g)_\mathfrak{p} = I_\mathfrak{p}$.

(c) Set $K_i = \mathfrak{a}_i \colon I$ and observe that $\mathfrak{a}_i I^t \colon I^t \subset \mathfrak{a}_i \colon I^t = (\mathfrak{a}_i \colon I) \colon I^{t-1} = K_i \colon I^{t-1}$. Since K_i is a i-geometric residual intersection and is unmixed of height i, it follows that I^{t-1} has positive grade modulo K_i. Thus, we conclude that $K_i \colon I^{t-1} = K_i$. Using (b), we have

$$\mathfrak{a}_i I^t \colon I^t \subset K_i \cap I = \mathfrak{a}_i,$$

as desired. We observe that the equality $K_i \cap I = \mathfrak{a}_i$ follows from [2, Proposition 3.4 (c)]. □

Our next goal is to relax the assumptions of Theorem 2.2(c) in the case $i = s$.

Theorem 2.3. *Let R be a local Gorenstein ring. Let I be an R-ideal of grade g, satisfying G_s for some $s \geq g$ and locally unmixed in codimension s. Suppose further that I is weakly $(s-1)$-residually S_2 and that $\operatorname{Ext}_R^{g+j}(R/I^j, R)_\mathfrak{p} = 0$ whenever $\mathfrak{p} \in V(I)$ with $\dim R_\mathfrak{p} \leq s$ and $1 \leq j \leq s-g$. Let $\mathfrak{a} \colon I$ be an s-residual intersection. Then for every $1 \leq t \leq s-g$*

$$\mathfrak{a} I^t \colon I^t = \mathfrak{a}.$$

Proof. Again, pick a generating sequence $a_1, \ldots, a_s$ of $\mathfrak{a}$ as in [11, Corollary 1.6 (a)] and we may assume $s \geq g+1$. Clearly, $\mathfrak{a} \subset \mathfrak{a} I^t \colon I^t$ for all t. Hence it is enough to show the equality at the associated primes of $R/\mathfrak{a}$ which have height at most s (see [2, Proposition 3.4(b)]). Let $\mathfrak{p}$ be any such prime. We may also assume that $\mathfrak{p} \in V(I)$, as otherwise the conclusion is immediate. Since $\mathfrak{a} \colon I$ is an s-residual intersection, it follows that if $\mathfrak{p}$ has height at most $s-1$, then $I_\mathfrak{p} = \mathfrak{a}_\mathfrak{p}$. The unmixedness assumption on I then forces $\mathfrak{p}$ to have height g. Therefore $I_\mathfrak{p} = \mathfrak{a}_\mathfrak{p} = (\mathfrak{a}_g)_\mathfrak{p}$, where $\mathfrak{a}_g = (a_1, \ldots, a_g)$ and $\mathfrak{a}_\mathfrak{p} I_\mathfrak{p}^t \colon I_\mathfrak{p}^t = (\mathfrak{a}_g)_\mathfrak{p} = \mathfrak{a}_\mathfrak{p}$.

In conclusion, we may assume that $\mathfrak{p}$ has height s and therefore, by the unmixedness assumptions on I, we have that $I_\mathfrak{p} \neq \mathfrak{a}_\mathfrak{p}$. After a change of notation, let R be a local Gorenstein ring of dimension s and I an R-ideal of grade g satisfying G_s and $\operatorname{Ext}_R^{g+j}(R/I^j, R) = 0$ for $1 \leq j \leq s-g$. Let '$-$' denote images modulo $K_{s-1} = \mathfrak{a}_{s-1} \colon I$. Then

$$\overline{\mathfrak{a} I^t \colon I^t} \subset \overline{\mathfrak{a} I^t} \colon \overline{I}^t = (\overline{a}_s)\overline{I}^t \colon \overline{I}^t \subset (\overline{a}_s)(\overline{I}^t \underset{Q}{:} \overline{I}^t),$$

where Q is the total ring of fractions of $\overline{R}$ and $\overline{a}_s$ is a non-zero-divisor in $\overline{R}$ (see [2, Proposition 3.1 and Proposition 3.3]). By [2, (c) in the proof of Theorem 4.1] we have that $(\overline{I}^{s-g})^{\vee\vee}$ is the canonical module $\omega_{\overline{R}}$ of $\overline{R}$, where $(\_)^\vee = \operatorname{Hom}_{\overline{R}}(\_, \omega_{\overline{R}})$. Thus, for $1 \leq t \leq s-g$ it follows that

$$\overline{I}^t \underset{Q}{:} \overline{I}^t \subset \overline{I}^{s-g} \underset{Q}{:} \overline{I}^{s-g} \subset (\overline{I}^{s-g})^{\vee\vee} \underset{Q}{:} (\overline{I}^{s-g})^{\vee\vee} = \operatorname{Hom}_{\overline{R}}(\omega_{\overline{R}}, \omega_{\overline{R}}) = \overline{R},$$

as $\overline{R}$ satisfies property S_2 (see [2, (a) in the proof of Theorem 4.1]). Thus $\overline{\mathfrak{a} I^t \colon I^t} \subset (\overline{a}_s)$ and, after lifting back to the ring R, we get

$$\mathfrak{a} I^t \colon I^t \subset (a_s) + K_{s-1} = \mathfrak{a} + K_{s-1}.$$

Using Theorem 2.2(b), for $1 \leq t \leq s-g$, we have that

$$\mathfrak{a} I^t \colon I^t \subset (\mathfrak{a} + K_{s-1}) \cap I = \mathfrak{a} + (K_{s-1} \cap I) = \mathfrak{a} + \mathfrak{a}_{s-1} = \mathfrak{a},$$

as desired. Again, we observe that the equality $K_{s-1} \cap I = \mathfrak{a}_{s-1}$ follows from [2, Proposition 3.4(c)]. □

A case of interest is the following 'global' formulation of Theorem 2.3.

Corollary 2.4. *Let R be a local Gorenstein ring. Let I be an unmixed R-ideal of grade g, satisfying G_s for some $s \geq g$. Suppose further that $\mathrm{Ext}_R^{g+j}(R/I^j, R) = 0$ for $1 \leq j \leq s-g$. Let $\mathfrak{a}\colon I$ be an s-residual intersection. Then for every $1 \leq t \leq s-g$*

$$\mathfrak{a}I^t \colon I^t = \mathfrak{a}.$$

Proof. The ideal I is $(s-1)$-residually S_2 by [2, Theorem 4.1]. The statement then follows from Theorem 2.3. □

Remark 2.5. (a) The vanishing of the Ext modules in Theorem 2.3 and in Corollary 2.4 are satisfied in particular if $\operatorname{depth} R/I^j \geq \dim R - g - j + 1$ for $1 \leq j \leq s-g$, which in turn holds if I is a strongly Cohen–Macaulay ideal (assuming that I satisfies G_s).

(b) For applications to projective varieties, a convenient reformulation of the vanishing of the Ext modules can be given in terms of vanishing of certain local cohomology modules, via local duality (see [2, Corollary 4.3] for more details).

(c) If we set $s = g+1$, then Corollary 2.4 recovers [5, Theorem 2.2].

2.2. **The module case.** For the reader's sake, we start recalling some facts from [10]. We will always assume that R is a local Gorenstein ring of dimension $d > 0$, infinite residue field and E is a finitely generated R-module with rank $e > 0$. The *Rees algebra* $\mathcal{R}(E)$ of E is the symmetric algebra $\mathcal{S}(E)$ of E modulo its R-torsion submodule. If, in addition, E is a submodule of a free R-module G, the above definition coincides with the one, given by other authors, of the Rees algebra of E being the image of the natural map $\mathcal{S}(E) \longrightarrow \mathcal{S}(G)$.

A submodule U of E is a *reduction* of E or, equivalently, E is *integral* over U if $\mathcal{R}(E)$ is integral over the R-subalgebra generated by U. Alternatively, the integrality condition is expressed by the equations $\mathcal{R}(E)_{r+1} = U \cdot \mathcal{R}(E)_r$ for $r \gg 0$. The least integer $r \geq 0$ for which this equality holds is called the *reduction number* of E *with respect to* U and denoted by $r_U(E)$. The *reduction number* $r(E)$ of E is defined to be the minimum of $r_U(E)$, where U ranges over all minimal reductions of E. Since the residue field is assumed to be infinite, the minimal number of generators of U is given by the analytic spread $\ell = \ell(E)$ of E, and it satisfies the inequalities: $e \leq \ell \leq d+e-1$.

In [10] the study of the Rees algebra of a module E is pursued via the notion of a Bourbaki ideal of E. By a *Bourbaki ideal* of a module E we mean an ideal I fitting into the short exact sequence

$$0 \to F \longrightarrow E \longrightarrow I \to 0,$$

with F a free R-module. For technical reasons, it is actually better to work with *generic* Bourbaki ideals, which are defined over a suitable faithfully flat extension of R. Having this tool at our disposal, one can then embark in the comparison of the Rees algebras of E to the one of I.

Similarly to what happens in the ideal case, also in many of the results of [10] a crucial role is played by conditions on the local number of generators of a module. In the module case, E is said to satisfy condition G_s, for an integer $s \geq 1$, if $\mu(E_{\mathfrak{p}}) \leq \dim R_{\mathfrak{p}} + e - 1$ whenever $1 \leq \dim R_{\mathfrak{p}} \leq s-1$.

We are ready to state our main result in the case of modules. In the same setting as Theorem 2.6, it is worth pointing out that other equivalent conditions to the Cohen–Macaulayness of $\mathcal{R}(E)$ are described in [10, Theorem 4.7].

Theorem 2.6. *Let R be a local Gorenstein ring with infinite residue field and let E be a finitely generated R-module with* $\operatorname{proj\,dim} E = 1$. *Write $e = \operatorname{rank} E$, $\ell = \ell(E)$ and assume that E satisfies $G_{\ell-e+1}$ and is torsionfree locally in codimension 1.*

If $\mathcal{R}(E)$ is Cohen–Macaulay, then for any minimal reduction U of E

$$U \cdot \mathcal{R}(E)_t \underset{E}{:} \mathcal{R}(E)_t = U$$

for $0 \le t \le \ell - e - 1$.

Proof. Write $U = \sum_{i=1}^{\ell} Ra_i$. Let z_{ij}, with $1 \le i \le \ell$ and $1 \le j \le e-1$, be variables and let

$$\widetilde{R} = R(\{z_{ij}\text{'s}\}), \qquad \widetilde{E} = \widetilde{R} \otimes_R E, \qquad \widetilde{U} = \widetilde{R} \otimes_R U, \qquad x_j = \sum_{i=1}^{\ell} z_{ij} a_i \in \widetilde{U}.$$

Consider $I \simeq \widetilde{E}/(x_1, \ldots, x_{e-1})$ and let J be the image of $\widetilde{U}$ in I. By [10, Theorem 3.5] one has that

- I is a perfect ideal of grade $g = 2$ (hence *licci*) satisfying $G_{\ell-e+1}$.
- Since I is licci, then $\operatorname{depth} R/I^j \ge \dim R - g - j + 1$ for $1 \le j \le (\ell - e + 1) - g + 1$.
- Since J is a reduction of I, one has that $J : I$ is an $(\ell - e + 1)$-residual intersection (see [11, Proposition 1.11] or [6, Remark 2.7]).
- $\mathcal{R}(I)$ is Cohen–Macaulay, with $\mathcal{R}(I) \simeq \mathcal{R}(\widetilde{E})/(x_1, \ldots, x_{e-1})$.

By Corollary 2.4 we conclude that

$$JI^t : I^t = J$$

for every $0 \le t \le (\ell - e + 1) - g = \ell - e - 1$.

Since $\mathcal{R}(I) \simeq \mathcal{R}(\widetilde{E})/(x_1, \ldots, x_{e-1})$ we obtain that

$$\widetilde{U} \subset \widetilde{U} \cdot \mathcal{R}(\widetilde{E})_t \underset{\widetilde{E}}{:} \mathcal{R}(\widetilde{E})_t \subset \widetilde{U} + ((x_1, \ldots, x_{e-1}) \cdot \mathcal{R}(\widetilde{E}))_1 \subset \widetilde{U},$$

where the last inclusion holds since $((x_1, \ldots, x_{e-1}) \cdot \mathcal{R}(\widetilde{E}))_1 \subset \widetilde{U}$. Thus

$$\widetilde{U} \cdot \mathcal{R}(\widetilde{E})_t \underset{\widetilde{E}}{:} \mathcal{R}(\widetilde{E})_t = \widetilde{U}.$$

The final result now follows as $\widetilde{R}$ is a faithfully flat extension of R. ☐

3. Applications to ideals

The results in this section recover [5, Corollary 2.3, Theorem 2.4, and Corollary 2.13]. Corollary 3.2 is a weaker version of [6, Corollary 5.5] and [12, Corollary 2.4(*c*)].

Corollary 3.1. *Let R be a local Gorenstein ring. Let I be an R-ideal of grade g, satisfying G_s for some $s \ge g+1$ and unmixed locally in codimension s. Suppose further that I is weakly $(s-1)$-residually S_2 and that $\operatorname{Ext}_R^{g+j}(R/I^j, R)_{\mathfrak{p}} = 0$ whenever $\mathfrak{p} \in V(I)$ with $\dim R_{\mathfrak{p}} \le s$ and $1 \le j \le s - g$. Let $\mathfrak{a} : I$ be an s-residual intersection and suppose that $\mathfrak{a}$ and I have the same radical (e.g. if s is the analytic spread of I and $\mathfrak{a}$ is a reduction of I). Then $I^n \subset \mathfrak{a}$ if and only if $I^{n+s-g} \cap \mathfrak{a} \subset \mathfrak{a} I^{s-g}$.*

Proof. The proof is the one of [5, Corollary 2.13]. If $I^n \subset \mathfrak{a}$, then $I^{n+s-g} = I^n I^{s-g} \subset \mathfrak{a} I^{s-g}$. Conversely, suppose $I^{n+s-g} \cap \mathfrak{a} \subset \mathfrak{a} I^{s-g}$. We know there exists N such that $I^N \subset \mathfrak{a}$. If $N \leq n$, then we are done. If $N-1 \geq n$, we have $I^{N-1} I^{s-g} \subset I^{n+s-g} \cap \mathfrak{a} \subset \mathfrak{a} I^{s-g}$ (here we need $s-g \geq 1$). Thus $I^{N-1} \subset \mathfrak{a} I^{s-g} \colon I^{s-g} = \mathfrak{a}$, by Theorem 2.3. Repeat the process until $I^n \subset \mathfrak{a}$. □

Corollary 3.2. *Let R be a local Gorenstein ring with infinite residue field. Let I be an R-ideal of grade g, satisfying G_ℓ, where ℓ is the analytic spread of I, and unmixed locally in codimension ℓ. Suppose further that I is weakly $(\ell-1)$-residually S_2 and that $\operatorname{Ext}_R^{g+j}(R/I^j, R)_\mathfrak{p} = 0$ whenever $\mathfrak{p} \in V(I)$ with $\dim R_\mathfrak{p} \leq \ell$ and $1 \leq j \leq \ell - g$. It follows that*

(*a*) *the reduction number of I is either zero or at least $\ell - g + 1$;*
(*b*) *if $I^{r+1} = JI^r$, with J a minimal reduction of I, then $I^{r-\ell+g+1} \subset J$.*

Proof. (*a*) Assume the reduction number r of I to be different from zero and let J be a minimal reduction of I. Then $K = J \colon I$ is an ℓ-residual intersection (see [11, Proposition 1.11] or [6, Remark 2.7]). By Theorem 2.3, we have that

$$JI^t \colon I^t = J$$

for every $1 \leq t \leq \ell - g$. Hence the statement.

(*b*) If $I^{r+1} = JI^r$, one has that $I^{r-\ell+g+1} I^{\ell-g} = JI^r \subset JI^{\ell-g}$, as $r \geq \ell - g + 1$ by (*a*). Thus $I^{r-\ell+g+1} \subset JI^{\ell-g} \colon I^{\ell-g} = J$ by Theorem 2.3. □

In the next corollary, '$-$' denotes the integral closure of an ideal.

Corollary 3.3. *Let R be a regular local ring with infinite residue field. Let I be a equidimensional R-ideal of grade g, satisfying G_ℓ, where ℓ is the analytic spread of I, and unmixed locally in codimension ℓ. Suppose further that I is weakly $(\ell-1)$-residually S_2 and that $\operatorname{Ext}_R^{g+j}(R/I^j, R)_\mathfrak{p} = 0$ whenever $\mathfrak{p} \in V(I)$ with $\dim R_\mathfrak{p} \leq \ell$ and $1 \leq j \leq \ell - g - 1$. If $I^{\ell-g}$ is unmixed locally in codimension ℓ, then $\overline{I^g} \subset J$, for any reduction J of I.*

Proof. We know that $I^{\ell-g}\overline{I^g} \subset \overline{I^\ell} \subset JI^{\ell-g}$. The last inclusion follows from [1, Theorem 3.3], the fact that $\operatorname{bight}(I) = \max\{\operatorname{ht}\mathfrak{p} : \mathfrak{p} \in \min(I)\} = g$ and the assumption of $I^{\ell-g}$ being unmixed. Thus we conclude that $\overline{I^g} \subset JI^{\ell-g} \colon I^{\ell-g} = J$, by Theorem 2.3. □

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