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RESIDUALLY FINITE DIMENSIONAL AND AF-EMBEDDABLE C^*-ALGEBRAS

HUAXIN LIN

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Abstract. We show that every separable nuclear residually finite dimensional C^*-algebras satisfying the Universal Coefficient Theorem can be embedded into a unital separable simple AF-algebra.

The problem of determining when a C^*-algebra can be embedded as a C^*-subalgebra of an AF-algebra has been studied for a while (for example, in [Pi], [Vo], [Sp], [Br] and [D]— to name a few). J. Spielberg showed in [Sp], among other related things, that every separable CCR C^*-algebra can be embedded into an AF-algebra. In [D], it is shown that a separable nuclear residually finite dimensional C^*-algebra can be embedded into an AF-algebra if it is homotopically dominated by an AF-algebra. Consequently, from a result of Spielberg, Dadarlat shows (in [D]) that the AF-embeddability of a residually finite dimensional C^*-algebra A (say in the "bootstrap" class of [RS]) depends only on the homotopy type of A. The renewed interest in residually finite dimensional C^*-algebras arises from the result of Blackadar and Kirchberg ([BK1] and [BK2]) that simple separable nuclear quasidiagonal C^*-algebras are inductive limits of nuclear RFD C^*-algebras.

Since a C^*-subalgebra of a nuclear C^*-algebra must be exact ([K1]), non-exact C^*-algebras cannot be embedded into AF-algebras. So, we will consider nuclear C^*-algebras only. In this short note, we use recent results ([Ln3]) in classification of nuclear C^*-algebras to show that, in fact, every separable nuclear residually finite dimensional C^*-algebra can be embedded into a simple AF-algebra if it satisfies the Universal Coefficient Theorem (for example, if it is in the so-called "bootstrap" class of C^*-algebras).

Definition 1. Let A be a C^*-algebra. An irreducible representation of A, π, is said to be finite dimensional, if $\pi(A)$ is finite dimensional. A C^*-algebra A is said be *residually finite dimensional* (RFD) if there is a separating family of finite dimensional irreducible representations of A.

A separable RFD C^*-algebra has a separating sequence of finite dimensional irreducible representations.

Definition 2. Denote by $\mathcal{N}_1$ the bootstrap class of C^*-algebras (see [RS]). Denote by $\mathcal{N}$ the class of C^*-algebras for which the Universal Coefficient Theorem (UCT)

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holds. Every C^*-algebra in $\mathcal{N}_1$ satisfies the UCT ([RS]). In fact, $A \in \mathcal{N}$ if A is kk-equivalent to a C^*-algebra in $\mathcal{N}_1$. $\mathcal{N}_1$ contains all separable type I C^*-algebras, and is closed under inductive limits, hereditary subalgebras, quotients, tensor products, cross products with $\mathbf{Z}$, as well as extensions.

3. Let $C \in \mathcal{N}$ be a unital separable residually finite dimensional C^*-algebra. Let $s(C)$ (with the weak-topology) be the set of tracial states defined by $tr \circ \pi$, where π is an irreducible representation of finite rank and tr is the standard tracial state on $\pi(C)$. There is a positive homomorphism $\delta_C : K_0(C) \to C(s(A))$ defined by $f \mapsto f(t)$ $(t \in s(A))$.

Let A be a unital separable stably finite C^*-algebra and $T(A)$ be its tracial state space. Let $Aff(T(A))$ be the continuous affine functions on $T(A)$. Define $\rho_A : K_0(A) \to Aff(T(A))$ by $f \mapsto f(t)$ $(t \in T(A))$.

Definition 4 (2.1 in [Ln2]). Let A be a unital simple C^*-algebra. We say that A is *tracially approximately finite dimensional* (TAF for brevity) if it satisfies the following: For any $\varepsilon > 0$ and any finite subset $\mathcal{F}$ of A which contains a non-zero element x_1 and a nonzero positive element $a \in A_+$ there exists a finite dimensional C^*-subalgebra $B \subset A$ with $p = 1_B$ such that

(1) $\|px - xp\| < \varepsilon$ for all $x \in \mathcal{F}$;

(2) $pxp \subset_\varepsilon F$ for all $x \in \mathcal{F}$ and $\|px_1p\| \geq \|x_1\| - \varepsilon$;

(3) $1 - p$ is equivalent to a projection in $\overline{aAa}$.

Simple TAF C^*-algebras are quasidiagonal, have real rank zero, stable rank one, and weakly unperforated K_0.

5. Let $C \in \mathcal{N}$ be a unital separable residually finite dimensional C^*-algebra. Fix a sequence of finite dimensional irreducible representations $\{\pi_k\}$ such that $\{\pi_k\}_{k=m}^{\infty}$ is separating for C for all m. We also assume that $\{tr \circ \pi_k\}_{k=m}^{\infty}$ is dense in $s(A)$. Suppose that π_k has rank $r(k)$. In what follows, we may not distinguish π_k from $\pi_k \otimes \mathrm{id}_K$ (on $C \otimes M_K$).

Let $\bar{r}(2) = r(1) + 1$, $\bar{r}(n+1) = (n)!(\bar{r}(n)r(n)(n+1))$, $n = 1, 2, \ldots.$ Set $R(i) = \bar{r}(i)r(i)$, $i = 1, 2, \ldots.$ Let $C_1 = C$, $C_2 = C_1 \otimes M_{\bar{r}(2)}, \ldots,$ $C_{n+1} = C_n \otimes M_{\bar{r}(n+1)}, \ldots.$ Define $j_{n,n+1} : C_n \to C_{n+1}$ by

$$f \mapsto diag(f, f, \ldots, f, \pi_n \otimes \mathrm{id}_{\bar{r}(n)}(f), \ldots, \pi_n \otimes \mathrm{id}_{\bar{r}(n)}(f)),$$

where $f \in C_n$ repeats $n!$ times and $\phi_n \otimes \mathrm{id}_{\bar{r}(n)}(f)$ repeats $n(n)!$ times. Also, the image of $\pi_n \otimes \mathrm{id}_{\bar{r}(n)}$ is in $(\mathbb{C} \cdot \mathrm{id}_{C_n}) \otimes M_{\bar{R}(n)}$. Define $A = lim_n(C_n, j_{n,n+1})$. In what follows, the map $C_n \to A$ will be denoted by j.

Lemma 6. *A is a unital separable nuclear simple TAF C^*-algebra with divisible $K_0(A)$ and a unique tracial state and which satisfies the UCT.*

Proof. Since A is a direct limit of C_n and each C_n is nuclear and satisfies the UCT, A is separable, nuclear and satisfies the UCT. It follows from 4.2, 4.3 and 4.4 in [Ln2] that A is a unital simple TAF C^*-algebra with a unique tracial state. We need only to show that $K_0(A)$ is divisible. It suffices to show that, for any projection $p \in M_K(A)$ (K is any positive integer) and any integer $k > 0$, there exists a projection $q \in M_K(A)$ such that $k[q] = [p]$ in $K_0(A)$. Replacing p by an equivalent projection, without loss of generality, we may assume that $p \in j(M_K(C_n))$ for some large n. There exists m such that $k|(n+m)$. Let $p' \in M_K(C_n)$ such that $j(p') = p$. From the construction in 5, it is clear that there exists $q' \in M_K(C_{n+m})$ such that $k[q'] = [j_{n,n+m}(p')]$ in $K_0(C_{n+m})$. This implies that $k[j(q')] = [p]$. □

7. Fix m and $n > 0$. Let $j_{m,m+n} = j_{m+n-1,m+n} \circ \cdots \circ j_{m,m+1}$. Note $C_{m+n} = C_m \otimes M_{K(n)}$, where $K(n) = \prod_{i=m}^{m+n-1}(R(i)i!(i+1))$. Thus

$$j_{m,m+n}(f) = diag(f, f, ..., f, \Pi_m(f), \Pi_{m+1}(f), ..., \Pi_{m+n-1}(f)),$$

where f repeats $\prod_{i=1}^{m+n}(m+i-1)$ many times, Π_m is

$$m(m!)\prod_{j=1}^{n-1}(R(m+j)(m+j)!(m+j+1))$$

copies of $\pi_m \otimes \mathrm{id}_{R(m)}$, Π_{m+1} is

$$(m+1)(m+1)!\prod_{j=2}^{n-1}(R(m+j)(m+j)!(m+j+1))$$

copies of $\pi_{m+1} \otimes \mathrm{id}_{R(m)}, ..., \Pi_{m+n-1}$ is $(m+n-1)(m+n-1)!$ copies of $\pi_{m+n-1} \otimes \mathrm{id}_{R(m)}$. Set $A = lim_n(C_n, j_{n,n+1})$.

Define

$$\begin{aligned} a_1^{(n)} &= (m(m)!\prod_{j=1}^{n-1}(R(m+j)(m+j)!(m+j+1))/K(n) = m/(R(m)(m+1), \\ a_2^{(n)} &= (m+1)(m+1)!\prod_{j=2}^{n}(R(m+j)(m+j)!(m+j+1))/K(n) \\ &= 1/R(m)m!(R(m+1)((m+1)+1)), ..., \end{aligned}$$

and

$$\begin{aligned} a_k^{(n)} &= (m+k)(m+k)!/[R(m+k)(m+k)!(m+k+1) \\ &\quad \times \prod_{j=1}^{k-1}(R(m+j)(m+j)+1)] \\ &= 1/[R(m+k-1)(m+k-1)!R(m+k)(m+k+1) \\ &\quad \times \prod_{j=1}^{k-2} R(m+j)(m+j)!(m+j+1)], ... \end{aligned}$$

Fix j, then $a_j^{(n)} = a_j^{(m+1)}$ for all $n > m$. Thus we set $a_j = a_j^{(n)}$. Note that $a_j^{(n)} \in \mathbb{Q}$. Let $p_1, p_2, ..., p_r \in C_m \otimes M_K$ be nonzero projections and tr be the normalized trace on $\pi_j \otimes \mathrm{id}_{\bar{R}(m)}(C_m)$ $(j = m, m+1, ..., m+n)$. Set $x_{ij} = tr(\pi_{m+j} \otimes \mathrm{id}_K(p_i))$, $i = 1, 2, ..., r$ and $j = 1, 2, ..., n$. So $x_{ij} \in \mathbb{Q}$. Let $P_{m,m+n} : C_m \to C_{m+n}$ be defined by

$$f \mapsto (0, \Pi_m(f), \Pi_{m+1}(f), ..., \Pi_{m+n}(f)).$$

We compute that

$$b_j^{(n)} = tr(P_{m,m+n} \otimes \mathrm{id}_K(p_i)) = \sum_{i=1}^{n} x_{ij}a_j.$$

It is important to note that $x_{ij}, a_j^{(n)}, b_j^{(n)} \in \mathbb{Q}$. Let τ be the unique tracial state of A. Set $\tau(j(p_j)) = z_j$. Note that

$$(\prod_{j=m}^{m+n}(m+j-1))/K(n) \to 0 \quad (\text{as } n \to \infty).$$

Therefore, it is easy to see that $b_j^{(n)} \to z_j$ as $n \to \infty$.

Lemma 8. *Let τ be the unique tracial state on A. Then $\tau|_{j(C_m)}$ is in the weak closure of the convex hull of $s(C_m)$. Consequently,* $\ker\tau|_{j(C_m)} \supset \ker\delta_{C_m}$.

Proof. First, if $t \in s(C_{m+n})$, it is easy to compute that $t|_{j_{m,m+n}(C_m)}$ is in the convex hull of $s(C_m)$. Take a sequence of $\{t_n\} \subset s(C_{m+n})$. Extend t_n to a state on A and let s be a weak limit of t_n (on A). Then s is a tracial state on A. Therefore $s = \tau$. This implies that $\tau|_{j(C_m)}$ is in the weak closure of the convex hull of $s(C_m)$. □

Lemma 9. *For any finitely generated subgroup $G \subset K_0(A)$, there is a finitely generated free subgroup $G_0 \subset K_0(C_m)$ for some $m > 0$ which has $\mathbb{Z}$-rank r and is generated by r positive $\mathbb{Z}$-linearly independent elements $f_1, ..., f_r \in G_0$ such that $\rho_A \circ j_*(G_0) \supset \rho_A(G)$ and $\rho_A \circ j_*$ is injective on G_0.*

Proof. Note that $\rho_A(K_0(A))$ is a divisible subgroup of $\mathbb{R}$. Let $g_1, g_2, ..., g_r$ be $\mathbb{Q}$-linearly independent elements in $\rho_A(K_0(A))_+$ such that $\bigoplus_{i=1}^r \mathbb{Z}g_i \supset \rho_A(G)$. Let $\imath : \oplus_{i=1}^r \mathbb{Z}g_i \to K_0(A)$ be an injection such that $\rho_A \circ \imath = \mathrm{id}_{\oplus_{i=1}^r \mathbb{Z}g_i}$. Note that $\imath(g_i)$ is positive ($i = 1, 2, ..., r$). Therefore there are $f_1, f_2, ..., f_r \in K_0(C_m)_+$ for some large m such that $j_*(f_i) = \imath(g_i)$, $i = 1, 2, ..., r$. Let G_0 be the subgroup generated by $f_1, ..., f_r$. We see that G_0 has rank r and $f_1, f_2, ..., f_r \in K_0(C_m)_+$ are $\mathbb{Z}$-linearly independent. □

Lemma 10. *Let F be a divisible ordered subgroup of $\mathbb{R}$. Let $\{x_{ij}\}_{0<i\le r, 0<j<\infty}$ be an $r \times \infty$ matrix having rank r and with each $x_{ij} \in \mathbb{Q}_+$, and let $\{a_j^{(n)}\}$ be sequences of positive rational numbers such that $a_j^{(n)} \to a_j (>0)$ as $n \to \infty$. For each n,*

$$(x_{ij})_{r\times n} v_n = y_n,$$

where $v_n = (a_j^{(n)})_{n\times 1}$ is an $n \times 1$ column vector and $y_n = (b_i^{(n)}) \in \mathbb{Q}^r$ is an $r \times 1$ column vector.

Suppose that $y_n \to z$ for some $z = (z_j)_{r\times 1} \in F^r$ with $z_j \ge 0$ (in $\mathbb{R}^r$ norm). Then for some sufficiently large n, there is $u = (c_j)_{n\times 1} \in F_+^n$ ($c_j > 0$) such that

$$(x_{ij})_{r\times n} u = z.$$

Proof. To save notation, without loss of generality, we may assume that $(x_{ij})_{r\times r}$ has rank r. Set $A_n = (x_{ij})_{r\times n}$ ($n \ge r$). Then there exists an invertible matrix $B \in M_r(\mathbb{Q})$ (which does not depend on n) such that $BA_n = C_n$, where $C = (c_{ij})_{r\times n}$, $c_{ii} = 1$ for $i = 1, 2, ..., r$, and $c_{ij} = 0$ if $i \ne j$, $j = 1, 2, ..., r$, and $c_{ij} \in \mathbb{Q}$. Let I_r be the $r \times r$ identity matrix. We may write

$$C_n = (I_r, D_n'),$$

where D_n' is a $r \times (n-r)$ matrix. Thus we have

$$C_n v_n = By_n \quad \text{and} \quad I_r v_n' = By_n - D_n v_n,$$

where $v_n' = (a_1^{(n)}, a_2^{(n)}, ..., a_r^{(n)})$ (as a column) and $D_n = (0, D_n')$ is a $r \times n$ matrix. Note that for any $n \times 1$ column vector v with the form $(t_1, t_2, ..., t_r, a_{r+1}^{(n)}, a_{r+2}^{(n)}, ..., a_n^{(n)})$, $D_n v = D_n v_n$. Since $a_j^{(n)} \to a_j > 0$, there is an $N_1 > 0$ such that

$$a_j^{(n)} \geq a_j/2 > 0$$

for all $n \geq N_1$ and $j = 1, 2, ..., r$. Let $0 < \varepsilon < min\{a_j/4 : j = 1, 2, ..., r\}$. There is $N_2 > 0$ such that

$$\|By_n - Bz\|_\infty < \varepsilon$$

if $n \geq N_2$. Set $N = max\{N_1, N_2\}$. Let $u' = (c_1, c_2, .., c_r)$ (column vector) satisfy the equation

$$I_r u' = Bz - D_n v_n.$$

Since $I_r v_n' = v_n'$ and $I_r u' = u'$, we have

$$\|u' - v'\|_\infty < \varepsilon$$

if $n \geq N$. Therefore $c_j > 0$ for $j = 1, 2, ..., r$. Set $u = (c_1, ..., c_r, a_{r+1}^{(n)}, a_{r+2}^{(n)}, ..., a_n^{(n)})$. Then

$$I_r u' = Bz - D_n u$$

$(n \geq N)$. Since $B \in M_r(\mathbb{Q})$, $D_n u = D_n u_n \in \mathbb{Q}^r$, $z \in F^r$ and F is divisible, we conclude that $u' \in F^r$. Since $D_n = C_n - I_r$, we have

$$C_n u = Bz.$$

Finally, since B is invertible, we have

$$A_n u = z.$$

□

11. Let B be a unital separable simple AF-algebra with $(K_0(B), K_0(B)_+, [1_B]) = (\rho_A(K_0(A)), \rho_A(K_0(A))_+, 1)$. Let $\alpha_0 : K_0(A) \to K_0(B)$ be the positive homomorphism defined by ρ_A. By the Universal Coefficient Theorem, there is $\alpha \in KL(A, B)_+$ such that $\alpha|_{K_0(A)} = \alpha_0$. As in [DL], we identify $KL(A, B)$ with $Hom_\Lambda(\underline{K}(A), \underline{K}(B))$. An element $\alpha \in KL(A, B)_+$ is an element such that $\alpha(K_0(A) \setminus \{0\}) \subset (K_0(A) \setminus \{0\})$.

Since $K_1(B) = 0$ and $Tor(K_0(B)) = 0$, from the following commutative diagram,

$$\begin{tikzcd}
K_0(A) \arrow[rr] \arrow[dr, "\alpha"] & & K_0(A, \mathbf{Z}/k\mathbf{Z}) \arrow[rr] \arrow[d, "\alpha"'] & & K_1(A) \arrow[dl, "\alpha"'] \arrow[ddd] \\
& K_0(B) \arrow[r] & K_0(B, \mathbf{Z}/k\mathbf{Z}) \arrow[r] & K_1(B) \arrow[d] & \\
& K_0(B) \arrow[u] & K_1(B, \mathbf{Z}/k\mathbf{Z}) \arrow[l] & K_1(B) \arrow[l] & \\
K_0(A) \arrow[uuu] \arrow[ur, "\alpha"] & & K_1(A, \mathbf{Z}/k\mathbf{Z}) \arrow[ll] \arrow[u, "\alpha"] & & K_1(A) \arrow[ll] \arrow[ul, "\alpha"']
\end{tikzcd}$$

(the map from $K_i(-)$ to $K_i(-)$ is the multiplication by k), we have

$$\alpha|_{K_1(A)} = 0 \quad \text{and} \quad \alpha|_{K_1((A,\mathbb{Z}/k\mathbb{Z})} = 0.$$

We claim that

$$\alpha|_{K_0(A,\mathbb{Z}/k\mathbb{Z})} = 0$$

for all $k > 0$.

Since $\rho_A(K_0(A)) = K_0(B)$ is divisible, $K_0(B)/kK_0(B) = 0$. Then (since $K_1(B) = 0$)

$$K_0(B, \mathbb{Z}/k\mathbb{Z}) = 0.$$

Therefore $\alpha|_{K_0(A,\mathbb{Z}/k\mathbb{Z})} = 0$.

In [Ln3], we show the following.

Theorem 12. *Let A and B be unital separable simple nuclear TAF C^*-algebras satisfying the UCT. Let $\alpha \in KL(A,B)_+$. Suppose that for every finite subset $\mathcal{G} \subset K_i(A, \mathbb{Z}/k\mathbb{Z})$ ($i = 0, 1$, $k = 0, 1, \ldots$), there exists a sequence of contractive completely positive linear maps $L_n : A \to B$ such that*

$$\|L_n(ab) - L_n(a)L_n(b)\| \to 0 \quad (as\ \ n \to \infty)$$

and

$$[L_n]|_G = \alpha|_G.$$

Then there is a homomorphism $h : A \to B$ such that $[h] = \alpha$. (see 1.8 in [Ln3] for the definition of $[L_n]|_G$).

13. We need only a version of Theorem 12 in which B is an AF-algebra. In the next theorem, we let B be a unital separable simple AF-algebra with

$$(K_0(B), K_0(B)_+, [1_B]) = (\rho_A(A), \rho_A(A)_+, 1).$$

Let $\alpha \in KL(A,B)_+$ be as in 11. From 11, we only need to consider $G \subset K_0(A)$.

Theorem 14. *Every unital separable residually finite dimensional C^*-algebra in $\mathcal{N}$ can be embedded into a unital simple AF-algebra.*

Proof. From the construction in 5, C maps into A. Since each $j_{n,n+1}$ is injective, we see that C is embedded into A. Let B be as in 11. It suffices to show that A can be embedded into B. Let α be as in 13. Let G be any finitely generated subgroup of $K_0(A)$. By 11, Theorem 12 and 13, to show that A can be embedded into B, it suffices to show that there is a sequence of contractive completely positive linear maps $L_n : A \to B$ such that

$$\|L_n(ab) - L_n(a)L_n(b)\| \to 0$$

for all $a, b \in A$ and

$$[L_n]|_G = \alpha|_G.$$

Let m, G_0 and $f_1, \ldots, f_r$ be as in Lemma 9. There are $z_1, z_2, \ldots, z_r \in \rho_B(B)_+ \setminus \{0\} = \rho_A(A)_+ \setminus \{0\}$ such that

$$\rho_A \circ j_*(f_i) = z_i \quad (i = 1, 2, \ldots, r\,).$$

Note that $(j_{m,m+l})|_{G_0}$ is injective. So we may assume that there are projections $p_1, p_2, \ldots, p_r \in C_m \otimes M_K$ such that $[p_i] = f_i$ in $K_0(C_m)$, $i = 1, 2, \ldots, r$. We now use the notation developed in 7. Since $f_1, \ldots, f_r$ are $\mathbb{Z}$-linearly independent, by Lemma

8, $\delta_{C_m}(f_1), ..., \delta_{C_m}(f_r)$ are $\mathbb{Z}$-linearly independent. Since $\{tr \circ \pi_n\}$ is dense in $s(C)$, we conclude that (x_{ij}) has rank r. By Lemma 10, (with $z = (z_1, z_2, ..., z_r)$, a column vector in $\rho_A(A)^r$), there is $u \in (\rho_A(A)^n)_+$ such that

$$(x_{ij})_{r\times n} u = z.$$

Let $D = \bigoplus_{i=m}^{m+n} \pi_i \otimes \mathrm{id}(C_m)$. Note $K_0(D) = \mathbb{Z}^n$. Define a positive map $\lambda : K_0(D) \to \rho_B(B) = \rho_A(A)$ by

$$(l_1, l_2, ..., l_n) = \sum_{j=1}^{n} (\frac{l_j}{R(m+j-1)}) c_j$$

($u = (c_1, c_2, ..., c_n) \in (\rho_A(A)^n)_+$). It is well known that there exists a homomorphism $h : D \to B$ such that

$$[h]|_{K_0(D)} = \lambda.$$

Let $L_n' = h \circ \bigoplus_{j=m}^{m+n} \pi_i : C_m \to B$. Note for each i,

$$(\bigoplus_{j=m}^{m+n} \pi_j)_*(f_i) = (R(m)x_{i1}, R(m+1)x_{i2}, ..., R(m+n)x_{in})$$

and

$$\rho_B \circ [L_n'](f_i) = \sum_{j=1}^{n} x_{ij} c_j = z_i.$$

Thus

$$[L_n']|_{G_0} = \alpha \circ (j)_*|_{G_0}.$$

For any $\varepsilon_n > 0$ and any finite subset $\mathcal{F} \subset C_m$, since C_m is nuclear, there exists a contractive completely positive linear map $L_n : A \to B$ (see 3.2 (2) in [Ln3]) such that

$$\|L_n(a) - L_n'(a)\| < \varepsilon_n$$

for all $a \in \mathcal{F}$. Thus (by considering the finitely many generators and by choosing sufficiently small ε_n—see 1.8 in [Ln3])

$$[L_n]|_G = \alpha|_G.$$

Since we can choose any large m to start, one sees that the theorem follows. □

Corollary 15 (cf. 7 in [D]). *Let A be a separable RAF C^*-algebra. Then the cone over A, CA and the suspension of A, SA, are AF-embeddable.*

Proof. First, we note that CA is a separable RAF C^*-algebra. Since $CA = C_0(0,1] \otimes A$ is homotopic to $\{0\}$, $CA \in \mathcal{N}$. So the corollary follows from Theorem 14. □

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Department of Mathematics, East China Normal University, Shanghai, China

Current address: Department of Mathematics, University of Oregon, Eugene, Oregon 97403-1222

E-mail address: `hxlin@noether.uoregon.edu`