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STRUCTURE OF CONTRACTIBLE LOCALLY C^*-ALGEBRAS

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ABSTRACT. A locally C^*-algebra is contractible iff it is topologically isomorphic to the topological cartesian product of a certain family of full matrix algebras.

0. INTRODUCTION

At the beginning of the 1970's a well-known series of papers by *J. L. Taylor* appeared, viewing his concepts of joint spectrum and multi-operator functional calculus under the light of homology. By the requirements of his theory [13], *J. L. Taylor* developed in [12] important homological methods in the general framework of topological algebras. In the latter paper, among other interesting results, Proposition 5.7 was presented, which loosely speaking, has the following meaning: A contractible Arens-Michael algebra is topologically isomorphic to the direct sum of the topological cartesian product of a certain family of full matrix algebras and of some hypothetical "badly behaving" algebra, which in the commutative case is always zero. Around the 1980's *A. Ya. Helemskii* traced back a gap in the proof of the preceding result, which was filled by himself in 1985 in the case where the algebra under consideration is moreover commutative; for a proof of this result, see [5, Theorem IV. 5.27]. The aforementioned Taylor's result initiates a more general problem of whether an arbitrary contractible Arens-Michael algebra is topologically isomorphic to the topological cartesian product of a certain family of full matrix algebras. This, among other questions, was considered by the school of Helemskii in Moscow, with successful answers in several cases. Nevertheless, the main problem still remains open, even in the normed case (see [5, p. 195, comments after Exercise 5.28]). More precisely, in 1996, positive answers were given by *Y. V. Selivanov* [11] for contractible semiprime Fréchet Arens-Michael algebras having a nice geometrical property, the so-called "approximation property" of *A. Grothendieck* (cf. e.g., [5, Definition II.2.33]). In this respect, see also Proposition 2.1. In a conversation with the author, *A. Ya. Helemskii* conjectured that Taylor's result holds for all contractible locally C^*-algebras. Indeed, this is true, and in this note the details

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of the proof are presented (Theorem 3.3). The new result cannot be taken either from the amended version of Taylor's result by *A. Ya. Helemskii* or from the result of *Y. V. Selivanov*, since an arbitrary locally C^*-algebra is neither commutative nor Fréchet. Take for instance the topological cartesian product of an uncountable family of full matrix algebras (needless to say that at least one of them must be of size > 1). The latter algebra is always contractible. *J. L. Taylor* had noticed in 1972 ([12, p. 181]; for a proof see [11, Lemma 11]) that a topological cartesian product of full matrix algebras is a contractible algebra.

1. Preliminaries

Throughout this paper we deal with complex associative algebras and Hausdorff topological spaces.

An *lmc* (locally m-convex) *algebra* is an algebra A endowed with a family of submultiplicative seminorms, i.e., with a family of seminorms $\{p\}$ satisfying the property

$$p(ab) \leq p(a)p(b), \quad \forall\, a, b \in A.$$

A complete lmc algebra is called *Arens-Michael algebra* [6, Definition I.2.4]. An Arens-Michael algebra A is called *B-complete* resp. *Fréchet*, whenever the underlying locally convex space of A is B-complete resp. Fréchet (see, e.g., [8, p. 183, 9.5]). Note that every Fréchet locally convex space is B-complete.

Now, a *locally convex C^*-algebra* is an involutive algebra A equipped with a family $\{p\}$ of C^*-seminorms; e.g., each seminorm p fulfils the *C^*-property*:

$$p(a^*a) = p(a)^2, \quad \forall\, a \in A.$$

Due to *Z. Sebestyén* [10] each C^*-seminorm p on an involutive algebra A is automatically submultiplicative and $*$-preserving (the latter means that $p(a^*) = p(a)$, $\forall\, a \in A$). Thus, *each locally convex C^*-algebra is an lmc algebra with continuous involution.* A complete locally convex C^*-algebra is called *locally C^*-algebra* (Inoue [7]). For the same concept various authors have used different terminology, such as, e.g., b^*-algebra, LMC^*-algebra, pro-C^*-algebra, etc.; for detailed references see, for instance, [3, p. 321].

In all the preceding cases, the family $\{p\}$ of the seminorms is denoted by Γ and assumed to be *saturated*.

We briefly mention some examples of locally C^*-algebras, which cannot be (normed) C^*-algebras:

(1) The algebra $\mathbb{C}^{\mathbb{N}}$ of all complex sequences with the cartesian product topology.

(2) The algebra $C[0,1]$ of all continuous functions on $[0,1]$ with the topology of uniform convergence on the countable compact subsets of $[0,1]$.

(3) The algebra $L(H)$, H a locally Hilbert space, due to *A. Inoue* [7, Section 5]. A *locally Hilbert space* H is the direct limit of an increasing family, say H_λ, $\lambda \in \Lambda$, of Hilbert spaces, endowed with the direct limit locally convex topology. Roughly speaking, $L(H)$ consists of all continuous linear operators $T : H \to H$, which are direct limits of operators belonging to the C^*-algebras $\mathcal{B}(H_\lambda)$, $\lambda \in \Lambda$, of bounded linear operators on H_λ's. In particular, $L(H)$ is a locally C^*-algebra, with respect to the family of the C^*-seminorms induced by the operator norms of the C^*-algebras $\mathcal{B}(H_\lambda)$, $\lambda \in \Lambda$. Algebras of the type $L(H)$ are very important, since *an arbitrary locally C^*-algebra always sits topologically in some $L(H)$, with H a locally Hilbert space* [7, Theorem 5.1].

If a locally C^*-algebra $(A, \Gamma = \{p\})$ is given, set $N_p \equiv \ker(p)$, $p \in \Gamma$, and denote by A_p the quotient A/N_p, endowed with the C^*-norm $\|\cdot\|_p$ induced by p, i.e.,

$$\|a_p\|_p := p(a), \quad \forall\ a_p \equiv a + N_p \in A/N_p.$$

A_p is automatically complete [1, Theorem 2.4], hence a C^*-algebra. In particular,

$$A = \varprojlim A_p \tag{1.1}$$

up to a topological isomorphism (cf. [6, 7, 9]). By a *topological isomorphism* between two Arens-Michael algebras A, B we understand a continuous algebraic isomorphism from A onto B with continuous inverse. If A, B have the additional structure of an involution *, a *topological $*$-isomorphism* between them is a topological isomorphism φ which preserves involution, i.e., $\varphi(a^*) = \varphi(a)^*$, $\forall\ a \in A$.

Now let $(A, \Gamma = \{p\})$ be an lmc algebra. A topological *left A-module* is a complete locally convex space X, such that X is a left A-module algebraically and the operation

$$m : A \times X \to X : (a, x) \mapsto ax$$

is jointly continuous, when $A \times X$ carries the cartesian product topology (cf., for instance, [5, Definition 0.3.9]). In the sequel, by *A-module* we shall always mean a topological left A-module in the preceding sense.

We recall that an lmc algebra $(A, \Gamma = \{p\})$ is said to be *contractible* iff A is biprojective and unital, that is A is unital and as an A-bimodule is projective (cf. [6, Assertion VII.1.72 and Definition VII.1.59]). Equivalently, we can say that A is contractible whenever all of its cohomological groups of positive dimension are trivial with all coefficients (see, e.g., [5, p. 406]). It is easily seen that a full matrix algebra, say M_n ($n \times n$ complex matrices), is a contractible C^*-algebra.

2. Some remarks on contractibility

In the case of a *commutative Banach algebra* and/or of a *C^*-algebra*, say A, *A. Ya. Helemskii* (1970) resp. *Y. V. Selivanov* (1978) characterized contractibility of A by any of the following statements, which in these particular cases are moreover all equivalent (cf. [6, p. 362]):

(1) *The projectivity of all A-modules.*

(2) *The projectivity of all* (algebraically) *irreducible A-modules.*

(3) *The representation of A* (up to a topological $(*)$-isomorphism) *by a finite cartesian product of full matrix algebras.*

From (1), contractibility of A accounts for the fact that A has *global dimension* zero, which in symbols is written by $dgA = 0$. From (3), contractibility of A is closely related to the "classical semisimplicity" (i.e., to Artin-Wedderburn theorem).

In general, it is known that *if A is a Banach algebra, then*:

$A \cong \prod_{\text{finite}} M_n \Rightarrow$ *A is contractible* $\Rightarrow dgA = 0 \Rightarrow$ *A is unital and all the irreducible A-modules are projective.*

Whether the inverse implications are valid is still an open question (ibid.).

On the other hand, if A is a C^*-algebra, then it is semisimple, so being additionally finite dimensional, it becomes classically semisimple, therefore topologically $*$-isomorphic to the direct sum of a finite number of full matrix algebras (see, for instance, [5, p. 27, 1.4.1^o]). Thus one has the next [5, p. 20 and p. 186, Corollary 4.12].

Proposition 2.1 (Y. V. Selivanov)**.** *Let A be a C^*-algebra. The following are equivalent:*

(1) *A is finite dimensional.*
(2) *A is contractible.*
(3) *A is classically semisimple.*
(4) *$dgA = 0$.*

From the above it now follows that:

Corollary 2.2. *A C^*-algebra is contractible iff it is topologically $*$-isomorphic to the direct sum of a finite number of full matrix algebras.*

As an illustration now let H be an infinite dimensional Hilbert space and $\mathcal{B}(H)$ the C^*-algebra of all bounded linear operators on H. It is evident from Corollary 2.2 that $\mathcal{B}(H)$ cannot be contractible. But clearly, $\mathcal{B}(H)$ is contractible iff H is finite dimensional.

3. Main result

Let $(A, \Gamma = \{p\})$ be an Arens-Michael algebra and X an A-module. Let $A\hat{\otimes}X$ be the completed projective tensor product of A, X. Then one very useful characterization of projectivity for X is given by claiming that the canonical (well defined) projection

$$\pi : A\hat{\otimes}X \to X : a \otimes x \mapsto ax \tag{3.1}$$

is a retraction, i.e., π has a right inverse morphism of A-modules, say ρ [6, Theorem VII.1.11].

Lemma 3.1. *Let $(A, \Gamma = \{p\})$ be a locally C^*-algebra for which all A-modules are projective. Then each A_p-module is projective, $\forall\ p \in \Gamma$.*

Proof. Let $p \in \Gamma$ and let X be an A_p-module. Then X is made into an A-module by defining

$$m : A \times X \to X : (a, x) \mapsto a_p x, \quad a_p \equiv a + N_p.$$

Since X is projective there is a right inverse, say ρ, for the canonical projection π (see (3.1)). If $\tau_p : A \to A_p : a \mapsto a_p$ is the natural $*$-morphism of A onto A_p, consider the following diagram

$$\begin{tikzcd}
A\hat{\otimes}X \arrow[r, shift left, "\pi"] \arrow[d, "\tau_p\otimes \mathrm{id}_X"'] & X \arrow[l, shift left, dashed, "\rho"] \arrow[dl, shift left, dashed, "\rho_p"] \\
A_p\hat{\otimes}X \arrow[ur, shift left, "\pi_p"] &
\end{tikzcd}$$

where π_p is the corresponding canonical map for $A_p\hat{\otimes}X$ and $\rho_p := (\tau_p \otimes id_X) \circ \rho$. Taking into account that $\pi_p \circ (\tau_p \otimes id_X) = \pi$, it follows easily that ρ_p is a right inverse morphism of A-modules for π_p. $\square$

Corollary 3.2. *Let $(A, \Gamma = \{p\})$ be a contractible locally C^*-algebra. Then each C^*-algebra A_p, $p \in \Gamma$, is contractible.*

Theorem 3.3. *Let $(A, \Gamma = \{p\})$ be a locally C^*-algebra. The following are equivalent:*

(1) *A is contractible.*

(2) *$dgA = 0$.*

(3) *A is topologically $*$–isomorphic to the topological cartesian product of a certain family of full matrix algebras.*

Proof. (1) $\Rightarrow$ (2) It suffices to show that each A-module X is projective. But by (1) A as an A-bimodule is projective. This implies that the A-module $A\hat{\otimes}_A X$ (tensor product of A-modules [6, pp. 330-335]) is projective (adopt the proof of the corresponding Banach algebra result [6, Proposition VII.1.57]). Since X is topologically isomorphic to $A\hat{\otimes}_A X$, projectivity of X follows.

(2) $\Rightarrow$ (3) By Lemma 3.1 and Proposition 2.1 we conclude that each C^*-algebra A_p is contractible, therefore (Corollary 2.2) topologically $*$–isomorphic to the direct sum of a finite number of full matrix algebras, i.e.,

$$A_p = \bigoplus_{\nu \in F_p} M_{n_\nu}, \quad p \in \Gamma, \tag{3.2}$$

where $M_{n_\nu} \equiv M_{n_\nu}(\mathbb{C})$ and F_p is a finite index set. We now proceed to specify the index set F_p by means of the structure space $Prim(A_p)$ of the C^*-algebra A_p, consisting of the kernels of all non-zero irreducible $*$–representations of A_p, $p \in \Gamma$. Since A_p is of the form indicated above, each of its non-zero irreducible $*$-representations is equivalent to one of the projections

$$\bigoplus_{\nu \in F_p} M_{n_\nu} \underset{\sigma_\nu}{\longrightarrow} M_{n_\nu} \cong \mathcal{B}(H_{n_\nu}), \quad H_{n_\nu} \equiv \mathbb{C}^{n_\nu}.$$

Therefore, $Prim(A_p)$ consists of exactly ν elements, which are the kernels of the preceding projections σ_ν, $\nu \in F_p$. So there is a (well-defined) correspondence

$$F_p \to Prim(A_p) : \nu \mapsto \ker(\sigma_\nu),$$

which is clearly a bijection. Hence the finite set F_p is identified with the structure space $Prim(A_p)$ of A_p, $p \in \Gamma$. We now show that

$$p \le q \ \text{ in } \ \Gamma \ \text{ implies some natural injection of } \ F_p \ \text{ into } \ F_q.$$

Let $I_p \equiv \ker(\pi_p) \in Prim(A_p)$, $p \in \Gamma$, with π_p a non-zero irreducible $*$–representation of A_p. Let $p \le q$ in Γ and let

$$\tau_{pq} : A_q \to A_p : a_q \mapsto a_p$$

be the corresponding connecting continuous $*$–morphism between A_q, A_p. Consider the diagram

$$\begin{tikzcd} A_q \arrow[r, "\tau_{pq}"] \arrow[dr, "\pi_q := \pi_p \circ \tau_{pq}"'] & A_p \arrow[d, "\pi_p"] \\ & \mathcal{B}(H_p) \end{tikzcd}$$

where H_p is the Hilbert space on which π_p acts. Clearly, π_q is a non-zero irreducible $*$-representation of A_q and the map

$$Prim(A_p) \to Prim(A_q) : I_p \mapsto I_q \equiv \ker(\pi_q)$$

is a well-defined injection. Therefore,

$$F_p \cong Prim(A_p) \underset{\to}{\subseteq} Prim(A_q) \cong F_q. \tag{3.3}$$

Thus we may consider the preceding increasing, up to set-theoretical isomorphisms, family of the sets F_p, $p \in \Gamma$, and take their corresponding direct limit, say Λ, i.e.,

$$\Lambda := \varinjlim F_p = \bigcup_{p \in \Gamma} F_p.$$

The set Λ will serve as the index set of the topological cartesian product of the full matrix algebras we need. We shall prove that

$$A = \prod_{\lambda \in \Lambda} M_{n_\lambda},$$

with respect to a topological $*$–isomorphism. We already have the following topological $*$–identifications (see (1.1) and (3.2))

$$A = \varprojlim A_p = \varprojlim \left(\bigoplus_{\nu \in F_p} M_{n_\nu} \right).$$

We must show that

$$\varprojlim \left(\bigoplus_{\nu \in F_p} M_{n_\nu} \right) = \prod_{\lambda \in \Lambda} M_{n_\lambda}, \tag{3.4}$$

up to a topological $*$-isomorphism.

Let $\mathcal{F}$ be the family of all finite subsets of Λ, directed by inclusion. An easy modification of a general result on topological spaces, which can be found, for instance, in [9, Lemma XII.2.1], shows that

$$\varprojlim \left(\bigoplus_{\lambda \in F} M_{n_\lambda} \right) = \prod_{\lambda \in \Lambda} M_{n_\lambda}, \quad F \in \mathcal{F}, \tag{3.5}$$

up to a topological $*$-isomorphism. But the family $\{F_p, p \in \Gamma\}$ is a cofinal subset of $\mathcal{F}$. Indeed: Let $F \in \mathcal{F}$. Then $F = \{\nu_1, \dots, \nu_k\}$ with $\nu_i \in F_{p_i}$, $i = 1, \dots, k$. Since Γ is saturated, there exists $p \in \Gamma$ such that $p_i \leq p$, $\forall\ i = 1, \dots, k$; therefore (3.3) implies that $F \subseteq F_p$. Consequently (see also [2, p. 80, Proposition 3]),

$$\varprojlim \left(\bigoplus_{\lambda \in F} M_{n_\lambda} \right) = \varprojlim \left(\bigoplus_{\nu \in F_p} M_{n_\nu} \right), \tag{3.6}$$

with respect to a topological $*$-isomorphism. So (3.4) follows from (3.5) and (3.6).

(3) $\Rightarrow$ (1) This implication is derived from [11, Lemma 11]. $\square$

Let $(A, \Gamma = \{p\})$ be a locally C^*-algebra and I a closed 2-sided ideal of A. Then I is a self-adjoint ideal and the quotient algebra A/I, endowed with the quotient C^*-seminorms induced by the C^*-seminorms $p \in \Gamma$, is a locally convex C^*-algebra [7, Theorem 2.7]. In the case where A is either B-complete or Fréchet, then A/I is also complete [8, p. 186, Theorem 1]; therefore A/I becomes a B-complete or Fréchet locally C^*-algebra. In this respect, we have the following.

Corollary 3.4. *Let $(A, \Gamma = \{p\})$ be a contractible B-complete or a contractible Fréchet locally C^*-algebra and I a closed 2-sided ideal of A. Then the quotient algebra A/I is represented, up to a topological $*$–isomorphism, as an arbitrary or resp. at most a countable topological cartesian product of full matrix algebras.*

Proof. As we noticed before A/I is a B-complete or Fréchet locally C^*-algebra according to our hypothesis for A. On the other hand, A/I is contractible since A has this property (cf. [6, Proposition VII.1.71 and comments in p. 361, before §1.5]); so the assertion follows from Theorem 3.3. □

Now let $(A, \Gamma = \{p\})$, $(B, \Gamma' = \{q\})$ be two locally C^*-algebras and α, β the *projective resp. injective tensorial locally C^*-topology* on $A \otimes B$, as they have been defined in [3, p. 322] and/or [4, p. 27]. Denote by $A\hat{\otimes}_\alpha B$, $A\hat{\otimes}_\beta B$ the locally C^*-algebras resulting by completing the tensor product of A, B under α, β resp. Then (cf. [4, Corollary 4.11])

$$A\underset{\alpha}{\hat{\otimes}}B = \varprojlim A_p \underset{\min}{\otimes} B_q, \quad A\underset{\beta}{\hat{\otimes}}B = \varprojlim A_p \underset{\max}{\otimes} B_q, \tag{3.7}$$

up to topological $*$-isomorphisms, where $A_p \otimes_{\min} B_q$, $A_p \otimes_{\max} B_q$ denote the completed tensor product of the C^*-algebras A_p, B_q under the minimum resp. maximum C^*-cross-norms $\|\cdot\|^{pq}_{\min}$, $\|\cdot\|^{pq}_{\max}$. In this regard, we have the following.

Corollary 3.5. *Let $(A, \Gamma = \{p\})$, $(B, \Gamma' = \{q\})$ be two contractible locally C^*-algebras. Then $A\hat{\otimes}_\alpha B$, $A\hat{\otimes}_\beta B$ are contractible locally C^*-algebras that coincide up to a topological $*$–isomorphism.*

Proof. From Corollaries 2.2, 3.2 and (3.7) we have that

$$\begin{aligned} A\underset{\alpha}{\hat{\otimes}}B &= \varprojlim \left(A_p \underset{\min}{\otimes} B_q \right) \\ &= \varprojlim \left(\left(\underset{\nu \in F_p}{\oplus} M_{n_\nu} \right) \underset{\min}{\otimes} \left(\underset{\mu \in F_q}{\oplus} M_{n_\mu} \right) \right) \\ &= \varprojlim \left(\underset{(\nu,\mu) \in F_p \times F_q}{\oplus} M_{n_\nu n_\mu} \right), \end{aligned} \tag{3.8}$$

with respect to topological $*$-isomorphisms, where the finite sets F_p, $p \in \Gamma$, F_q, $q \in \Gamma'$, as in the proof of Theorem 3.3, are specified by the structure spaces $Prim(A_p)$, $Prim(B_q)$ of the C^*-algebras A_p, B_q resp. Thus the finite set $F_p \times F_q$ is now identified (up to a set-theoretical isomorphism) with the structure space $Prim(A_p \otimes_{\min} B_q)$ of the C^*-algebra $A_p \otimes_{\min} B_q$. On the other hand, for $p \leq r$ in Γ and $q \leq s$ in Γ' one clearly gets

$$\|a \otimes b\|^{pq}_{\min} = p(a)q(b) \leq r(a)s(b) = \|a \otimes b\|^{rs}_{\min}, \quad \forall\ a \in A, \quad b \in B;$$

hence (cf. (3.3))

$$F_p \times F_q \cong Prim(A_p \underset{\min}{\otimes} B_q) \underset{\rightarrow}{\subset} Prim(A_r \underset{\min}{\otimes} B_s) \cong F_r \times F_s.$$

Put $F_{pq} \equiv F_p \times F_q$, $(p, q) \in \Gamma \times \Gamma'$. As in the proof of Theorem 3.3, we may define

$$\Lambda := \varinjlim F_{pq} = \bigcup_{(p,q) \in \Gamma \times \Gamma'} F_{pq}.$$

Since Γ, Γ' are saturated families of C^*-seminorms, the same will be true for the family of the C^*-seminorms defining α. So $\{F_{pq}, (p, q) \in \Gamma \times \Gamma'\}$ will be cofinal in the family of all finite subsets of Λ. Arguing again as in the proof of Theorem 3.3, we conclude that (see (3.8))

$$\begin{aligned} A\underset{\alpha}{\hat{\otimes}}B &= \varprojlim \left(\bigoplus_{\nu \in F_{pq}} M_{n_\nu} \right) \\ &= \varprojlim \left(\bigoplus_{\lambda \in \Lambda} M_{n_\lambda} \right) \\ &= \prod_{\lambda \in \Lambda} M_{n_\lambda}, \end{aligned}$$

up to topological $*$-isomorphisms. Consequently $A\hat{\otimes}_\alpha B$ is contractible as the topological cartesian product of a family of full matrix algebras (see [12, p. 181] and/or [11, Lemma 11]). Since A, B are contractible, it follows from (3.7) (see also Corollary 3.2 and Proposition 2.1) that $A\hat{\otimes}_\alpha B$, $A\hat{\otimes}_\beta B$ are topologically $*$-isomorphic. $\square$

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