

## SOME DESCRIPTIVE SET-THEORETIC PROPERTIES OF THE ISOMORPHISM RELATION BETWEEN BANACH SPACES

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ABSTRACT. Consider the space  $\mathcal{V}(E)$  of closed linear subspaces of a separable Banach space  $E$  equipped with the standard Effros Borel structure. The isomorphism relation between Banach spaces being elements of  $\mathcal{V}(E)$  determines a partition of  $\mathcal{V}(E)$ . In this note we prove a result describing the complexity of analytic subsets of  $\mathcal{V}(E)$  intersecting a large enough number of the above-mentioned parts of  $\mathcal{V}(E)$ .

### 1. INTRODUCTION

Given a separable Banach space  $E$ , we shall denote by  $\mathcal{V}(E)$  the collection of closed linear subspaces of  $E$  equipped with the Effros Borel structure. The Effros structure is the  $\sigma$ -algebra generated by the sets  $\{F \in \mathcal{V}(E) : F \cap U \neq \emptyset\}$ ,  $U$  being open in  $E$ . The Effros Borel space  $\mathcal{V}(E)$  is standard, i.e., it is Borel isomorphic to the unit interval  $I$ ; cf. [Chr], [Ke].

Recall that a set in a finite product  $\mathcal{V}(E)^m$  is analytic if it is the projection of a Borel set in  $\mathcal{V}(E)^m \times I$ .

Bossard [Bo] investigated descriptive set-theoretic properties of the isomorphism relation on  $\mathcal{V}(E)$  (we call Banach spaces  $X$  and  $Y$  isomorphic if  $X$  is linearly homeomorphic to  $Y$ ; cf. [L-T]). Bossard proved that the isomorphism relation is analytic in  $\mathcal{V}(E) \times \mathcal{V}(E)$  and, if  $E$  is universal, the relation is not Borel and admits no analytic selectors ([Bo], Theorem 2).

The aim of this note is to prove the following.

**1.1. Theorem.** *Let  $\mathcal{A}$  be an analytic set in the Effros Borel space  $\mathcal{V}(E)$  such that each Banach space of continuous functions on a countable compact topological space has an isomorphic copy in  $\mathcal{A}$ . Then  $\mathcal{A}$  contains  $2^{\aleph_0}$  pairwise isomorphic elements which are isomorphic to a Banach space of continuous functions on some countable compact space.*

The result, strengthening Bossard's theorem on selectors, is related to some topics concerning Lusin's constituents. Let us comment on this aspect of the subject. Let  $2^{\mathbb{Q}}$  be the Cantor set of all subsets of the rationals  $\mathbb{Q}$  (identified with the characteristic functions), and let us consider in  $2^{\mathbb{Q}} \times 2^{\mathbb{Q}}$  the relation of having the same order type. The equivalence class of a well-ordered set of type  $\alpha$  is the  $\alpha$ th Lusin

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constituent; cf. [Ku], [Ke]. V. G. Kanovei [Ka], answering an old question of Lusin, demonstrated that no analytic set in  $2^{\mathbb{Q}}$  can hit every Lusin constituent in exactly one point (cf. [CGP] for a simpler proof). Analogous facts concerning the Cantor-Bendixson derivative were discussed in [Ch-P]. Theorem 1.1 discloses a similar phenomenon for the isomorphism relation between Banach spaces with separable dual (note that this collection forms a coanalytic but not Borel set in  $\mathcal{V}(E)$ , provided  $E$  is universal; see [Bo], Corollaire 7, and [Ke], 33.24).

In fact, our proof of Theorem 1.1 given in section 3, will be based on the results about the Cantor-Bendixson derivative. In section 2 we shall clarify some notions and establish some background needed in this proof.

## 2. TERMINOLOGY AND SOME BACKGROUND

Our terminology concerning descriptive set theory follows Kechris [Ke] and Kuratowski [Ku]. For the material related to Banach spaces and ordinals we refer to Semadeni [Se]. Some additional facts about ordinals we shall use can be found in [K-M].

**2.1. Borel spaces and analytic sets.** A Borel (or measurable) space is a pair  $(A, \mathcal{S})$ , where  $A$  is an arbitrary set and  $\mathcal{S} \subset 2^A$  is a  $\sigma$ -algebra of subsets of  $A$ . The elements of  $\mathcal{S}$  are called Borel sets. We usually denote a Borel space  $(A, \mathcal{S})$  just as  $A$ . In this case a  $\sigma$ -algebra  $\mathcal{S}$  is assumed to be already defined or is not important. A Borel space  $(A, \mathcal{S})$  is standard if there exists a separable completely metrizable topology on  $A$ , such that the Borel sets generated by this topology are exactly the elements of  $\mathcal{S}$ . According to this definition every topological space can be regarded as a Borel space and every separable complete metric space can be regarded as a standard Borel space. Every Borel subset of a standard Borel space equipped with a relative Borel structure is a standard Borel space.

A map  $f : A \rightarrow B$  between Borel spaces  $(A, \mathcal{S})$  and  $(B, \mathcal{T})$  is Borel if  $f^{-1}(T) \in \mathcal{S}$  for  $T \in \mathcal{T}$ .

A set  $K \subset B$  is analytic in a standard Borel space  $(B, \mathcal{T})$  if it is the projection of a Borel set in  $(B \times I, \mathcal{T} \otimes \mathcal{B})$ , where  $I$  is a unit interval and  $\mathcal{T} \otimes \mathcal{B}$  is the  $\sigma$ -algebra generated by the rectangles  $T \times B$  with  $T \in \mathcal{T}$  and  $B$  open in  $I$ .

**2.2. The Effros Borel space  $\mathcal{V}(E)$ .** We shall fix some notation and mention a few basic facts concerning the space of closed linear subspaces of a Banach space and its Borel structure.

Let  $E$  be a separable Banach space. We shall denote by  $\mathcal{F}(E)$  the Borel space of nonempty closed subsets of  $E$ . Borel structure of  $\mathcal{F}(E)$  is an Effros structure and is defined as a  $\sigma$ -algebra generated by sets of the form

$$B_U = \{F \in \mathcal{F}(E) : F \cap U \neq \emptyset\},$$

where  $U$  is an open subset of  $E$ . It is well known that  $\mathcal{F}(E)$  is a standard Borel space. In  $\mathcal{F}(E)$  we can consider a subset  $\mathcal{V}(E)$  consisting of closed Banach subspaces of  $E$ .  $\mathcal{V}(E)$  is Borel in  $\mathcal{F}(E)$ ; hence  $\mathcal{V}(E)$  equipped with the relative Borel structure is a standard Borel space. If  $E$  is a Banach algebra with a unit we shall also consider the subset  $\mathcal{H}(E) \subset \mathcal{V}(E)$  consisting of subalgebras with a unit. The set  $\mathcal{H}(E)$  is Borel in  $\mathcal{F}(E)$  and is a standard Borel space (after equipping with the relative Borel structure). Our main area of interest is the Banach algebra  $C(2^{\mathbb{N}})$  (i.e., the space of continuous functions on the Cantor set), so denote  $\mathcal{F} = \mathcal{F}(C(2^{\mathbb{N}}))$ ,  $\mathcal{V} = \mathcal{V}(C(2^{\mathbb{N}}))$  and  $\mathcal{H} = \mathcal{H}(C(2^{\mathbb{N}}))$ .

The Banach space  $C(2^{\mathbb{N}})$  is universal in the class of separable Banach spaces, i.e.,  $C(2^{\mathbb{N}})$  is separable and for every separable Banach space  $E$  there exists a closed subspace in  $C(2^{\mathbb{N}})$  isomorphic to  $E$ . Therefore, the Borel space  $\mathcal{V}(E)$  can be considered as a Borel subspace of  $\mathcal{V}$ .

### 2.3. The Cantor-Bendixson derivative and the Cantor-Bendixson rank.

For any separable metric space  $X$  the hyperspace  $\mathcal{K}(X)$  is the space of compact subsets of  $X$  with the Hausdorff metric; cf. [Ke]. If  $X$  is a separable complete metric space, so is  $\mathcal{K}(X)$ . We shall denote by  $\mathcal{K}$  the hyperspace  $\mathcal{K}([-1, 1]^{\mathbb{N}})$  of the Hilbert cube.

The Cantor-Bendixson derivative of  $K \in \mathcal{K}(X)$  is the set  $K'$  of all accumulation points of  $K$ . If  $\xi$  is an ordinal, we define the  $\xi$ th iterated Cantor-Bendixson derivative by the transfinite induction:  $D^0(K) = K$ ,  $D^\xi(K) = (D^\zeta(K))'$  if  $\xi = \zeta + 1$  and  $D^\xi(K) = \bigcap_{\zeta < \xi} D^\zeta(K)$  if  $\xi$  is a limit ordinal.

The Cantor-Bendixson rank on a hyperspace  $\mathcal{K}(X)$  is a map  $\delta_{CB} : \mathcal{K}(X) \rightarrow \omega_1 \cup \{\infty\}$ , such that  $\delta_{CB}(K)$  is the minimal ordinal  $\xi$  with  $D^{\xi+1}(K) = \emptyset$ , or  $\delta_{CB}(K) = \infty$  if  $D^\xi(K) \neq \emptyset$  for every ordinal  $\xi$ .

We shall denote by  $\mathcal{K}_{<\infty}$  the collection of countable subsets of the Hilbert cube, i.e.,  $\mathcal{K}_{<\infty} = \{K \in \mathcal{K} : \delta_{CB}(K) < \infty\}$ .

**2.4. The ordinals.** We shall denote the minimal infinite ordinal by  $\omega$ , and the minimal uncountable ordinal by  $\omega_1$ . As usual, an ordinal  $\xi$  will be identified with the segment  $\{\alpha : \alpha < \xi\}$ , and the segment will be considered with the order topology. By the Mazurkiewicz theorem, each countable compact space  $K$  is homeomorphic to an ordinal  $\omega^\xi n + 1$ , with  $\xi$  countable and  $n$  natural, and the Cantor-Bendixson rank of  $K$  is then equal to  $\xi$ , while  $D^\xi(K) = n$ ; cf. [Se].

We shall need a theorem of Bassaga and Pelczyński [B-Pe] providing an isomorphical classification of the Banach spaces  $C(\xi + 1)$  of continuous functions on countable ordinals.

**2.4.1. Theorem.** *For any two countable infinite ordinals  $\xi$  and  $\zeta$  Banach spaces  $C(\xi + 1)$  and  $C(\zeta + 1)$  are isomorphic if and only if  $\xi < \zeta^\omega$  and  $\zeta < \xi^\omega$ .*

Moreover, for any infinite  $\xi < \omega_1$  the space  $C(\xi + 1)$  has dual  $l_1$ .

A set of ordinals  $A \subset \omega_1$  is called a club if it is closed and unbounded in  $\omega_1$ . An intersection of countably many clubs is a club.

Let  $\Lambda \subset \omega_1$  be the club of fixed points of the function  $\xi \mapsto \omega^\xi$ , i.e.,  $\Lambda = \{\xi < \omega_1 : \xi = \omega^\xi\}$ ; cf. [K-M]. If  $\lambda \in \Lambda$ ,  $\xi < \lambda$  and  $n \in \mathbb{N}$ , then  $(\omega^\xi n)^\omega \leq (\omega^{\xi+1})^\omega = \omega^{(\xi+1)\omega} \leq \omega^{\omega^{\xi+1}\omega} = \omega^{\omega^{\xi+2}} \leq \omega^{\omega^\lambda} = \omega^\lambda$ . Therefore, by Theorem 2.4.1 we get

**2.4.2. Corollary.** *For any  $\lambda \in \Lambda$ ,  $\xi < \lambda$  and  $n \in \mathbb{N}$  the Banach space  $C(\omega^\lambda + 1)$  is not isomorphic to  $C(\omega^\xi n + 1)$ .*

**2.5. A result concerning the Cantor-Bendixson rank (cf. [Ch-P]).** We shall use the following fact concerning the Cantor-Bendixson rank  $\delta_{CB} : \mathcal{K}(X) \rightarrow \omega_1 \cup \{\infty\}$ ; cf. section 2.3. The fact is a simple corollary of the main result in [Ch-P].

**2.5.1. Theorem.** *For any metrizable compact space  $X$  and any separable completely metrizable space  $T$  let  $\delta : T \times \mathcal{K}(X) \rightarrow \omega_1 \cup \{\infty\}$  be a function defined by*

$$(1) \quad \delta(t, K) = \delta_{CB}(K) \text{ for } t \in T \text{ and } K \in \mathcal{K}(X),$$

where  $\delta_{CB}$  is the Cantor-Bendixson rank on  $\mathcal{K}(X)$ . Let  $A \subset T \times \mathcal{K}(X)$  be an analytic set satisfying  $A \cap \delta^{-1}(\{\xi\}) \neq \emptyset$  for  $\xi$  in some club in  $\omega_1$ . Then, for all  $\xi$  in some club, any  $F_\sigma$ -set containing  $A \cap \delta^{-1}(\{\xi\})$  intersects  $A \cap \delta^{-1}(\{\alpha\})$  for some  $\alpha < \xi$ .

### 3. PROOF OF THEOREM 1.1

Before proving the main theorem let us make a few observations.

First observe that it is enough to prove Theorem 1.1 with  $E = C(2^\mathbb{N})$ . Indeed, let  $E$  be arbitrary and let  $\mathcal{A} \subset E$  satisfy the assumption of Theorem 1.1. We can embed  $E$  into  $C(2^\mathbb{N})$  as a closed subspace. Then  $\mathcal{V}(E)$  is Borel in  $\mathcal{V} = \mathcal{V}(C(2^\mathbb{N}))$  and  $\mathcal{A}$  is analytic in  $\mathcal{V}$ . This reduces our problem to the case  $E = C(2^\mathbb{N})$ .

We shall define a Borel map  $\Phi : \mathcal{H} \rightarrow \mathcal{K}$ , such that for any  $H \in \mathcal{H}$  the algebra  $C(\Phi(H))$  is isometric to  $H$ . To this end we shall apply the Kuratowski–Ryll–Nardzewski theorem (cf. [Ke], Theorem 12.13) which allows one to select for each  $H \in \mathcal{H}$  a collection of elements  $d_n^H \in H$ ,  $n \in \mathbb{N}$ , dense in the unit ball of  $H$  such that each map  $d_n : \mathcal{H} \rightarrow C(2^\mathbb{N})$  is Borel. (Note that the map  $\mathcal{H} \rightarrow \mathcal{F}$  assigning to a closed subalgebra of  $C(2^\mathbb{N})$  its intersection with the closed unit ball in  $C(2^\mathbb{N})$  is Borel.)

The map  $\Phi$  is defined as follows:

$$(2) \quad \Phi(H) = \{(d_1^H(x), d_2^H(x), \dots) : x \in 2^\mathbb{N}\}.$$

Obviously  $\Phi(H)$  is a non-empty closed subset of the Hilbert cube  $[-1, 1]^\mathbb{N}$ , for  $H \in \mathcal{H}$ .

**3.1. Lemma.** *For any  $H \in \mathcal{H}$ , the algebra  $C(\Phi(H))$  is isometrically isomorphic to the algebra  $H$ .*

*Proof.* For any  $h \in H$  and  $y \in \Phi(H)$  let  $\psi(h)(y) = h((d_1^H, d_2^H, \dots)^{-1}(y))$ . It is easy to see that  $\psi : H \rightarrow C(\Phi(H))$  is well defined and it is a norm preserving homomorphism of the algebras. To check that it is surjective observe that  $\psi(H)$  contains constant functions and that  $\psi(H)$  separates the elements of  $\Phi(H)$ , so  $\psi(H) = C(\Phi(H))$ , by the Stone-Weierstrass theorem.  $\square$

**3.2. Remark.** As a matter of fact the space  $\Phi(H)$  is homeomorphic to a quotient space  $2^\mathbb{N} / \simeq$ , where  $x_1 \simeq x_2$  if and only if  $h(x_1) = h(x_2)$  for all  $h \in H$ . Indeed, the map  $\Phi$  is identifying exactly these points of Cantor set  $2^\mathbb{N}$  which are in relation  $\simeq$  and it is easy to verify that a topology of  $\Phi(H)$  is a topology of quotient space.

**3.3. Lemma.** *The map  $\Phi : \mathcal{H} \rightarrow \mathcal{K}$  is Borel.*

*Proof.* The  $\sigma$ -algebra of Borel sets in  $\mathcal{K}$  is generated by the sets  $B_{U_1, \dots, U_k} = \{K \in \mathcal{K} : K \cap (U_1 \times \dots \times U_k \times [-1, 1]^\mathbb{N}) \neq \emptyset\}$ , where  $k \in \mathbb{N}$  and  $U_1, \dots, U_k$  are open in  $[-1, 1]$ . Let us fix a countable dense subset  $M$  of  $2^\mathbb{N}$ . Then

$$\begin{aligned} \Phi^{-1}(B_{U_1, \dots, U_k}) &= \{H \in \mathcal{H} : \exists_{x \in 2^\mathbb{N}} d_1^H(x) \in U_1, \dots, d_k^H(x) \in U_k\} \\ &= \{H \in \mathcal{H} : \exists_{x \in M} d_1^H(x) \in U_1, \dots, d_k^H(x) \in U_k\} \\ &= \bigcup_{x \in M} \{H \in \mathcal{H} : d_1^H(x) \in U_1, \dots, d_k^H(x) \in U_k\} \\ &= \bigcup_{x \in M} \bigcap_{i=1}^k \{H \in \mathcal{H} : d_i^H(x) \in U_i\}. \end{aligned}$$

The sets  $\mathcal{U}_{i,x} = \{H \in \mathcal{H} : d_i^H(x) \in U_i\}$  are Borel because, for any fixed  $x \in M$ ,  $\mathcal{U}_{i,x}$  is the preimage of  $U_i$  under the composition of the Borel map  $H \mapsto d_i^H$  and the evaluation map  $f \mapsto f(x)$ . Therefore,  $\Phi^{-1}(B_{U_1, \dots, U_k})$  is Borel as a countable sum of finite intersections of Borel sets  $\mathcal{U}_{i,x}$ .  $\square$

The Borel space  $\mathcal{V}$  is standard and we can fix a completely metrizable separable topology on  $\mathcal{V}$  such that the Borel structure of  $\mathcal{V}$  is the  $\sigma$ -algebra generated by the open sets. In the sequel, we shall refer to this topology. Let  $\mathcal{R} \subset \mathcal{V} \times \mathcal{V}$  be the collection of pairs of Banach subspaces of  $C(2^{\mathbb{N}})$  that are isomorphic. By the result of Bossard [Bo],  $\mathcal{R}$  is an analytic set.

We shall define a rank  $\delta : \mathcal{V} \times \mathcal{K} \rightarrow \omega_1 \cup \{\infty\}$  by the formula

$$\delta(L, K) = \delta_{CB}(K) \text{ for } L \in \mathcal{V} \text{ and } K \in \mathcal{K},$$

where  $\delta_{CB} : \mathcal{K} \rightarrow \omega_1 \cup \{\infty\}$  is the Cantor-Bendixson rank on the hyperspace  $\mathcal{K}$  of the Hilbert cube.

We can now pass to the proof of Theorem 1.1.

*Proof of Theorem 1.1* (the case  $E = C(2^{\mathbb{N}})$ ). Let  $\mathcal{A}$  be an analytic set in  $\mathcal{V}$  containing isomorphic copies of every separable Banach space of continuous functions on a countable compact topological space and let

$$\widehat{\mathcal{A}} = (\mathcal{A} \times \mathcal{K}) \cap (\text{Id}_{\mathcal{V}} \times \Phi)(\mathcal{R} \cap (\mathcal{V} \times \mathcal{H})),$$

where  $\text{Id}_{\mathcal{V}}$  is the identity on  $\mathcal{V}$  and  $\Phi$  is defined by (2). According to this definition and Lemmas 3.1 and 3.3 set  $\widehat{\mathcal{A}}$  is analytic in  $\mathcal{V} \times \mathcal{K}$ , and it consists of pairs  $(L, K) \in \mathcal{V} \times \mathcal{K}$  such that  $L \in \mathcal{A}$ ,  $L$  is isomorphic to  $C(K)$  and there exists  $H \in \mathcal{H}$  satisfying  $\Phi(H) = K$ .

For any infinite  $\xi \in \omega_1$  set  $\mathcal{A}$  contains an element  $L_{\xi}$  isomorphic to  $C(\omega^{\xi} + 1)$  and there exists an algebra  $H_{\xi} \in \mathcal{H}$ , such that  $\Phi(H_{\xi})$  is homeomorphic to  $\omega^{\xi} + 1$ . To see that define  $H_{\xi}$  as  $\{f \circ h : f \in C(\omega^{\xi} + 1)\}$ , where  $h : 2^{\mathbb{N}} \rightarrow \omega^{\xi} + 1$  is a continuous surjection and apply Remark 3.2. We have  $(L_{\xi}, \Phi(H_{\xi})) \in \widehat{\mathcal{A}}$  and  $\delta(L_{\xi}, \Phi(H_{\xi})) = \xi$ . This implies that for any infinite  $\xi \in \omega_1$  the intersection  $\widehat{\mathcal{A}} \cap \delta^{-1}(\{\xi\})$  is non-empty and we can apply Theorem 2.5.1 with  $\widehat{\mathcal{A}}$  analytic in  $\mathcal{V} \times \mathcal{K}$ . The conclusion of Theorem 2.5.1 states that there exists a club  $\Theta \subset \omega_1$  such that for any  $\xi \in \Theta$  every  $F_{\sigma}$ -set containing  $\widehat{\mathcal{A}} \cap \delta^{-1}(\{\xi\})$  intersects  $\widehat{\mathcal{A}} \cap \delta^{-1}(\{\zeta\})$  for some  $\zeta < \xi$ .

We shall show that if  $0 < \xi \in \Lambda \cap \Theta$  (note that  $\Lambda \cap \Theta$  is a club in  $\omega_1$  and hence is uncountable), then  $\mathcal{A}$  contains uncountably many isomorphic copies of  $C(\omega^{\xi} + 1)$ . Aiming at a contradiction, assume that there exists  $0 < \xi \in \Lambda \cap \Theta$ , such that  $\mathcal{I}_{\xi} = \{L \in \mathcal{A} : L \text{ is isomorphic to } C(\omega^{\xi} + 1)\}$  is countable. For every  $K \in \mathcal{K}$  satisfying  $\delta(K) = \xi$  the set  $K$  is homeomorphic to  $\omega^{\xi}n + 1$  with  $n < \omega$  and due to Theorem 2.4.1  $C(K)$  is isomorphic to  $C(\omega^{\xi} + 1)$ . Hence the  $F_{\sigma}$ -set  $\mathcal{I}_{\xi} \times \mathcal{K} \subset \mathcal{V} \times \mathcal{K}$  contains  $\widehat{\mathcal{A}} \cap \delta^{-1}(\{\xi\})$  and it must intersect  $\widehat{\mathcal{A}} \cap \delta^{-1}(\{\zeta\})$  for some  $\zeta < \xi$ . The last statement means that there exists  $L \in \mathcal{I}_{\xi}$  isomorphic to  $C(\omega^{\zeta}n + 1)$  with  $n \in \mathbb{N}$  and consequently  $C(\omega^{\xi} + 1)$  is isomorphic to  $C(\omega^{\zeta}n + 1)$ . This contradicts Theorem 2.4.2. The above contradiction proves that  $\mathcal{I}_{\xi}$  is uncountable and since it is an analytic set in the standard Borel space  $\mathcal{V}$  it contains  $2^{\aleph_0}$  elements.  $\square$

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